Carmel Schettino, Ph.D.
NCTM Annual Conference April 2018
Metacognitive Journals: Lessons from 20 Years of Student Writing

[Information sheet students received from 2012-present]

Journal Writing in Math Class

What's this All About?
I've never written in math class before!!

Possibly for the first time in your life, you will be expected to keep a mathematics journal this year. You are to write 1 entry every 2 weeks. Your journal will be collected every other Friday – keep your eye on moodle to see when it is due.

What do you do with a math journal? It should be a place where you write down your **ideas about problems** that we have completed in class (correctly) and write a written commentary about the methods used, new topics learned, theorems proven, extension questions it posed for you, and other entries that are related to the problems and topics we cover **in your own words.** It can also be a place where you question topics or specific problems that you don't understand. This gives the opportunity for you and I to have a little conversation in writing about what is confusing to you.
It is important that you write about your own thought process. Begin by writing down the problem – what page it was on and what the problem asked for. Then think about what questions it raised for you.

1. What was the question actually asking you to do?
2. Did you know how to do it right away?
3. Was this question easy for you or did you have to reach into your prior knowledge and really think about how it connected to other problems you had done?
4. Did you have to wait until we went over it in class to understand it fully?
5. What exactly did the person presenting it in class say that made you understand?
6. How did that conversation in class help you to understand it better?
7. Was it a drawing or diagram that helped or was it seeing another person's perspective or insight that made the difference?
8. Finally, what did you learn from the problem and did that concept connect to any other problem(s) we have done?

In your writing make sure that go into these details to help you to see your process of learning the idea, problem or concept and this will help you to understand your own learning better.
So what are the big expectations for me?
For the term, the following are the major expectations:

1. Entries will be dated as of the date the topics were discussed.
2. Entries will include written commentary on problems, formulas, class discussion and interpretation of ideas.
3. This will be an ongoing process, not to be done all in one night before the journal is due.
4. You will be able to use your journal as a resource during in-class problem sets.
5. This notebook will NOT be a collection of formulas and problems. If you choose to put a formula in your journal, it must be accompanied by an explanation or proof of the formula, and an example.
6. This will NOT be the same notebook in which you do your homework.
7. Your journal grades will all count as a single assessment grade at the end of the semester/midsemester.

At any time during the term, if you would like feedback on your journal writing, please don't hesitate to ask. It is supposed to serve as a learning tool for you and a communication device for you and I.

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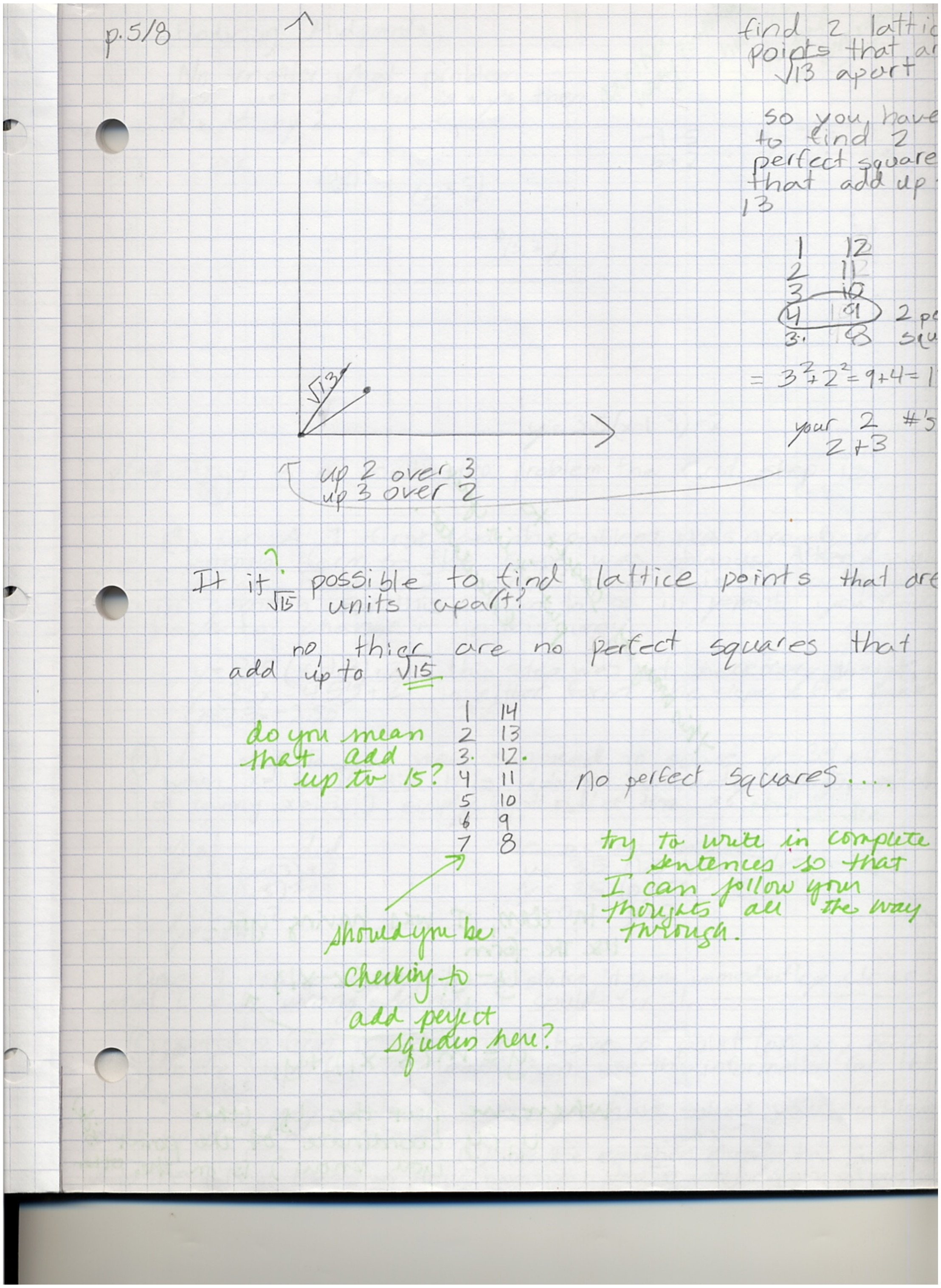
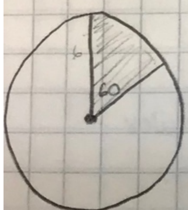
Sample 1. Hard to Grade



Sample 2: Just how to do it, little reflection



how to find the area of a:
Sector

the shaded area in this circle is a sector

following the porportion:

$$\frac{\text{central } \measuredangle}{360^\circ} = \left(\frac{\text{arclength}}{\text{circumference}}\right) = \frac{\text{sector area}}{\text{circle area}}$$

so because the central ∡ is 60 and the circle area is πr^2 or in this case $\pi 36$ this porportion can be written as:

$$\frac{60}{360} = \frac{x}{36\pi}$$ → basically saying its 1/6 of the whole circle

cross multiply and solve for x to find the area of the sector: (note: π can be treated like a varible)

$$\frac{60}{360} = \frac{x}{36\pi} \qquad 360x = 2160\pi \qquad x = \frac{2160\pi}{360}$$

The fraction should be simplified but π can be left as π

2160π = 216π = 108π = 54π = $\boxed{6\pi}$ = the sector length

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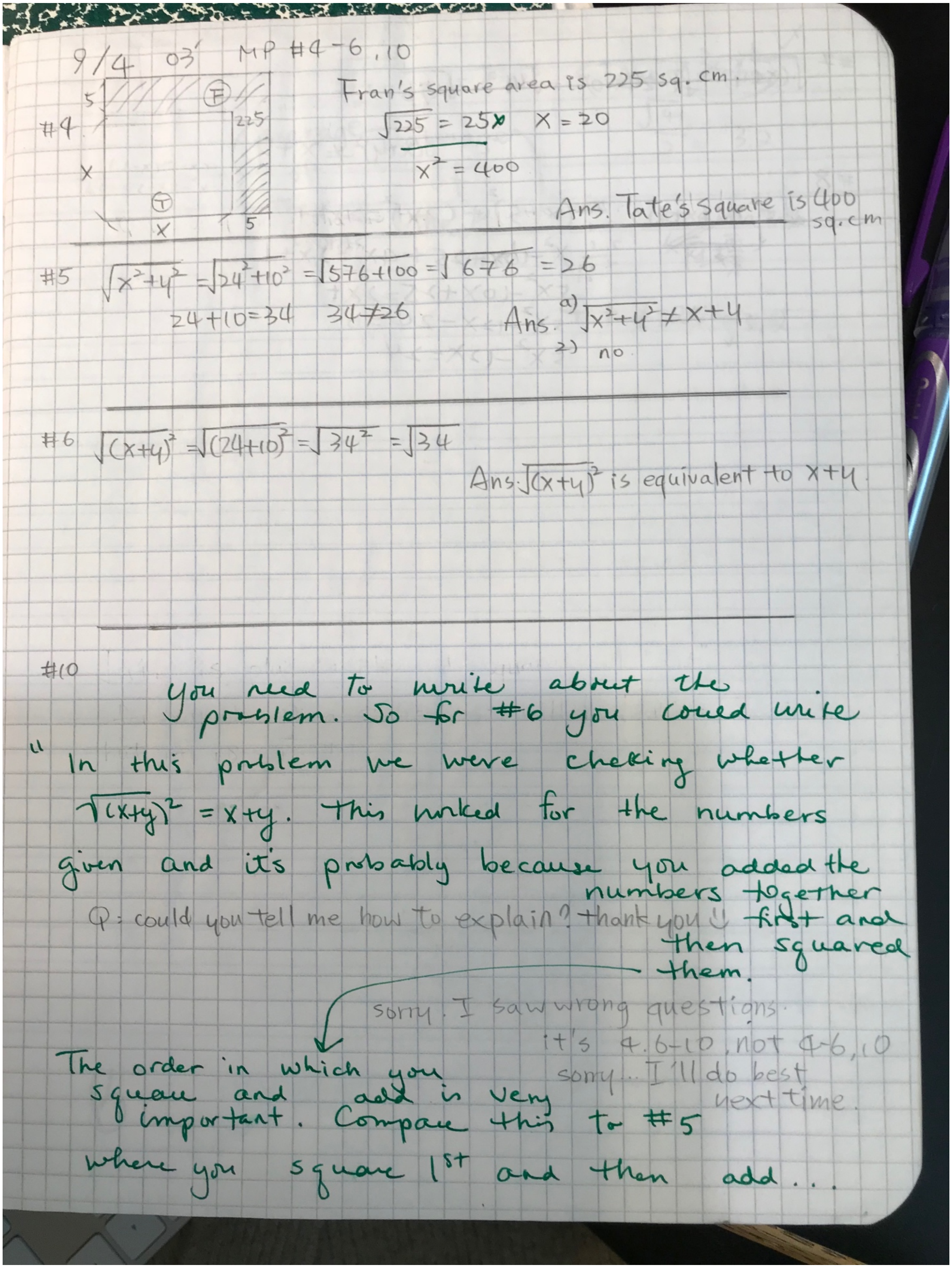
Sample 3: ELL Student early in the year



9/4 03' MP #4-6, 10

#4 Fran's square area is 225 sq. cm.

$\sqrt{225} = 25$ $x = 20$

$x^2 = 400$

Ans. Tate's square is 400 sq. cm

#5 $\sqrt{x^2+4^2} = \sqrt{24^2+10^2} = \sqrt{576+100} = \sqrt{676} = 26$

$24+10=34$ $34 \neq 26$

Ans. a) $\sqrt{x^2+4^2} \neq x+4$

2) no

#6 $\sqrt{(x+4)^2} = \sqrt{(24+10)^2} = \sqrt{34^2} = \sqrt{34}$

Ans. $\sqrt{(x+4)^2}$ is equivalent to $x+4$.

#10

You need to write about the problem. So for #6 you could write "In this problem we were checking whether $\sqrt{(x+y)^2} = x+y$. This worked for the numbers given and it's probably because you added the numbers together first and then squared them.

Q: could you tell me how to explain? thank you

sorry. I saw wrong questions. it's 4, 6-10, not 4-6, 10 sorry... I'll do best next time.

The order in which you square and add is very important. Compare this to #5 where you square 1st and then add...

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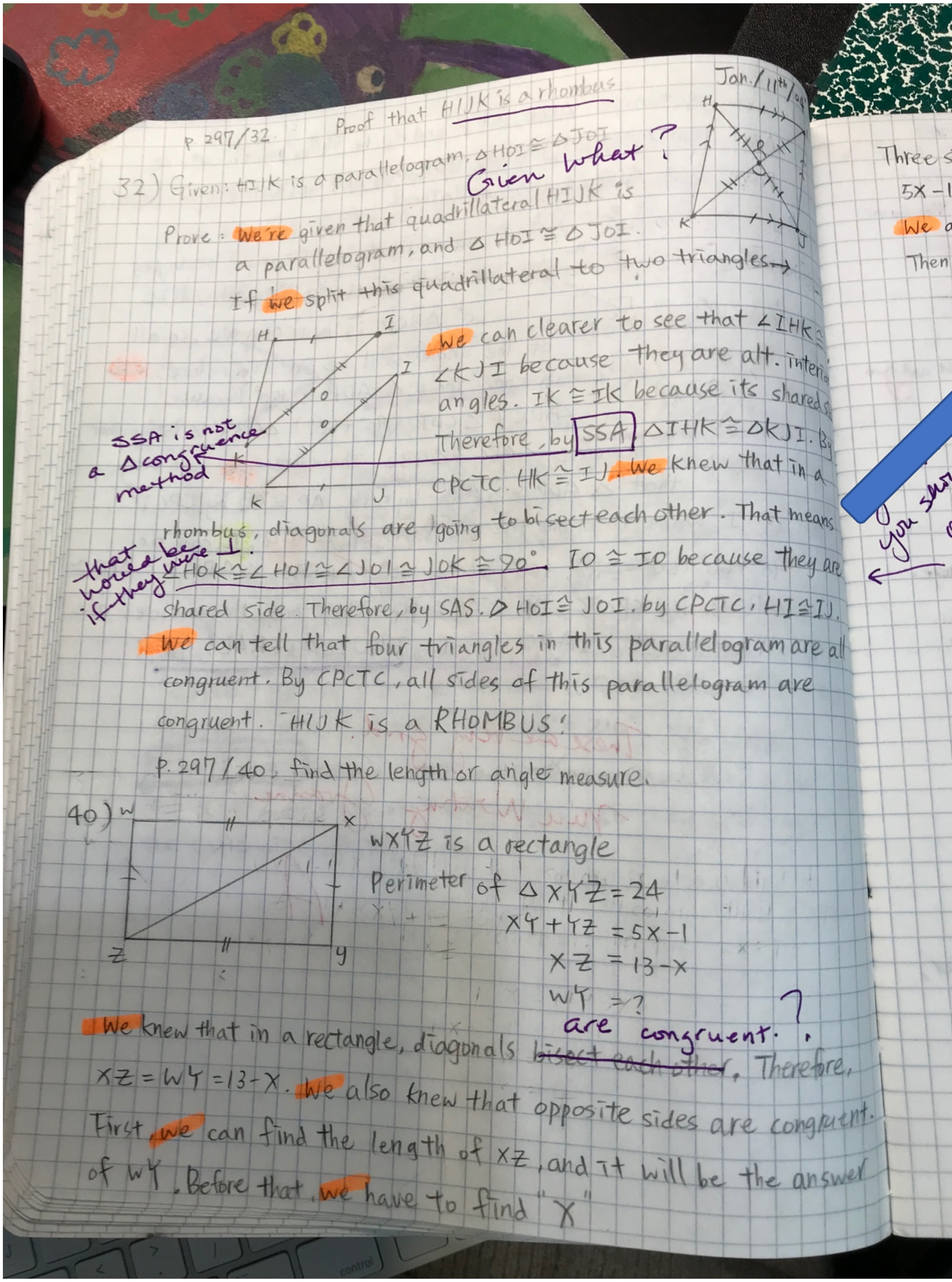
Sample 4: Same ELL student later in the year


Jan./11th/

p. 297/32 Proof that HIJK is a rhombus

32) Given: HIJK is a parallelogram, △HOI ≅ △JOI Given what?

Prove: We're given that quadrillateral HIJK is a parallelogram, and △HOI ≅ △JOI. If we split this quadrillateral to two triangles →

We can clearer to see that ∠IHK ≅ ∠KJI because they are alt. interior angles. IK ≅ IK because its shared side. Therefore, by SSA, △IHK ≅ △KJI. By CPCTC, HK ≅ IJ. We knew that in a rhombus, diagonals are going to bisect each other. That means ∠HOK ≅ ∠HOI ≅ ∠JOI ≅ JOK ≅ 90°. IO ≅ IO because they are shared side. Therefore, by SAS, △HOI ≅ JOI. by CPCTC, HI ≅ IJ. We can tell that four triangles in this parallelogram are all congruent. By CPCTC, all sides of this parallelogram are congruent. HIJK is a RHOMBUS!

SSA is not a △ congruence method

that would be if they were ⊥.

P. 297/40 find the length or angle measure.

40) WXYZ is a rectangle
Perimeter of △XYZ = 24
XY + YZ = 5X − 1
XZ = 13 − X
WY = ?

We knew that in a rectangle, diagonals ~~bisect each other~~ are congruent. Therefore, XZ = WY = 13 − X. We also knew that opposite sides are congruent. First, we can find the length of XZ, and it will be the answer of WY. Before that, we have to find "X"

Three s…
5X − 1…
We al…
Then…

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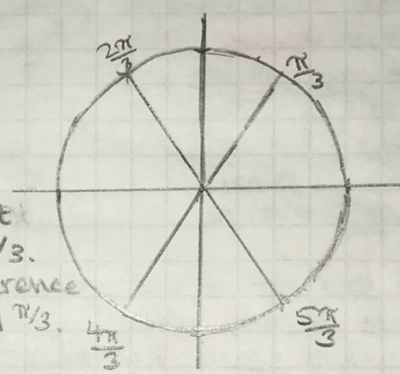
Sample #5: Some reflection but mostly on own understanding

Entry # 4

I FINALLY get the concept of reference angles! Admittedly it took me one quiz and a problem set to do so, but whatever. I get it now! EXAMPLE:

"Name an angle that has the same sine value as $4\pi/3$ radians. Explain briefly how you know they have the same sine value WITHOUT using a calculator. What about their cosine values? Are they the same as well? Explain."

$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$



$4\pi/3$ is in the third quadrant and its reference angle is $\pi/3$. Three other angles have a reference angle of $\pi/3$: $2\pi/3$, $5\pi/3$, and $\pi/3$. Because $\pi/3$ and $2\pi/3$ are in the first and second quadrants, the y-values of their coordinates are positive, which means that their sine values are the opposite of $4\pi/3$'s. That leaves $5\pi/3$. $5\pi/3$ lies in the fourth quadrant; therefore, its sine value is negative, and it shares a reference angle with $4\pi/3$, so their sine values are the same.

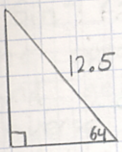
$\sin \frac{4\pi}{3} = \sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

However, because the x-coordinate of $4\pi/3$ is negative while the x-coordinate of $5\pi/3$ is positive, the cosine values of $5\pi/3$ and $4\pi/3$ are NOT the same. They are opposites. ✓

I want to say a BIG thank you to Ms. Schettino for helping me to FINALLY grasp this concept!

You did it with determination

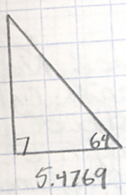
Sample 6: Explanation of Methods



Lets say we want to find the adjacent side of this triangle. We would use the previously stated formula:

$\cos(x) = \frac{adj}{hyp} \rightarrow 12.5 \cdot \cos(64) = \frac{x}{12.5} \cdot 12.5 \rightarrow 5.4796 = x$

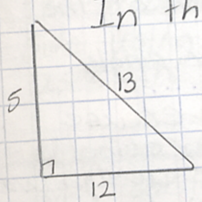
Oh, okay! This is all starting to make sense! Can you solve for the hypotenuse too? Of course! Lets try the same problem again!



$\cos(x) = \frac{adj}{hyp} \rightarrow x \cdot \cos(64) = \frac{5.4769}{x} \cdot x \rightarrow \frac{x \cos(64)}{\cos(64)} = \frac{5.4769}{\cos(64)} \rightarrow x = 12.5!$

So it works either way! great! But... I'm still confused about one thing. What is inverse cosine? Well, it's very similar to inverse sine and inverse tangent. It can find angle measurements in a right triangle, but just uses different sides to find the answer. Hmm... that kind of makes sense. Lets solve a problem using inverse cosine, sine, and tangent!

In this problem, find both acute angles.



$\tan^{-1}\left(\frac{opp}{adj}\right) = \angle \quad \sin^{-1}\left(\frac{opp.}{hyp.}\right) = \angle \quad \cos^{-1}\left(\frac{adj.}{hyp.}\right)$

$\tan^{-1}\left(\frac{5}{12}\right) \quad \sin^{-1}\left(\frac{5}{13}\right) \quad \cos^{-1}\left(\frac{12}{13}\right)$

$\tan^{-1}\left(\frac{5}{12}\right)$ **22.6199** $\quad \sin^{-1}\left(\frac{5}{13}\right)$ **22.6199** $\quad \cos^{-1}\left(\frac{12}{13}\right)$ **22.6199**

To find the other acute angle, you would do 180-90-22.6199 which is: 67.380!

watch out for your notation

Woah! It works no matter which function you use! In the end though, it really just comes down to what you're comfortable with and what information your given. If you know the opp. and adj. sides, you would want to use tan; if you know the opp. and hyp. sides, you would want to use sine; while if you know the adj. and hyp sides, you would want to use cosine. On the other hand, if you had a particular aversion to one, you could just use the pythagorean theorem to find the third side, enabling you to use any function. Choose your favorite!

Good Point!

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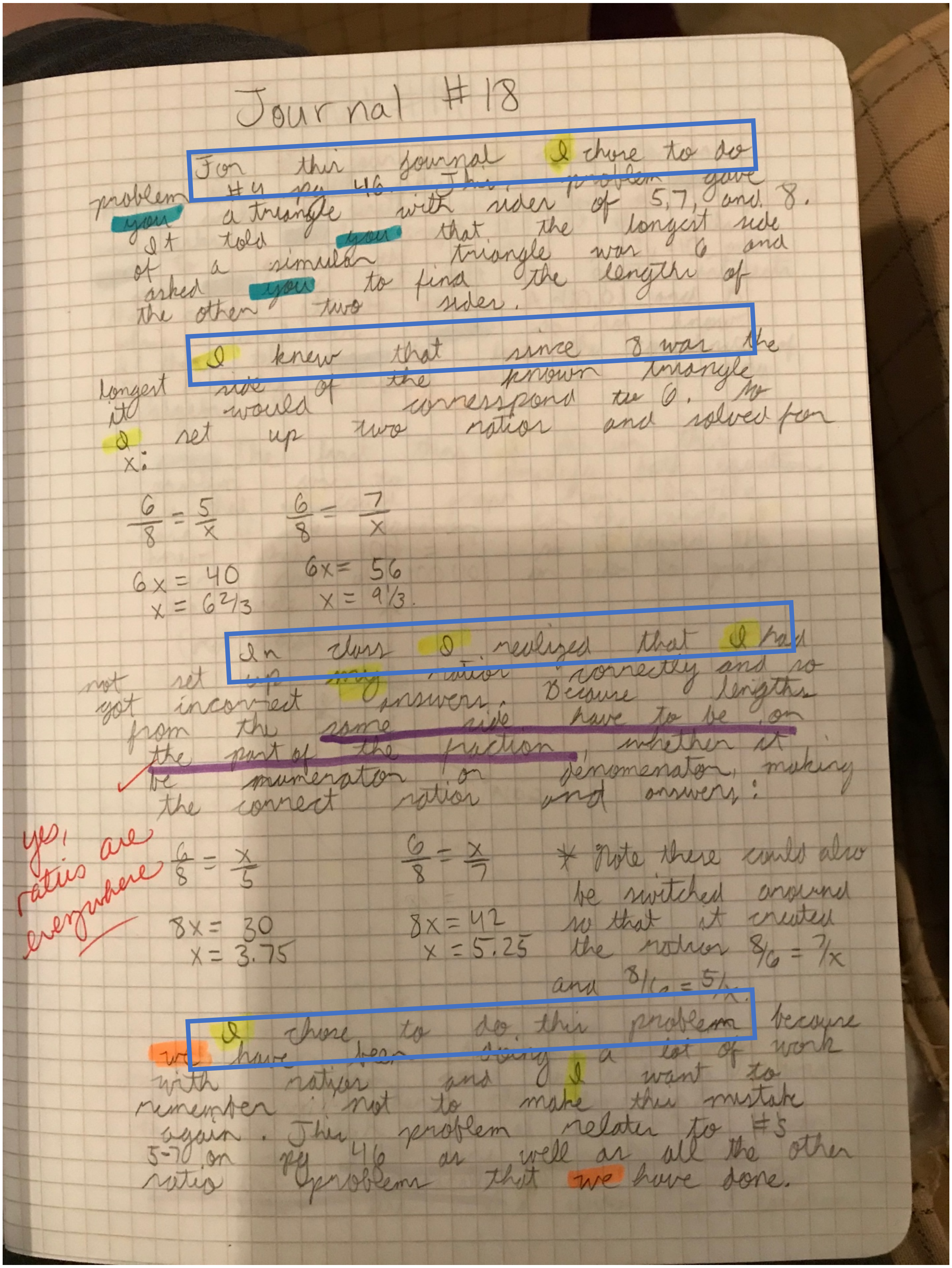
Sample 7 & 8 – Student found a "formula" for entries that worked for them.



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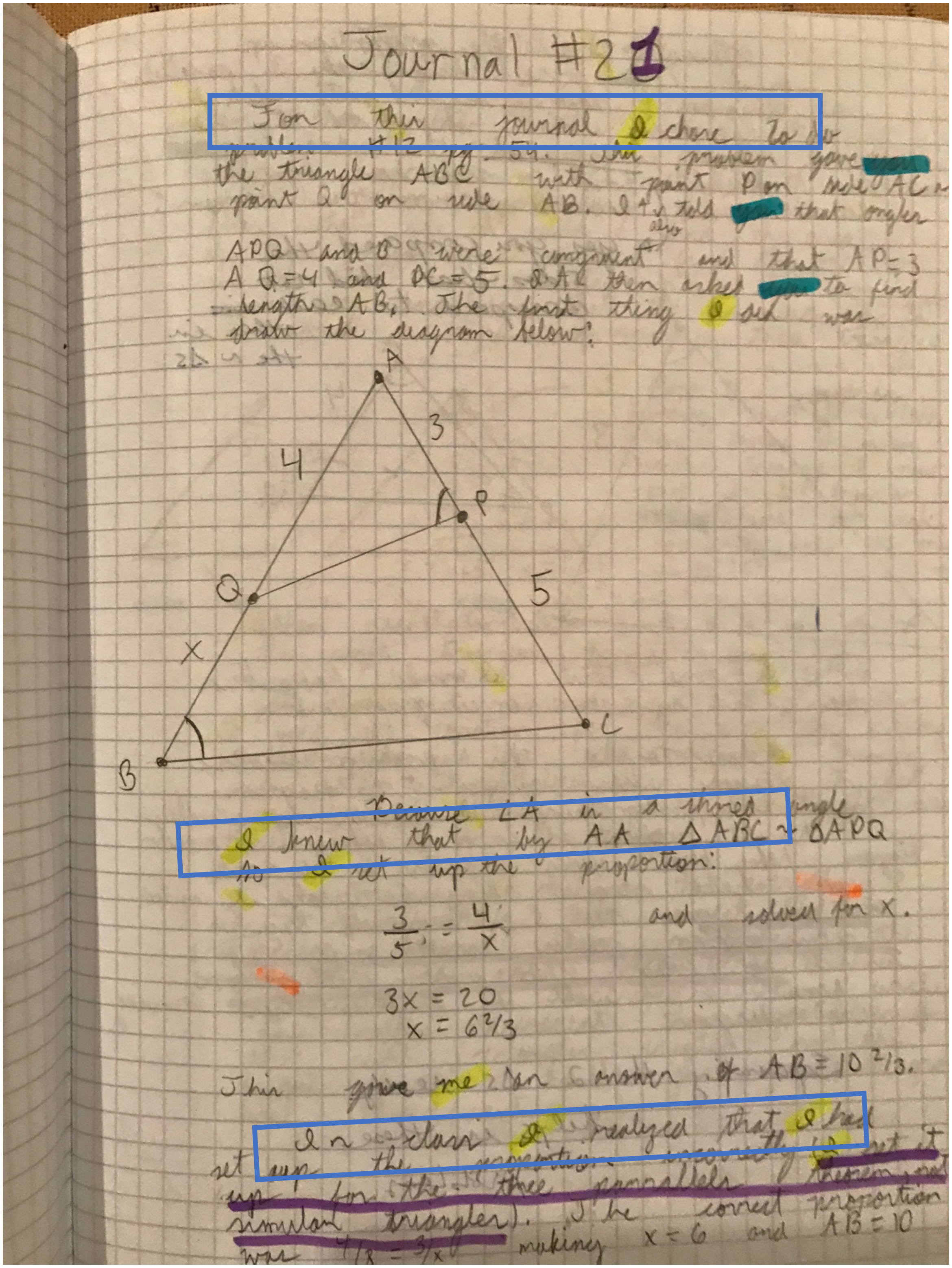
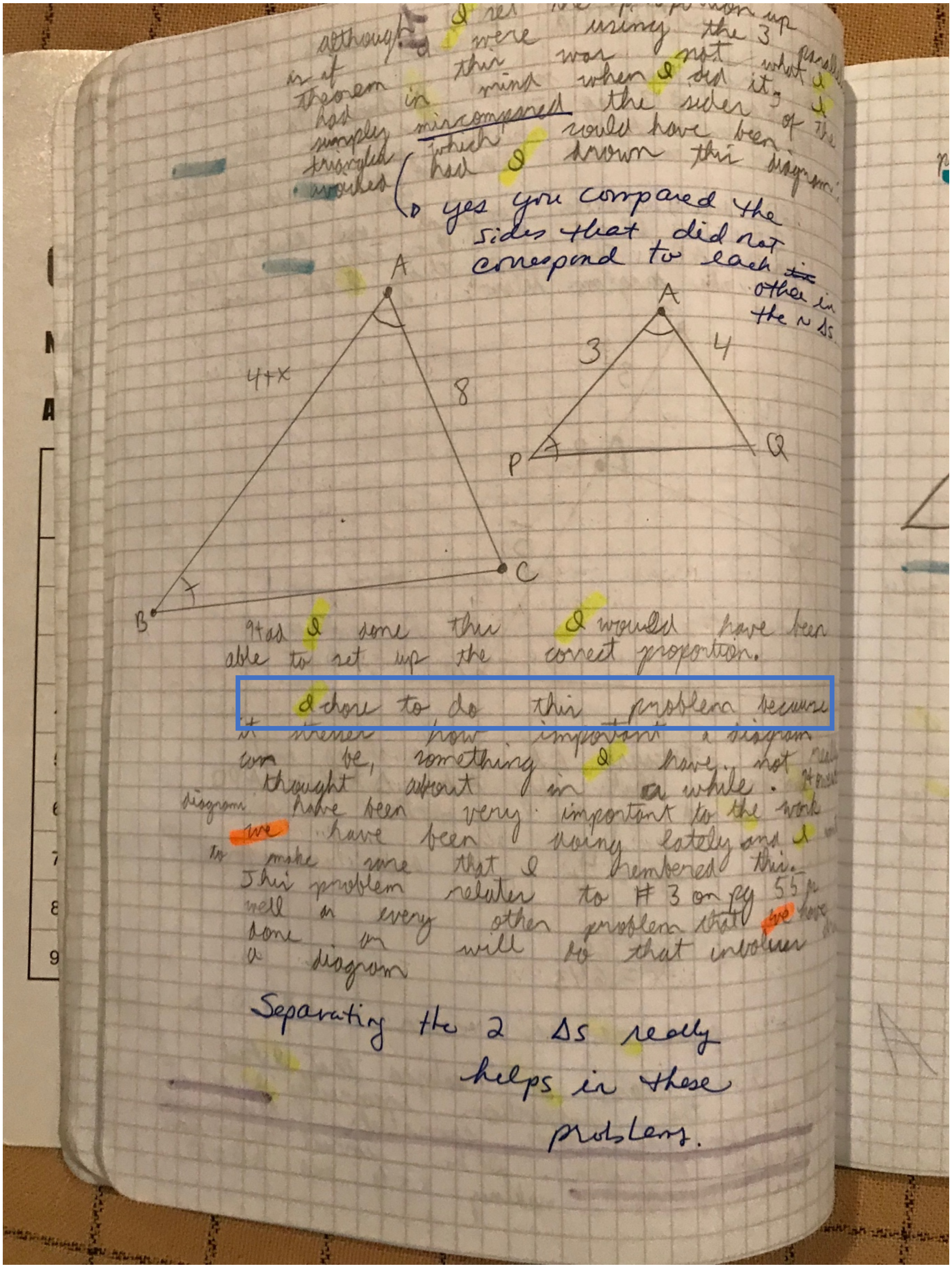
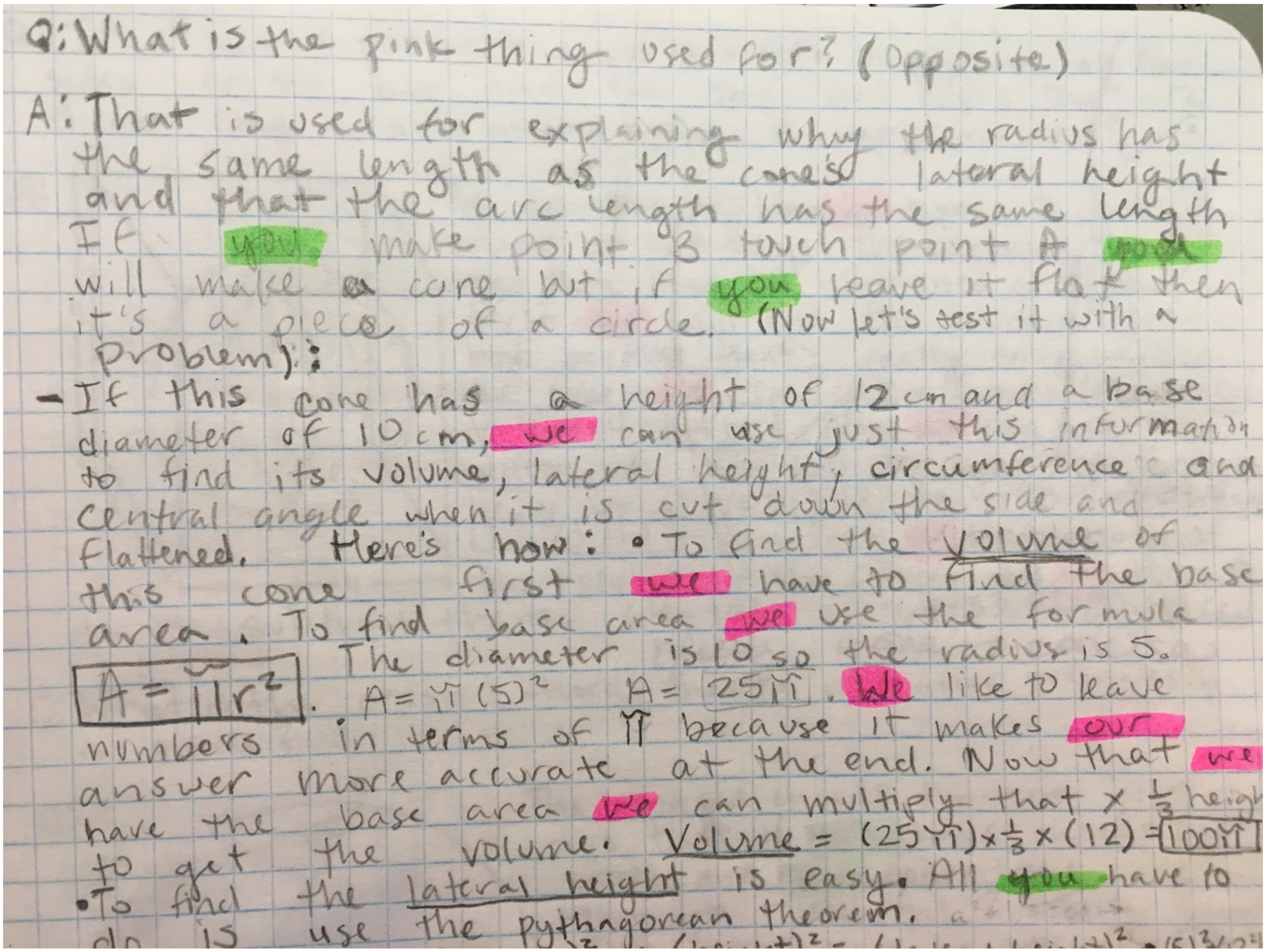



were using the 3
theorem in mind when I did it, I
simply miscompared the sides of the
triangles which would have been
had I drawn this diagram.
yes you compared the sides that did not correspond to each other in the ~ Δs
A
A
3
4
4+x
8
P
Q
B
C
Had I done this I would have been able to set up the correct proportion.
I chose to do this problem because
it stresses how important a diagram
can be, something I have not really thought about in a while.
diagrams have been very important to the work we have been doing lately and I
to make sure that I remembered this.
This problem relates to # 3 on pg 55
well as every other problem that we have done or will do that involves a diagram
Separating the 2 Δs really helps in these problems.

Sample 9 – (on slide)

Sector entry -



Q: What is the pink thing used for? (Opposite)

A: That is used for explaining why the radius has the same length as the cone's lateral height and that the arc length has the same length If you make point B touch point A you will make a cone but if you leave it flat then it's a piece of a circle. (Now let's test it with a problem):

- If this cone has a height of 12 cm and a base diameter of 10 cm, we can use just this information to find its volume, lateral height, circumference and central angle when it is cut down the side and flattened. Here's how: • To find the volume of this cone first we have to find the base area. To find base area we use the formula $A = \pi r^2$. The diameter is 10 so the radius is 5. $A = \pi(5)^2$ $A = 25\pi$. We like to leave numbers in terms of π because it makes our answer more accurate at the end. Now that we have the base area we can multiply that $\times \frac{1}{3}$ height to get the volume. Volume $= (25\pi) \times \frac{1}{3} \times (12) = 100\pi$

•To find the lateral height is easy. All you have to do is use the pythagorean theorem.

Grading Rubric for Journal Entries – This rubric is considered on a continuum and +/- is included when improvement and/or growth throughout the year has occurred.

Grade	Description of Work
A (90-100)	Your entries include well-written commentary on problems, formulas and class discussions and you select relevant and appropriate problems that incorporate multiple concepts and often integrate complex processes. Your writing strives to describe not only your thought process in your problem solving, but also in your original take on the problem – including errors made and different perspectives learned. Your writing is organized with persuasive arguments that use relevant formulas and terms. You write in complete sentences and diagrams are neatly drawn. You justify each statement and often make connections between concepts and from problem to problem.
B(80-89)	Your entries include written commentary on problems, formulas and class discussions. When writing about a formula, you provide an explanation or proof, and an example. You write down solutions, sometimes procedurally, without consistently justifying your work. You write in complete sentences and provide diagrams. You select problems that are relevant but often only on a single concept or process.
C(70-79)	Your entries include solution to problems, formulas discussed in class and other topics from class discussions. You do not justify your steps with persuasive arguments or mathematical reasoning. You are inconsistent about using complete sentences or providing diagrams with your entries. You make statements that are not valid and/or your solutions are often difficult to comprehend. There is little to no evidence of reflection on the problem solving process.
D (60-69)	Your entries look like class notes or homework. You provide little justification for your work or show no work at all. You have little to no commentary written on your work for each problem and there is no evidence of reflection on your problem solving process.
No Credit (<60)	You do not complete the assignment in a timely manner or in an acceptable way at all.