Carmel Schettino

Transition to a Problem-Solving Curriculum

"How are we supposed to learn anything if you don't tell us what we need to know?" My new students sat there staring at me, possibly considering whether I was insane. They were about to embark on an interesting and challenging adventure with me as I attempted to expose them to a problem-solving curriculum (PSC) for the first time. "What do you mean these problems aren't from the book?" and "You can't think I can do any math problems without having a teacher explain them to me first" were statements that I often heard during those first few weeks. I was asking students who were accustomed to a traditionally run mathematics classroom to think creatively and independently; and some of them were being asked to do so for the first time. My goal is to share this experience, which has been enlightening, educational, and at times sobering. However, I have been encouraged and pleased that, for my students, a transition to a PSC is under way.

CONSIDERING A PROBLEM-SOLVING CURRICULUM

"How are we supposed to learn anything if you don't tell us what we need to know?"

There has been much talk in recent years about the benefits of a PSC. Magdalene Lampert's method, as discussed in *Teaching Problems and the Problems of Teaching* (Lampert 2001), offers such an approach. While I read this book, I saw her grappling at the elementary school level with the same issues that I encountered in my high school classes. I also greatly appreciated her comment that in writing her large case study, she sought "to contribute to a conversation about the nature of the work that school teachers do" and not to "argue in favor of a particular approach to teaching" (Lampert 2001, p. 7) I wish to offer my experience in the same spirit.

What does having a PSC mean? In my view, it entails teaching students that they have the freedom to solve problems with a set of given tools and knowledgeable guidance and that the goal is to further develop their mathematical tool kit. It means having problems that lead students in the right direction for discussion and practice of new topics discovered.

To commit to this method, an educator must first commit to the premise that helping students develop their ability to solve problems independently is the major goal of mathematics education. This premise comes first, before any curricular or content priorities; otherwise, the method conflicts in too many ways with time, curriculum, or dictated district standards. If we can assume that independent problem-solving skills are truly important, we can clearly understand what is necessary to attain that goal. In fact, we can more easily justify to students why they are studying mathematics in the first place. The question "when will we ever use this?" becomes answerable in a life-skills way. When daily topics in mathematics are introduced as problems to solve, students no longer believe that we are spoon-feeding or talking at them. Their ideas that are relevant to solving problems begin to count as part of the formation of their knowledge and their deeper understanding of the topics. Formulas no longer act as abstract lists of signs and symbols that are waiting for numbers to be plugged in but are the outcomes of students' individual and collaborative work. The ownership linked to these ideas creates a confidence and understanding in students that is contagious and invigorating.

INITIATING A PSC

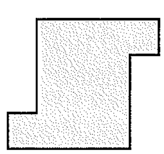
In fall 2001, I began teaching at a school that did not traditionally use a PSC in its mathematics classes. I had come from a school where a PSC was the teaching method of choice for the entire mathematics department. I knew that my decision to continue to use a PSC would be a challenge for me, but as an educator I needed to be true to myself and to my own beliefs about teaching mathematics. One of my first challenges in beginning the transition for my new students was to find a way to incorporate the problems from the PSC at my old school without making the experience tremendously uncomfortable for my students. I believed that they needed to transition at a comfortable pace to this new way of learning.

Carmel Schettino, cschetti@emmawillard.org, teaches at Emma Willard School in Troy, NY 12180, where she is the chair of the mathematics division. She is interested in studying and developing problem-solving curricula and programs for high school mathematics education.

I decided that for the first year, eliminating the textbook altogether was probably too radical. As a compromise, students had a textbook to use as a resource when they believed that using it was necessary, but the main focus of the curriculum came from what I called *motivational problems*. I had been using the PSC problems developed by the faculty at Phillips Exeter Academy. These problems explore topics concurrently, instead of using the typical textbook approach, which has chapters outlined by topic. As part of each homework assignment, the PSC problems served as a source to stimulate class discussions the following day. These assignments encouraged students to grapple with new ideas or to practice concepts that we had discussed in class. Although I strongly believe that learning topics concurrently is an important part of a PSC, the change would be a large one for students transitioning to this way of learning. I followed the basic outline of the course, but I gave my motivational problems instead; some were from the Phillips Exeter Academy curriculum, and some were my own problems.

A typical homework assignment consisted of five to seven problems from the motivational problems sheets or the problem book from the PEA curriculum. The purpose of these problems varied. Some obviously introduced new ideas in the curriculum, some tested whether students could apply ideas learned, and some were practice. The students' assignment was to come to class the following day prepared to discuss their own ideas and the directions in which those ideas led them. I required students to provide some evidence in writing that they had made an attempt to work on every problem. This requirement allowed for the fact that a problem's goal might only have been to start students thinking about a topic. For example, early in the year in a ninth- and tenth-grade geometry class, before the class discussed transformations or symmetry, I assigned the following problem:

> Figures that have exactly the same shape and size are called *congruent*. Dissect the region shown below into two congruent parts. How many different ways of doing this can you find? (Phillips Exeter Academy)



This problem introduced students to the term *congruent,* and they started investigating ways to dissect the region. The next day, many students had drawn lines that partitioned the figure into two congruent parts. Discussion followed:

Ms. S. How many lines did you find that would create congruent parts of this figure?

Susan. I found five lines. (*Susan then went to the board to draw the picture and share with the class the five lines she found that worked; Alex went to the board and added a few more lines.*)

Joyce. All these lines seem to intersect at like a "center" point in this figure.

Susan. That means there are 360 of them.

(*This statement caused much confusion and dissent among the students in the class, and Susan clarified how she arrived at her conclusion—she knew that 360 degrees are in a full circle.*)

Annie. But there are an infinite number of positions that the line can be in, not just 360.

Ms. S. (*after Annie clarified her statement*) What do you think is the importance of that "center" point? (*After some discussion, the students concluded that all lines that create congruent parts go through the same point.*)

Ms. S. (*following up on their group decision*) How can we say that there are an infinite number of lines that will create congruent regions?

What does it mean to have a PSC?

This conversation finished with the students discussing the rotation that occurs around that center point justifying any line through the center, thereby introducing to the class rotation as an isometry. I made sure that this idea was summarized.

That night's homework assignment included some problems in which students practiced the new idea, rotation as an isometry, that they discovered that day in class, as well as two or three more problems to spur the class discussion the next day. Such "homework" is more than practice. It provides important time for reflecting on concepts learned, as well as experience in independent problem solving while students struggle with new concepts in preparation for discussion the following day.

OVERCOMING THE HURDLES OF A PSC

In my experience transitioning to a PSC during the past eight years, I have encountered many hurdles. One of those hurdles is that the choice of assessments must be appropriate to evaluate progress toward independent problem solving. I found myself rewriting tests to make questions more "authentic" (Wiggins 1990), so that I could assess what I was valuing in my classroom. I base test questions on ideas discussed in class, but they are not mere repetitions of homework. Since I place high value on the problem-solving process, I give less credit for a lone correct answer than for the ideas, concepts, and work shown as a valid problem-solving method.

I value students' original thoughts, so I began having them reflect on their problem-solving processes in journals. These reflections also add opportunities for improving written descriptions of their problem solving. Oral assessment has become an important daily interaction between the students and me. To allow for this assessment opportunity, I have restructured class time so that students have regular opportunities to speak and write their ideas, and I give them constructive feedback and interactive guidance.

Another large obstacle that I encountered was my own comfort level with all aspects of mathematics. I had to be ready for the possibility that a student would come up with an idea that I had not considered and for which I might not be able to give valid feedback. I needed to be able to admit when I did not know the answer to a question. I am now very good at doing so. I became exceptionally resourceful and became a better problem solver. Inherent in my understanding of a PSC is the knowledge that students are constantly using the tools that they have and discovering new ones. If the teacher is not secure in all levels of mathematics, the process of questioning and answering becomes extremely difficult and even misleading for students. The teacher's job includes helping students identify productive ideas and set aside other ideas.

Students needed to transition to this new way of learning at a comfortable pace

I have also had to become very comfortable with the risk-taking skills needed for good problem solving and to model that behavior in class discussion. I had to clearly indicate to my students that I value the importance of a wrong answer, as well as students' ability to recognize errors on their own. One of my favorite teaching methods is using an incorrect method as a learning tool. A student who has made an error in front of his or her peers then has an opportunity to correct it; or in an atmosphere of openness, respect, and trust, he or she receives constructive criticism from the class. As the confidence of the class increases, problem solving becomes a more natural part of the daily routine. Students do not magically become able to discover new ideas or solve problems themselves, but they form the habit of inquiry. The stereotype of mathematics as a solely computational discipline begins to be eliminated, and the plug-and-chug setting changes to an open discussion forum.

I had to be confident enough in my goals to be flexible in planning lessons. In a geometry class, for example, a homework assignment introduced some transformations. I had planned on focusing the next day's class on finding image points by reflecting points over a line. Discussing this multistep process was scheduled to take up most of the class time that day. However, one homework problem asked students to identify simple reflections of a line segment over the x- and y-axes. When we discussed this question, students needed a visual tool, and I could see that some dynamic work on the computer might illustrate these reflections. Soon, this problem led students to discuss the possibility that the composition of two reflections was a rotation. Eventually, the students were deciding how to find the center of rotation in any rotation. During this evolving lesson, I became concerned that we were not going to address the planned work about images under reflection, but that concern did not last. I could easily see that the students' discussion was not only productive but was also in-depth and student-initiated. This result was exactly what I wanted from my students and was in line with my goals for a discussion-based PSC.

DISADVANTAGES OF A PSC

Of course, a PSC has some disadvantages. Without the structure of a lecture, the class period can easily fall into chaos if it is not organized in a way that invites students into the problem-solving process together. I often remind students to take notes in my classes, because they become so engrossed in the discussion that they do not keep track of the ideas that they are discussing. Journal writing serves as an organizational tool for the important ideas or problems that have been discussed in class. It allows the students the necessary time for reflection. Also, I believe that to have a successful lesson, I must create closure on a problem. I need to be sure that I clearly articulate the important pieces of the problem-solving process that were discovered or make sure that a student offers a clear and correct summary.

Many students are accustomed to regular practice in applying newly discovered mathematics topics. A PSC might not include such practice. Although rote repetition of similar problems may not instill understanding, it at least reinforces a method that a student needs to add to her or his tool kit. If a PSC is based on carefully sequenced and well-written problems, the need for repetition should not occur, since an idea learned will be used again in future problems. Students must be able to recall an idea, apply it, and not merely repeat the same process. For example, an initial problem in a geometry lesson whose purpose is to introduce the use of the distance formula to show whether a triangle is isosceles is as follows:

> The sides of a triangle are formed by the graphs of $9y - 4x = 22$, $2y + 3x = 1$, and $y = x - 2$. Is the triangle isosceles? How do you know? (Phillips Exeter Academy)

In a subsequent problem, students encounter a brief definition of the altitude of a triangle and are asked to do some algebra:

> Consider the triangle whose vertices are $A = (0, 0)$, $B = (8, 0)$, and $C = (4, 12)$. Find the equation for the line that contains the altitude

from A to segment BC and the equation for the line BC. Find the coordinates of the point where the altitude from A meets segment BC. What is the length of that altitude? (Phillips Exeter Academy)

In solving that problem, the student may recall a by-hand, algebraic method to find the intersection point of the line that contains the altitude with the line that contains the side, or he or she may recall a method that uses a graphing calculator. Either way, finding an intersection point is necessary to complete this problem.

A SUCCESS STORY

Improved creativity is a hidden benefit of a PSC. In my precalculus class, I assigned a problem that required the students to simplify the expression $(4 + 3i)/(1 + i)$. We had discussed using the conjugate of a complex number to rewrite reciprocals of complex numbers in $a + bi$ form, but I had not given an example using conjugate multiplication as the method of "dividing" complex numbers. However, we had discussed the transformational ramifications of the multiplication of complex numbers being a rotation and dilation of the complex number. Stephanie and Kara went to the board with their solution, although they were sure that their method was incorrect. They had collaboratively extended those transformational ideas to division, assuming that the simplified complex number resulted from a clockwise rotation (subtracting the angles) and reduction of the radius (the new radius was the quotient of the two old ones). The rest of the class was impressed with the students' ingenuity and the use of the tools to which they had already been exposed. Catie then suggested using conjugate multiplication, and Stephanie and Kara were surprised that the two answers were the same in rectangular form. This anecdote illustrates the creativity that comes with this type of curriculum, since Stephanie and Kara had really come a long way from the beginning of the year. They were known for saying, "I have no clue," but they had been able to develop an innovative method for solving the problem.

CONCLUSION

Of course, each teacher will find different challenges in transitioning to a PSC. The ability and maturity levels of students can affect the success of such a program, and the size of a class can affect the success of a structure around discussion-based learning. At the schools in which I have taught, I have had the freedom to try new ideas and research different types of pedagogy. More important, I have had the time and support for this type of exploration. I have attempted this transition with a range of levels, including geometry, a two-year algebra 2 sequence, precalculus, and calculus. Overall, I believe that the PSC has been successful in developing independent problem solvers who are interested in and curious about mathematical questions. It instills the value of what *Principles and Standards for School Mathematics* calls the *Learning Principle:* "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM 2000, p. 11).

I have seen students adapt well and take advantage of the way that a PSC allows them to learn. Students appreciate the responsibility that a PSC gives them for their own learning, and they value the outcomes that it so obviously holds as important. I am still experimenting and learning about what does and does not work. I look forward to many new challenges and rewards as my transition continues.

Improved creativity is a hidden benefit of a PSC

REFERENCES

Lampert, Magdalene. *Teaching Problems and the Problems of Teaching*. New Haven, Conn.: Yale University Press, 2001.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, Va.: NCTM 2000.

Phillips Exeter Academy. "Mathematics Department at Phillips Exeter Academy." Available at math .Exeter.edu. World Wide Web.

Wiggins, Grant. "The Case for Authentic Assessment." *ERIC Digest*, 1990. MT