A bout seven years ago, the mathematics teachers at my secondary school came to the conclusion that they were not satisfied with our rather traditional geometry textbook. I had already begun using a problem-based approach to teaching geometry in my classes, a transition for me and my students that inspired me to write about the differences in the methodology and classroom practice (Schettino 2003).

This transition led me on a journey toward researching and learning about problem-based learning (PBL) in mathematics education at the secondary school level. My work began to intrigue my colleagues. When it came time to change our geometry textbook, instead of looking for a new textbook for the course, we decided to create our own. We followed the lead of the original writers of many of the problems I had been using (the faculty at Phillips Exeter Academy), making sure that the curriculum met the needs of the students at our independent all-girls school. Our student body has a range of educational preparation and ability, and we teach the geometry course in classes that are not “tracked.” This decision led us to create our own course; it helped inspire many of us to think about teaching and learning in new ways and launched us into the new world of Problem-Based Learning (PBL).

WHAT IS PROBLEM-BASED LEARNING?

In reading about PBL, we realized that there was not one universally accepted definition for what we wanted to do. We wanted a curriculum that empowered students and incorporated connections with prior mathematical knowledge; we wanted problems that spiraled and that would provide built-in reinforcement situated within posed problems. We also wanted a curriculum that made use of the twenty-first-century skills of communication, collaboration, and technology literacy (Partnership for Twenty-first Century Skills 2007). In addition, PBL fosters many of the skills needed for the Standards for Mathematical Practice that the Common Core State Standards call for, such as perseverance in problem solving and developing abstract and quantitative reasoning skills (Common Core State Standards Initiative 2010). Further, PBL provides for diversity in learning and potential for using more alternative pedagogical approaches in the future. It stresses the value of discourse and allows us to consider problems from multiple perspectives. Without a textbook that might “specify the appropriate tool to be used for the given problem,” we make room for the “crucial ethical moment of reflecting on whether the means suit the ends” (de Freitas 2008).

I found many studies showing that PBL allowed students to attain equal or greater achievement on standardized testing as students taught by direct instruction (Savery 2006) and to do better on problem-solving and long-term knowledge retention (Strobel and van Barneveld 2009). One study even showed that PBL is more effective with lower-ability students (Ridlon 2009).

In general, problem-based learning is defined as “an instructional (and curricular) learner-centered approach that empowers learners to … integrate theory and practice and apply knowledge and skills to develop a viable solution to a defined problem” facilitated by a teacher who “guides the learning process and conducts a thorough debriefing at the conclusion of the learning experience” (Savery 2006).

However, I define problem-based learning as follows: an instructional approach of curriculum and pedagogy where student learning and content material is constructed (and co-constructed) through the use, facilitation, and experience of contextual problems in a decompartmentalized, threaded topic format in a discussion-based classroom setting where student voice, experience, and prior knowledge are valued.
Our curriculum begins with multiple topics running in parallel, and these threads introduce new concepts through scaffolded problems. Because some problems are theoretical in nature (e.g., connecting the Pythagorean theorem and the distance formula), not all problems are situated in real-life contexts, so our definition differs from a more project-based curriculum.

The problems are designed so that homework assignments motivate class discussion for the next day. Students must plan deliberately, choose specific problems to lead into future problems sequentially. Each homework assignment consists of seven or eight problems—perhaps fewer early in the year. This new type of “homework,” where full, correct answers are not always expected, serves many purposes:

- It provides an opportunity for students to review material from past courses.
- It triggers prior knowledge for an upcoming problem.
- It inspires construction of new knowledge.
- It introduces new terminology.
- It allows students to practice a new skill. It challenges the more able students (differentiated instruction).
- It enables students to see the same new idea represented differently.

PBL homework is very different from homework in a traditional, direct-instruction class. For all the reasons listed above, it is important for students to form a different understanding of what the purpose of homework is. Teachers must send the message that they value risk taking and intuition even when the answer is not known.

For example, students assume that the pizza has ten slices. When pressed for a reason, they may respond that they divided 360 by 36 without further clarification. If asked to consider a central angle of 24°, the answer 15 might not come as quickly. Students need to hear others’ ideas to articulate the concept that the number of degrees in the central angle is a factor of 360. When another student explicitly states that 36 physically fits into 360 ten times, the concept is made more visual.

With PBL, a student’s solution often includes either a physical or a virtual model of a pizza. Because this question was posed in a problem, the students are more visually driven. The teacher facilitates open and honest student discussion to the formula for the area of a circle: $\text{Area} = \pi r^2$. The students must have the freedom to explore their ideas, as even students who are not strong in working with fractional parts can have the freedom to explore.

An upcoming topic in the curriculum extends the central angle idea to a proportional algorithm for arc length. Students need to see the central angle idea to a proportional algorithm for arc length and see overarching relationships. Another way to see PBL in action is through a discussion about finding the area of parallelograms before students were given the explicit formula. One student said, “Well, all we were looking at was how the sides related to each other. There was a 27° angle and a 90° angle, so she just made a similar triangle like in that problem we did yesterday.”

This student was making the connections between similar triangles and the sine function. A PBL curriculum enables students to make such connections and see overarching relationships.

Another way to see PBL in action is through collaboration. In a conversation about finding the area of parallelograms before students were given an explicit formula, one student was reluctantly presenting her solution to a problem. The question was to find the area of parallelogram ABCD given its vertices, as shown in figure 1.

**Teacher** [addressing student who is doing presentation] OK, Stephanie, so what did you do?

**Stephanie** OK, uh, I thought that because, like, a quadrilateral would be, like, have twice height, so I just took the length, and I got the distance from A to B and then the distance from A to D, and then I times them together…

**Annie** I definitely did not do that…

**Teacher** We have a student who is doing it differently. As shown in figure 1.

![Figure 1](image.png)

PBL IN ACTION

An advantage of this type of curriculum is that students have opportunities to make connections themselves. At one point in the curriculum, multiple threads occur simultaneously—dilations as transformations, a review of ratios and proportions, similar triangles, and an introduction to trigonometry.

In a problem introducing the sine ratio, students are asked to construct, using a ruler and a protractor, a right triangle with a hypotenuse of 15 cm and an angle of 27°. They are then asked to measure the side opposite the constructed angle as accurately as possible and find what percentage of the hypotenuse that side is. During the class discussion, students compared their answers and found that they were all around the same—around 45%. This result allowed me to introduce the sine function.

One student commented that she had done the problem incorrectly because she had misread the directions and had calculated the hypotenuse of her triangle 10 cm instead of 15 cm. “Why did I still get the right answer?” she asked. I turned that question to the class for discussion. It was at that time, one student offered, “Well, all we were looking at was how the sides related to each other. There was a 27° angle and a 90° angle, so she just made a similar triangle like in that problem we did yesterday.”

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In this discussion, Stephanie shares her (incorrect) conjecture about how to find the area of the parallelogram and has heard the responses of some of her classmates. Their responses prompt her to question her method. She thinks about what was done in a previous problem where students calculated area by finding an altitude algebraically. After receiving the feedback, Stephanie corrects her mistake and rethinks the problem. Although Annie and Leah think that Stephanie’s idea of how to find the area of the parallelogram was incorrect, the teacher tries to allow them to decide which method was correct by comparing Stephanie’s with one that Annie had used in a previous problem. By comparing different methods side-by-side and having students present their ideas to the class, the teacher can allow students’ prior knowledge and conjecture to reach a conclusion about the correct way to find the area of the parallelogram.

After Stephanie finished, another student, Becca, wondered whether her own approach was correct. She proceeded to show her diagram (see fig. 2). Becca described her approach: She drew a rectangle around the parallelogram, found the area of the four right triangles at the corners, and subtracted the four triangular areas from the large rectangle, resulting in the correct area.

Becca’s geometric approach for finding the area was very different from Stephanie’s algebraic approach, which entailed writing equations of altitudes and using the distance formula to find the necessary lengths. However, at the end of the discussion of this problem, all the students had an opinion about which method they appreciated more. They had been engaged and interested in seeing which would work better for them. The PBL curriculum gives students the opportunity to create their own solution methods and the freedom to express their own ideas; it also shows them the value of risk taking and efficiency in problem solving. However, it is the teacher’s responsibility to summarize the methods and lay out the advantages and disadvantages of each so that students can feel empowered to make those problem-solving decisions for themselves.

MOVING FORWARD
Our anecdotal research has found that this type of curriculum has met these goals. Reflecting on the effects of this curricular change, one student said: You can see people … asking you a question wherever you are in mathematics. … You can see that everyone here wants to figure out how they got a certain problem. There’s more of an interest than just getting something right, they want to understand how.

We still need further evidence to support achievement and retention of material for sequential courses. The benefits to student learning in mathematics that we have observed—indeed in problem solving, improved communication, empowerment in student voice, and agency in learning—far outweigh the traditional concerns that many teachers feared. This is not to say that there were no obstacles [see Schettino [2003] for more on obstacles in transitioning]. However, teachers working together with open minds, communicating in professional development settings, and visiting classrooms have allowed this mathematics department to find a balance in this transition. As a result, we have met our goals for teaching geometry that are consistent with both the NCTM Standards, the Common Core State Standards, and our hopes for the future of our students as independent problem solvers.

ACKNOWLEDGMENTS
The author is grateful to her former colleagues at Phillips Exeter Academy and at Emma Willard School for inspiring her work with the problem-based learning curriculum and supporting her exploration and research.

Laura: Yeah.
Stephanie: But I’m not sure if you needed to get the altitude?
Meghan: Oh, that’s what I did …

Teacher: How is that different from what Annie just said in number 9? What’s the difference between just multiplying AD and AB together …— the sides together—and multiplying base times height?

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