Mathematics 201: Geometry
Deerfield Academy
2014-2015

Problem Book

To MAT201 Students

Members of the Deerfield Academy, Emma Willard School and Phillips Exeter Academy Mathematics Department have created the material in this book. As you work through it, you will discover that algebra, geometry, and trigonometry have been integrated into a mathematical whole. There is no Chapter 5, nor is there a distinct section on right triangles. The curriculum is problem-based, rather than chapter-oriented.

A major goal of this course is have you practice thinking mathematically and to learn to become a more independent and creative problem solver. Problem solving techniques, new concepts and theorems will become apparent as you work through the problems, and it is your classroom community's responsibility to make these conclusions together. Your responsibility is to keep appropriate notes for your records — there are no boxes containing important theorems. There is no index as such, but the reference section at the end of the problems should help you recall the meanings of key words that are defined in the problems (where they usually appear italicized).

I. The Mathematical Thinking Process

1. Stay/Think/Say/Draw
    a. Reading each question carefully and repeatedly is essential, especially since definitions, highlighted in italics, are routinely inserted into the problem texts. Check the reference section regularly.
    b. It is important to make accurate diagrams whenever appropriate.
2. Talk/Use Resources
    a. Talk out loud, speak to friends, ask questions, email your teacher, use Voicethread groups to get feedback on your ideas
    b. Your prior knowledge – what you know already or have forgotten that you know – is your best resource.
    c. Use your notes, the internet
3. Estimate
    a. Before you try any mathematical formulas at all, you should have some idea of what the answer should be – is really large like 3000? Or should it be something small like .05?
4. Mathematize
    a. Formulas(Pythagorean theorem, quadratic formula, equations of lines), concepts (area, linear motion, what a triangle is, the sum of the angles in a triangle) and rules of mathematics (two points determine a line, all numbers squared are positive) can be used at this point in the process.
5. Try/Refine/Revise
    a. If something does work, see why it didn't work
    b. Change the method
    c. Try something else!

II. **<u>Problem Solving as Homework</u>**: You should approach each problem as an **<u>exploration.</u>** **You are not expected to come to class every day with every problem completely solved. Your presentations in class are expected to be unfinished solutions.**

- Useful strategies to keep in mind are:
  - create an easier problem
  - guess and check
  - work backwards
  - make use of prior knowledge
  - recall a similar problem.
- It is important that you work on each problem when assigned, since the questions you may have about a problem will likely motivate class discussion the next day. In other words, doing homework to get ahead is not a good idea since class discussion will help you prepare for future problems.
- Try to justify each step you do – ask *why* not just *how*. Justification is more important than the answer on a nightly basis.
- Problem-solving requires persistence as much as it requires ingenuity. When you get stuck, or solve a problem incorrectly, back up and start over. Keep in mind that you're probably not the only one who is stuck, and that may even include your teacher.
- If you have taken the time to think about a problem, <u>you should bring to class a written record of your efforts</u>, not just a blank space in your notebook. There should be a diagram, equation, reference to similar problem, evidence of your work or questions you had on the problem. This is what will get you credit for doing your homework!!

The methods that you use to solve a problem, the corrections that you make in your approach, the means by which you test the validity of your solutions, and your ability to communicate ideas are just as important as getting the correct answer. <u>You are not to spend more than the allotted time for that night's homework on any one nightly assignment, so please manage your study hall time carefully!</u>

Most importantly, be patient with yourself – learning to problem solve independently takes time, courage and practice.

III. **<u>About technology</u>**: Many of the problems in this book require the use of technology (graphing calculators or computer software) in order to solve them. Moreover, you are encouraged to use technology to explore, and to formulate and test conjectures. Keep the following guidelines in mind:

- write before you calculate, so that you will have a clear record of what you have done
- store intermediate answers in your calculator for later use in your solution
- pay attention to the degree of accuracy requested
- be prepared to explain your method to your classmates, including bringing your laptop to class with the file on it (or emailing it to your teacher the night before) in order to project your solution to the class

IV. Keeping a Mathematics Journal:

As part of this curriculum you will be asked to write about your problem solving processes on a regular basis. This will help you to organize your thoughts around not only problem solving, but the content of the course. When you write your journal entries you should keep in mind a few things:

- Write in complete sentences as if you were explaining to yourself or to another student how to do the problem
- Justify the steps of your process and explain to yourself why you chose the methods you used in the problem
- Make connections between why you chose a certain step in the process and ideas that have been discussed in class
- Make connections between problems – see if patterns emerge in how the problems are laid out in the curriculum
- Draw diagrams that help you to understand the problem better, even if a student used that diagram in class and explain why it helped your understanding

At any time during the year, if you have questions about journal writing or want more feedback, do not hesitate to speak with your instructor, or see your instructor's grading rubric for journal entries.

V. Classroom Contribution:

Learning in a PBL classroom is very different for most students for many different reasons. What is valued in the PBL classroom and what is considered successful takes time to understand, so most importantly you should come with an open mind and be ready to openly communicate. Be sure to communicate your learning needs to your teacher throughout the year. Here are some comments from past students:

About presenting homework solutions:

"The fact that we have to get up in front of the class helped in my learning"

"The accumulative mixture of problems the book had really helped me see the connections between concepts"

"I got more comfortable with taking mathematical risks"

"This curriculum has made me a better problem solver"

"It helped challenge me and taught me even if I didn't think I was learning"

"Make sure you try all of the problems – even if you can't get them."

About doing journal writing:

"Keeping a journal has really helped make reviewing and preparing for tests very easy"

"Journals totally helped, although having them on the test is useless. Once you've done a journal you know the subject."

"Although I never fully bought into keeping a journal, it gave me a good resource for studying."

About communication in class:

"I loved being able to discuss issues with classmates"

"It helps when the teacher summarizes what we learn"

"I liked finding more than one way to do something"

About getting support:

"Meeting with my teacher really helped"

"Asking questions is sign of strength not weakness"

"I liked how it was focused on yourself figuring out the problem – though that was hard for me to adjust to – however it's made me much more independent math-wise"

Becoming a better independent problem solver is not an easy journey, but it does need your whole-hearted curiosity and effort. The mathematics department is here to support you through this year so please make use of the support systems that are available if you feel you need them.

1. *Some terminology*: In a right triangle, the *legs* are the sides adjacent to the right angle. The *hypotenuse* is the side opposite to the right angle. Given the two points A(3, 7) and B(5, 2) find C so that triangle ABC is a right triangle with the right angle at C. How long are legs? How long is the hypotenuse?

2. The length of a rectangle is $(3x - 4)$ and the width is $(2x + 1)$. Find the perimeter and area of this rectangle. What is your definition of a rectangle? Write out a sentence in your own words.

3. Let $A = (0, 0)$, $B = (7, 1)$, $C = (12, 6)$, and $D = (5, 5)$. Plot these points and connect the dots to form the *quadrilateral ABCD*. Verify that all four sides have the same length. Such a figure is called *equilateral*.

4. *Factor:* $x^2 - 5x + 6$

5. If the hypotenuse of a right triangle is 12 and one of the legs is 4, what is the length of the other leg? What is the simplest form in which you can express your answer?

6. In the book (and later movie) *Flatland*, by Edwin Abbott, we are introduced to a world and its inhabitants that take place in a *plane*. The main character's name is Arthur T. Square and all activities and actions happen in the plane. However, Arthur (who is a square) happens to run into the "King of Pointland" talking out loud about himself to himself. Here is an excerpt of what Arthur hears:

> "Infinite beatitude of existence! It is; and there is none else beside It. It fills all space and It fills, It is. What It thinks, that It utters; and what It utters, that It hears; and It itself is Thinker, Utterer, Hearer, Thought, Word, Audition; it is the One and yet the All in All." (*Flatland*, p.99)

Watch the Video Clip from Flatland the Movie where Arthur T. Square meets the King of Pointland here https://www.youtube.com/watch?v=NeNvSCTbVVs and answer these questions:

   a. Why does the King of Pointland talk the way he does?
   b. Why does he describe himself in the movie as zero-dimensional?
   c. Why does he describe himself in the book as "filling all space"?

7. Two different points on the line $y = 2$ are both exactly 13 units from the same point (7, 14). Draw a picture of this situation, and then find the coordinates of these points.

1. The general notation in geometry is that points are labeled with capital letters and coordinates are defined with lowercase letters. Given the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ what do the subscripts on *x* and *y* represent? If triangle ABC is right triangle with C being the right angle.
    a. Find possible coordinates for point C in terms of $A(x_1, y_1)$ and $B(x_2, y_2)$.
    b. How could you express the length of the sides BC and AC?
    c. Now use a geometric idea you already know to find an expression for side AC.

2. Find the coordinates on a plane for the vertices of a quadrilateral that form a rectangle that is not a square. Can you find coordinates for vertices of a rectangle that does not lie horizontally or vertically?

3. Later in *Flatland*, Arthur T. Square meets the King of Lineland and has a whole conversation with him (this King can actually acknowledge others' presence). Here is some of Arthur's recounting of that meeting:

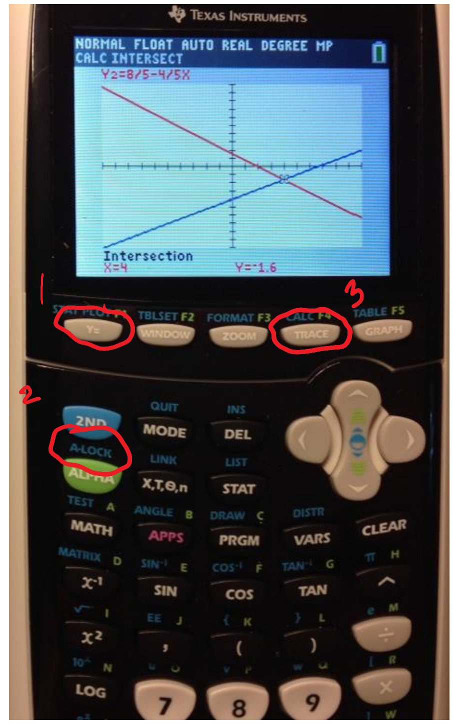
> His subjects – of whom the small lines were men and the Points, women – were all alike confined in motion and eyesight to that single straight line, which was their world. It need scarcely be added that the whole of their horizon was limit to a point; nor could anyone ever see anything but a point. Man, woman, child, thing – each was a point to the eye of a Linelander. Only by the sound of the voice could sex or age by distinguished. Moreover, as each individual occupied the whole of the narrow path, so the speak, which constituted his universe, and no one could move to the right or left to make way for passers by, it followed that no Linelander could ever pass another. Once neighbors, always neighbors. (*Flatland*, p.59)

Listen to the conversation between Arthur and the King of Lineland in the movie version of Flatland at this link https://www.youtube.com/watch?v=76lUZR6z3OQ and then answer these questions:

a. What does it mean that "each individual occupied the whole of the narrow path"?
b. Why does the King say that "length and space are the same"?
c. Is dimension the same as direction?
d. How many dimensions does the line have?

4. In algebra you spent a great deal of time learning to describe lines in the plane. How many points do you need to write the equation of a line? Justify your answer. Give an example.

5. Give an example of a point that is the same distance from (3, 0) as it is from (7, 0). Find lots of examples. Describe the configuration of all such points. In particular, how does this configuration relate to the two given points?



1. To find the intersection of two lines using your calculator, first graph the lines (press Y=) and make sure that you can see the point of intersection in the window. If you can't, be sure to change the window accordingly. Then bring up the CALC menu (2^{nd} trace) and choose intersect (#5). Using the arrow keys, move the cursor along one of the lines close to the point of intersection and press ENTER. The cursor will jump to the other line and then press ENTER on the other intersecting line. Then, press ENTER once more to get the coordinates of the intersection point from the bottom of your screen. Now, find the intersection of $y = 0.6x - 4$ and $4x + 5y = 8$.

2. Find two points on the line x=3 that are 5 units away from the point (6, 2).

3. *Some terminology*: When two angles fit together to form a straight angle (a 180-degree angle, in other words), they are called *supplementary angles* and either angle is the *supplement* of the other. When an angle is the same size as its supplement (a 90-degree angle), it is called a *right angle*. When two angles fit together to form a right angle, they are called *complementary* angles and either angle is the *complement* of the other. Two lines that form a right angle are said to be *perpendicular*. Draw a diagram for each definition.

4. Verify that the hexagon formed by $A = (0, 0)$, $B = (2, 1)$, $C = (3, 3)$, $D = (2, 5)$, $E = (0, 4)$, and $F = (-1, 2)$ is equilateral. Is it also *equiangular*?

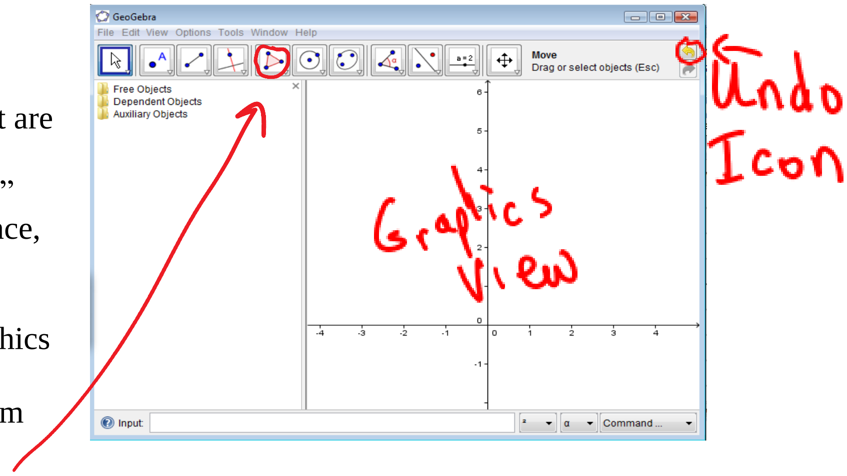
5. Draw a 20-by-20 square *ABCD*. Mark *P* on *AB* so that $AP = 8$, *Q* on *BC* so that $BQ = 5$, *R* on *CD* so that $CR = 8$, and *S* on *DA* so that $DS = 5$. Find the lengths of the sides of quadrilateral *PQRS*. Is there anything special about this quadrilateral? Explain.

6. Given the two points A(-2, 1) and B(4, 7) describe two different methods to find the distance between A and B. Which method do you prefer and why?

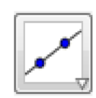
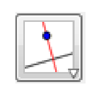
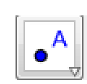
7. You may have learned in the past that the sum of the angles in any triangle is 180°. We will prove this more rigorously later on. For now, given this, what can we say about the two non-right angles of a right triangle?

8. How would you proceed if you were asked to verify that $P = (1, -1)$ is the same distance from $A = (5, 1)$ as it is from $B = (-1, 3)$? It is customary to say that *P* is *equidistant* from *A* and *B*. Find two more points that are equidistant from *A* and *B*. By the way, to "find" a point means to find its *coordinates*. Can points equidistant from *A* and *B* be found in every *quadrant*?

GeoGebra Lab #1

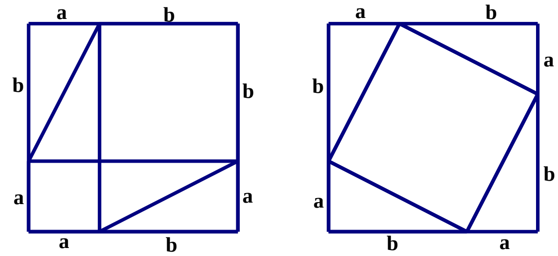


If you have never used a dynamic software program before, you should be aware of the difference between properties and objects that are “constructed” (using either a tool or a CONSTRUCT command) rather than “drawn” freehand with a tool. To observe this difference, complete the following activity

A. Right click and select axes in the graphics This will hide the axes.
B. Click on the polygon tool. It is 5th from the left and looks like a triangle. It is circled in the above diagram.
C. You are going to *draw* a triangle that looks right by doing the following: click somewhere on the drawing space (Graphics View) to start drawing the triangle, then click three more times (two more points) to draw a triangle that looks like a right triangle, finishing it by clicking on the first point you placed.
D. Press Escape (ESC), or click on the selection tool (the one that looks like an arrow) and click on any endpoint of a segment and move the triangle around. You will notice that the triangle changes, and is no longer a right triangle. You can undo, if you like, by pressing the yellow arrow at the top right of the screen.
E. Now you will *construct* a right triangle. In the bottom right-hand corner of the 3rd toolbox from the left (pictured at right) click on the bottom right corner of the button to expand it (the corner looks a little like a tiny white triangle, but changes to red when you can click it). Choose the tool that says “Segment between two points.” Click once for each endpoint of the segment you want to create.

F. Now choose the 4th toolbox (pictured at right) which is the Perpendicular Line. Click on an endpoint of the segment and the segment itself.

G. Place a point on the perpendicular line using the Point tool (pictured at right), 2nd from the left. When you are on the line, it is highlighted.


H. Choose the Polygon tool and connect your three points, clicking on each point, and making sure to again click the first point you choose at the end.
I. At the left-side of your screen you have a list of what is on your Graphics View. This section of your screen is called the Algebra View. Find the line equation – it will have *y* in it. Click on the circle to the left of the equation to hide the line. Bonus: Right-click the equation and you can turn it from Standard Form to y = mx + b form.
J. Drag the vertices of this right triangle. How is this right triangle different from the one you drew in part C? What do you think is the difference between drawing and constructing objects in GeoGebra?

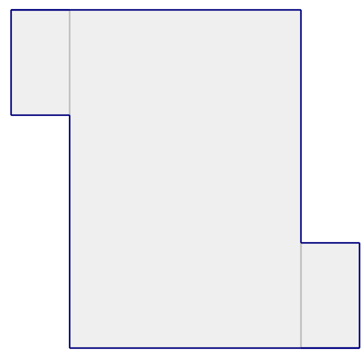
1. Alex was given the number $\sqrt{112}$ but knew there was another way to write it in a simpler form. Remembering that the number inside the square root can be split up into factors, Alex decided to try to write 112 as the product of two numbers, one that was at perfect square. Find the numbers and simplify the radical in the way that Alex might have.



2. The two-part diagram at right, which shows two different dissections of the same square, was designed to help *prove* the Pythagorean Theorem. Provide the missing details.

3. Find both points on the line $y = 3$ that are 10 units from (3, -3).

4. On a number line, where is $\frac{1}{2}(p + q)$ in relation to p and q? Try it with some values first.



5. Some terminology: Figures that have exactly the same shape and size are called *congruent*. Dissect the region shown at right into two congruent parts using only one cut. How many different ways of doing this can you find?

6. Let $A = (2, 4)$, $B = (4, 5)$, $C = (6, 1)$, $T = (7, 3)$, $U = (9, 4)$, and $V = (11, 0)$. Triangles ABC and TUV are specially related to each other in some way. Make calculations to discover and justify the relationship that you claim. Write a few words to describe what you discover.

7. If you were writing a geometry book, and you had to define a mathematical figure called a *kite*, how would you word your definition? Draw some examples and be as specific as possible in your definition.

8. Can you design a figure constructed with other familiar shapes (i.e. squares, rectangles, triangles) that has rotational symmetry?

9. A triangle that has at least two sides of equal length is called *isosceles*. Make up an example of an isosceles triangle, one of whose vertices is (3, 5). Give the coordinates of the other two vertices. Try to find a triangle that does not have any horizontal or vertical sides.

10. Let $A = (1, 5)$ and $B = (3, -1)$. Show that the point $P = (8, 4)$ is equidistant from A and B. Find at least two more points that are equidistant from A and B. How could you describe all such points?

11. Find two different rectangles with perimeter 26. Find a general way of writing the sides of the rectangle.

GeoGebra Lab #2

In this lab, you will explore the various ways to create circles.

A. First, click the x in the top right corner of the Algebra View. This will close it, and give you more space to draw. You can re-open it in the View menu. Also, right click in the Graphics View window and select Axes to turn them off.

B. The tools to create circles are in the toolbox with the circle icon. It is right next to the polygon toolbox. Try out the "Circle with Center Through Point" tool by following the instructions it gives you, at the top right part of your screen. Press ESC or select the Move tool (looks like small axes) and move the two points around to see what happens. Is there any way to move the circle that doesn't change it?

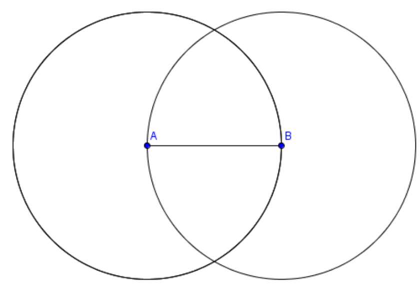
C. Right-click on the circle itself, select Object Properties, and notice that this gives you its name and how it was constructed. It also gives you several other options. Rename this circle "Circle_1." Notice that this creates a subscript.

D. Select Object Properties again. Explore some of the options in the box that pops up. Especially look through the Style tab.

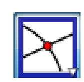
E. Now, try the Circle with Center and Radius tool. How is it different? Name this circle, "Circle_2" and change its color.

F. Right-click Circle_1 and choose Show Object. Do the same thing for Circle_2. This should hide the circles. Do this for any other objects on the screen.

G. Construct a segment. With the Compass tool, the third Circle tool, click on one endpoint of the segment, then the other endpoint to set the compass radius, then select one of the endpoints of this segment as the center of the circle. Repeat, using the other endpoint of the segment as its center. Your diagram should look like this:



H. Create the intersection points using the Intersect Two Objects tool in the Point toolbox (pictured at right). Then, connect these intersections with a line. What do you think is special about this line? Hint: How do the points on the line relate to the endpoints of the segment?



I. Find a way to check your conjecture.

1. Solve for x: $\sqrt{x+1} = \sqrt{2x-3}$. Hint: you can square both sides to eliminate the radical.

2. Find two points on the *y*-axis that are 9 units from (7, 5).

3. Using GeoGebra, plot the points P(3, 5), Q(0, 0) and R(-5, 3). Measure angle PQR, being careful to select the points in a clockwise manner. Create the segments PQ and QR. Use the Slope tool in the same toolbox as the Angle tool to find the slope of segment PQ. Do the same thing for segment QR. Make a conjecture about how these slopes are related. Verify by calculating the slopes by hand.

4. A *lattice point* is a point whose coordinates are *integers*. For example, (2, 3) is a lattice point, but (2.5, 3) is not. Find two lattice points that are 5 units apart but do not form a horizontal or vertical line.

5. (Continuation) Find a point that is 13 units away from (-1,4)

1. Graph the lines $2x - y = 5$ and $x + 2y = -10$ on a piece of graph paper on the same set of axes. Using your protractor, measure the angle of intersection.

2. Given that $2x - 3y = 17$ and $4x + 3y = 7$, and without using paper, pencil, or calculator, find the value of *x*.

3. Blair is walking along the sidewalk and sees a bird walking along the telephone wire that passes over the street. Draw a picture of this scenario. Are their paths *parallel*? Why or why not?

4. The point on segment *AB* that is equidistant from *A* and *B* is called the *midpoint* of *AB*. For each of the following, find coordinates for the midpoint of *AB*:
   (a) *A* = (-1, 5) and *B* = (-1, -7) (b) *A*= (-1,5) and *B* = (2,-7)

5. A unique line exists through any two points. In one form or another, this statement is a fundamental *postulate* of Euclidean geometry – accepted as true, without proof. Taking this for granted, then, what can be said about three points?

6. A river bank runs along the line $x = 3$ and a dog is tied to a post at the point D=(10,5). If the dog's leash is 25 units long (the same units as the coordinates), and if a fence were going to be placed at the edge of the river along $x = 3$, name the two coordinates along the river where it would be safe for the fence to end so that the dog could not fall in the river even though he is tethered at D. How long would the fence be?

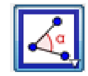
GeoGebra Lab #3

In this lab you will explore some of the graphing and measurement features of GeoGebra for segments and angles. Answer all questions in a text box in your ggb file.

A. Open GeoGebra. Notice that a coordinate axes does not always appear in the Graphics View. Right-click and choose Axes or Select Axes from the View Menu. Currently the x and y axes are in a 1:1 ratio meaning that the scales are equal.
B. Press and hold down the shift key and click on the y axis simultaneously. A label will appear stating the scale "x:y = 1:1". Now drag the y axis up and down. What happens to that scale definition? To return the scale definition to 1:1 you can simply right click on the graphic window and choose **x axis: y axis -> 1:1**.
C. Now, choose OPTIONS→Point Capturing→Snap to Grid. This will allow drawn points to snap to lattice points.
D. With your cursor, click on the Input Command Line and simply type (4, 5) and press enter. The point (4, 5) should be plotted and labeled A.
E. Plot the points (7, 3) and (1, 0) in the same way. GeoGebra automatically labels in alphabetical order and also keeps a record of your objects in the Algebra View to the left. If the Algebra View is not there, select Algebra View from the View menu.
F. Select the Move tool (farthest to the right) and move the Graphics View down so that the first quadrant is in full view in the window.

GeoGebra Lab #3 Continued

G. Press shift and select the y axis (as described in part B). What happens to points A, B and C? Is this what you expected? Do the coordinates of A, B and C change?
H. Construct the sides of triangle ABC using the Segment between Two Points tool which is the second tool in the Line toolbox.
I. What type of triangle does ABC appear to be? Measure the angles of triangle ABC using the Angle tool (fourth from the right, pictured at right). Select the three points in a clockwise order with the vertex of the angle to be measured as the second point.



J. What information is provided in the Algebra View at this time?
K. You can easily rotate your triangle when you know a center of rotation and an angle. With the point tool, draw a point on the Graphics View. Choose the Rotate tool from the Transformation Toolbox (third from the right) and follow the directions to do the rotation using your new point D as the center. When prompted for the angle of rotation use 70 degrees and be sure to include the degree symbol in the notation. If you accidentally erase the given degree symbol, you can find it in the first drop down menu. In what direction does the rotation take your triangle?
L. Repeat step K choosing -40 degrees for your angle of rotation. Which direction does it rotate now?

1. It is possible to write an equation that says that the distance from $A = (-1, 5)$ to $P = (x, y)$ is equal to the distance from $P = (x, y)$ to $B = (5, 2)$. How would you do this? This equation, when simplified represents a line that is called the *perpendicular bisector of AB*. If you were to simplify that two-sided radical equation (which you don't have to do) you would get the line 12x - 6y = 3. Verify line is the perpendicular bisector of AB by calculating two slopes and one midpoint.

2. How would you find the midpoint of the two points with coordinates $A = (m, n)$ and $B = (k, l)$

3. Find the slope of the line through:
   (a) (3, 1) and (-7, -4) **(b)** $(2, y)$ and $(x, 10)$.

4. Is it possible for a line $ax + by = c$ to lack a y-intercept? To lack an x-intercept? Explain.

5. Factor: **(a)** $x^2 - 16$; **(b)** $x^2 + 8x + 16$; **(c)** $x^2 + 6x - 16$. Describe any patterns that you see.

6. Let $P = (a, b)$, $Q = (0, 0)$, and $R = (-b, a)$, where a and b are positive numbers. Prove that angle PQR is right, by introducing two congruent right triangles into your diagram by connecting points P and Q to the x-axis. Using these two triangles, verify that the slope of *segment QP* is the *opposite reciprocal* of the slope of segment QR.

7. Find the point of intersection of the lines $3x + 2y = 1$ and $-x + y = -2$. Choose a method that is new to you.

1. The sides of a triangle are formed by the graphs of $3x + 2y = 1$, $y = x - 2$ and $-4x+9y=22$ Use technology to discover if the triangle is isosceles. How do you know?

2. Consider the linear equation $y = 3.5(x - 1.3) + 2$.
   (a) What is the slope of this line?
   (b) What is the value of y when $x = 1.3$?
   (c) This equation is written in *point-slope* form. Explain the terminology.
   (d) Use your calculator to graph this line.
   (e) Find an equation for the line through (4.2, -2.5) that is parallel to this line. Leave your answer in point-slope form.
   (f) Describe how you would graph by hand a line that has slope -2 and that goes through the point (-7, 3).

3. Given the points A = (-2, 7) and B = (3, 3), find two points P that are on the perpendicular bisector of AB. In each case, what can be said about the triangle PAB?

GeoGebra Lab #4: Investigating Point-Slope Form

A. Open the GeoGebra file Point-Slope Lab. You can find it where you find your class' assignments.
B. How did the slope slider change the line? What is the difference between negative and positive values?
C. How did the xcorr slider change the line? What is the difference between positive and negative values? How are these reflected in the equation of the line?
D. How did the ycorr slider change the line? What is the difference between positive and negative values? How are these reflected in the equation of the line?

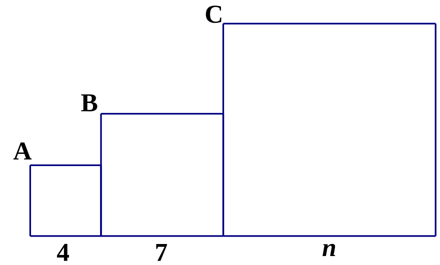
4. For each of the following questions, fill in the blank with always true (A), never true (N), or sometimes true (S). Please write a few sentences explaining your choice. Recall that italicized words are defined in the reference section.
   (a) Two parallel lines are ________ *coplanar*.
   (b) Two lines that are not coplanar _______ intersect.
   (c) Two lines parallel to the same plane are ________ parallel to each other.
   (d) Two lines parallel to a third line are ________ parallel to each other.
   (e) Two lines perpendicular to a third line are ________ perpendicular to each other.

5. A slope can be considered to be a rate of change (how one quantity changes in relation to another). Explain this interpretation and give an example. Explain the difference between a line that has *undefined slope* and a line whose slope is zero.

6. A five-foot tall Deerfield student casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

7. (Continuation) How far from the streetlight should the student stand in order to cast a shadow that is exactly as long as the student is tall?

1. Three squares are placed next to each other as shown. The vertices *A*, *B*, and *C* are *collinear*. Find the dimension *n*.



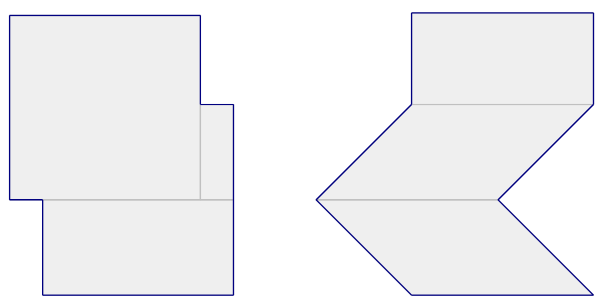
2. An airplane 27000 feet above the ground begins descending at the rate of 1500 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?

3. (Continuation) Graph the line $y = 27000 - 1500x$, using an appropriate window on your calculator. With the preceding problem in mind, explain the significance of the slope of this line and its two intercepts.

4. In a dream, Blair is shrunken down to the size of a pixel and is confined to a coordinate plane on a computer screen, moving along a line with a constant speed. Blair's position at 4 am is (2, 5) and at 6 am it is (6, 3). What is Blair's position at 8:15 am when the alarm goes off?

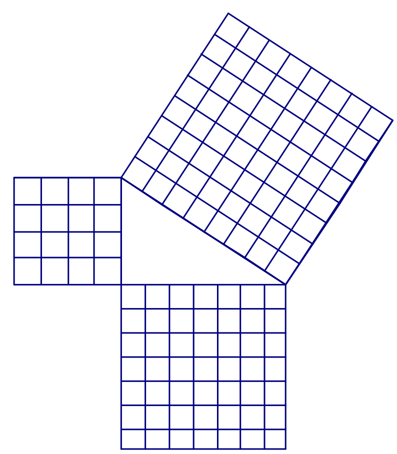
5. Find a way to show that points $A = (-4, -1)$, $B = (4, 3)$, and $C = (8, 5)$ are collinear.

6. Find as many ways as you can to dissect each figure at right into two congruent parts.



7. An airplane is flying at 36000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.

8. Is there enough evidence in the given diagram at the right to conclude that the triangle is right? Explain why or why not?



9. Golf balls cost \$0.90 each at Leonard's Club, which has an annual \$25 membership fee. At Alex & Taylor's sporting goods store, the price is \$1.35 per ball for the same brand. Where you buy your golf balls depends on how many you wish to buy. Explain, and illustrate your reasoning by drawing a graph.

10. Given the line $y = \frac{3}{4}(x+3) - 2$ and the point (9, 2). Using point-slope form, write equations for the lines parallel and perpendicular to this line through the given point.

1. Draw the following segments. What do they have in common?
   (a) From (3, -1) to (10, 3); (**b)** From (1.3, 0.8) to (8.3, 4.8);
   (**c)** From (-5,-2) to (2,2).

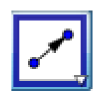
2. (Continuation) The above segments all have the *same* length and the *same* direction. Each represents the *vector* [7, 4]. The horizontal *component* of the vector is positive 7 and the vertical component is positive 4.
   (a) Find another example of two points that represent this vector. The initial point of your segment is called the *tail* of the vector, and the final point is called the *head*
   (b) Which of the following segments represents vector [7, 4]? From (-2, -3) to (5, -1); from (-3, -2) to (11, 6); from (10, 5) to (3, 1); from (-7, -4) to (0, 0).

3. Show that the triangle formed by the lines $y = 2x - 7$, $x + 2y = 16$, and $3x + y = 13$ is isosceles. Show also that the lengths of the sides of this triangle fit the Pythagorean equation. Can you identify the right angle just by looking at the equations?

4. A clock takes 3 seconds to chime at 3 pm, how long does it take to chime at 6 pm? Hint: The answer may be based on your interpretation of the question.

GeoGebra Lab #5

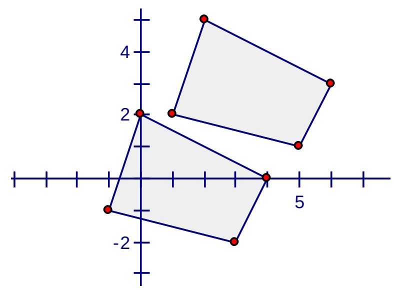
Using GeoGebra, you can facilitate visualization of vector translations.

A. Open a new GeoGebra file and hide the axes in the Graphics View.
B. Select the regular polygon tool from the polygon toolbox and construct a small regular pentagon.
C. Draw a vector somewhere else in the Graphics View, using the fifth tool in the Line toolbox (pictured at right). This vector is going to serve as your vector for a translation of the regular polygon, so the length of the vector will be how far the polygon is moved and the slope of the vector is the slope it will be moved by. The direction of the vector is denoted by the arrow at its head.

D. Now select the Translate tool for the Transformation toolbox (fourth toolbox from the right) and follow the directions to translate the regular pentagon by the vector you had drawn.
E. Now press escape (to give you back the Move tool) and drag the original vector by the point at the head (arrow) end. What happens to the translated pentagon? Does anything happen to the original pentagon? Why or why not?

1. A triangular plot of land has boundary lines of 45 meters, 60 meters, and 70 meters. The 60 meter boundary runs north-south. Is there a boundary line for the property that runs due east-west?

2. Using GeoGebra, plot the points A = (-5, 0), B = (5, 0), and C = (2, 6), then the points K = (5, -2), L = (13, 4), and M = (7, 7). Find the lengths of each side and the measure of each angle of the triangles ABC and KLM. It is customary to call two triangles *congruent* when all corresponding sides and angles are the same.

3. (Continuation) Are the triangles related by a vector translation? Why?



4. Let $A = (1, 2)$, $B = (5, 1)$, $C = (6, 3)$, and $D = (2, 5)$. Let $P = (-1, -1)$, $Q = (3, -2)$, $R = (4, 0)$, and $S = (0, 2)$. Use a vector to describe how quadrilateral *ABCD* is related to quadrilateral *PQRS*.

5. Let $K = (3, 8)$, $L = (7, 5)$, and $M = (4, 1)$. Find coordinates for the vertices of the triangle that is obtained by using the vector [2, -5] to slide triangle *KLM*.

6. Let $A = (2, 4)$, $B = (4, 5)$, and $C = (6, 1)$. Draw three new triangles as follows:
   (a) ΔPQR has $P = (11, 1)$, $Q = (10, -1)$, and $R = (6, 1)$;
   (b) ΔKLM has $K = (8, 10)$, $L = (7, 8)$, and $M = (11, 6)$;
   (c) ΔTUV has $T = (-2, 6)$, $U = (0, 5)$, and $V = (2, 9)$.
   These triangles are not obtained from *ABC* by applying a vector translation. Instead, each of the appropriate transformations is described by one of the suggestive names *reflection*, *rotation*, or *glide-reflection*. Decide which is which and explain your answers.

7. The senior grass can be approximated by the triangle seen in the picture at the right. The juniors are jealous of the seniors and they want to copy the senior grass onto the lawn behind the MSB. They have a limited amount of time so they measure <u>one of the sides</u> and create a congruent segment on the lawn. If they do not do any more measurements does this guarantee that the junior triangle will be congruent to the senior triangle? Sketch a diagram of this scenario. Another group measured only one angle and created a congruent angle on the field. If they do not do any more measurements does this guarantee that the junior triangle will be congruent to the senior triangle? Sketch a diagram.

8. In baseball, the bases are placed at the corners of a square whose sides are 90 feet long. Home plate and second base are at opposite corners. How far is it from home plate to second base to two decimal places?

9. Give the components of the vector whose length is 5 and that points in the opposite direction of [-4, 3].

1. A right triangle has one leg twice as long as the other and the perimeter is 18. Find the three sides of the triangle.

2. Let $A = (0, 0)$, $B = (2, -1)$, $C = (-1, 3)$, $P = (8, 2)$, $Q = (10, 3)$, and $R = (5, 3)$. Plot these points. Angles *BAC* and *QPR* should look like they are the same size. How would you argue that these angles are the same size without measuring them?

3. The juniors realize that copying a single measurement will not guarantee an exact copy of the senior triangle. They decide to try measuring two parts (i.e two angle or one side and one angle, etc.). Are there any combinations of two measurements that ensure that the copy will be congruent to the original?

4. An equilateral quadrilateral is called a *rhombus*. A square is a simple example of a rhombus. Find a non-square rhombus whose *diagonals* and sides are *not* parallel to the rulings on your graph paper. Use coordinates to describe its vertices. Write a brief description of the process you used to find your example.

5. A bug is initially at (-3, 7). Where is the bug after being displaced by vector [-7, 8]?

6. Plot points $K = (0, 0)$, $L = (7, -1)$, $M = (9, 3)$, $P = (6, 7)$, $Q = (10, 5)$, and $R = (1, 2)$. Show that the triangles *KLM* and *RPQ* are congruent. Show also that neither triangle is a vector translation of the other. Describe how one triangle has been transformed into the other.

7. What is the slope of the line $ax + by = c$? Find an equation for the line through the origin that is perpendicular to the line $ax + by = c$.

8. Let $A = (3, 2)$ and $B = (7, -10)$. What is the displacement vector that moves point *A* onto point *B*? What vector moves *B* onto *A*?

9. Realizing that one or two corresponding parts do not ensure congruent triangles the juniors conjecture that they must use three parts to create a new junior triangle. Create a table of the possibilities. Which do you think will work and why?

10. (Continuation) Your teacher will post some links on your moodle site entitled “Triangle Congruence Criteria Links.” For each link, answer the following questions:
    a. Do you believe that this combination of three parts of a triangle are enough to create a unique triangle? If yes, why and if no, why not?
    b. How does this GeoGebra applet help justify your conjecture?

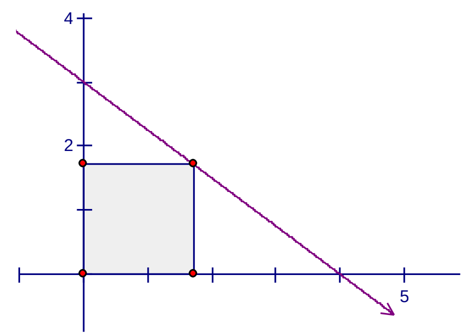
11. Without plotting points, let $M = (-2, -1)$, $N = (3,1)$, $M' = (0, 2)$, and $N' = (5, 4)$. Use the distance formula to show that segments *MN* and *M'N'* have the same length. Explain why this could be expected. (After you find the distances, graph the segments to confirm your explanation).

1. Two of the sides of a right triangle have lengths 360 and 480. Find the possible lengths for the third side.

2. If two figures are congruent, then their parts *correspond*. In other words, each part of one figure has been matched with a definite part of the other figure. Given congruent triangles KLM and RPQ, in the triangle *RPQ*, which angle corresponds to angle *M*? Which side corresponds to *KL*? In general, what can be said about corresponding parts of congruent figures? How might you confirm your hunch experimentally?

3. A nice acronym to shorten the statement about corresponding parts of congruent triangles can be written as CPCTC. What do you think these letters represent?

4. The diagram at right shows the graph of $3x + 4y = 12$. The shaded figure is a square, three of whose vertices are on the coordinate axes. The fourth vertex is on the line. Find
   (a) the *x*- and *y*-intercepts of the line;
   (b) the length of a side of the square.
   (c) Show that your point is equidistant from the coordinate axes.



5. A triangle has six principal parts – three sides and three angles. The SSS criterion states that three of these items (the sides) determine the other three (the angles). What other combinations of three parts determine the remaining three? In other words, if the class is given three measurements with which to draw and cut out a triangle, which three measurements will guarantee that everyone's triangles will be congruent?

6. The vector that is defined by a segment *AB* is often denoted $\overrightarrow{AB}$. Given A(1, 1) and B(3, 5); (**a)** Use the midpoint formula to find the midpoint of $\overline{AB}$; (**b)** find the vector $\overrightarrow{AB}$ and multiply by $\frac{1}{2}$. **(c)** Translate A by $\frac{1}{2}\overrightarrow{AB}$. What do you notice about your results?

7. Blair is in another dream on the coordinate plane and is walking along the line y=3x-2. A bug starts walking towards Blair perpendicularly from the point (5, 3). What is the equation of the line that describes the bug's path?

8. Let $A = (1, 4)$, $B = (0, -9)$, $C = (7, 2)$, and $D = (6, 9)$. Prove that triangles *DAB* and *DCB* are congruent.

9. Plot the three points $P = (1, 3)$, $Q = (5, 6)$, and $R = (11.4, 10.8)$. Verify that $PQ = 5$, $QR = 8$, and $PR = 13$. What is special about these points?

10. Sidney calculated three distances of the collinear points *A*, *B*, and *C*. She reported them as $AB = 29$, $BC = 23$, and $AC = 54$. What do you think of Sidney's data, and why?

1. After drawing the line $y = 2x - 1$ and marking the point $A = (-2, 7)$, Kendall is trying to decide which point on the line is closest to A. The point $P = (3, 5)$ looks promising. To check that P really is the point on $y = 2x - 1$ that is closest to A, what would help Kendall decide? Is P closest to A?

2. Let $A = (1, 4)$, $B = (0, -9)$, $C = (7, 2)$, and $D = (6, 9)$. Prove that angles *DAB* and *DCB* are the same size. Can anything be said about the angles *ABC* and *ADC*?

3. When two lines intersect, four angles are formed. It is not hard to believe that the nonadjacent angles in this arrangement are congruent. If you had to prove this to a skeptic, what reasons would you offer? These pairs of angles are called *vertical angles*.

4. Find a point on the line $2x + y = 8$ that is equidistant from the coordinate axes. How many such points are there?

5. Let $A = (2, 9)$, $B = (6, 2)$, and $C = (10, 10)$. Verify that segments *AB* and *AC* have the same length. Use GeoGebra to measure angles *ABC* and *ACB*. On the basis of your work, propose a general statement that applies to any triangle that has two sides of equal length.

6. A line goes through the points (2, 5) and (6, -1). Let P be the point on this line that is closest to the origin. What does it mean for the point on the line to be closest to a point not on the line? Calculate the coordinates of P.

7. We have conjectured that in an Isosceles Triangle the angles opposite the congruent sides seem to always be congruent. Write an argument supporting this assertion, which might be called the *Isosceles Triangle Theorem*.

8. Let $A = (-6, -4)$, $B = (1, -1)$, $C = (0, -4)$, and $D = (-7, -7)$. Show that the opposite sides of quadrilateral *ABCD* are parallel. Such a quadrilateral is called a *parallelogram*.

9. Let $A = (0, 0)$, $B = (4, 2)$, and $C = (1, 3)$, find the exact size of angle *CAB*. Justify your answer without your protractor or the use of technology.

10. Find components for the vectors $\overrightarrow{AB}$ where $A = (1, 2)$ and $B = (3, -7)$.

11. If $C = (-2, 5)$ and $D = (-3, 9)$, find components for the vector that points
    (a) from C to D **(b)** from D to C

12. If M is the midpoint of segment *AB*, how are vectors $\overrightarrow{AM}$, $\overrightarrow{AB}$, $\overrightarrow{MB}$, and $\overrightarrow{BM}$ related?

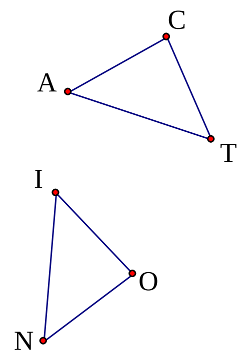
13. Show that the lines $3x - 4y = -8$, $x = 0$, $3x - 4y = 12$, and $x = 4$ form the sides of a rhombus.

14. Given the points D, A, and Y with the property $DA = 5$, $AY = 7$, and $DY = 12$. What can be said about these three points? What would be true if DY is less than 12?

1. You know that a triangle with sides of 3, 4 and 5 is a right triangle. What would happen if the third side changed from 5 to 4? From 5 to 2? From 5 to 1? Can it be less than 1?

2. Describe a transformation that carries the triangle with vertices (0, 0), (13, 0), and (3, 2) onto the triangle with vertices (0, 0), (12, 5), and (2, 3). Where does your transformation send the point (6, 0)? If you cannot find the exact coordinates make your best guess.

3. Suppose that triangle *ACT* has been shown to be congruent to triangle *ION*, with vertices *A*, *C*, and *T* corresponding to vertices *I*, *O*, and *N*, respectively. It is customary to record this result by writing $\Delta ACT \cong \Delta ION$. Notice that corresponding vertices occupy corresponding positions in the equation. Let $B = (5, 2)$, $A = (-1, 3)$, $G = (-5,-2)$, $E = (1, -3)$, and $L = (0, 0)$. Using only these five labels, find as many pairs of congruent triangles as you can, and express the congruences accurately.



4. (Continuation) How many ways are there of arranging the six letters of $\Delta ACT \cong \Delta ION$ to express the two-triangle congruence?

5. What can be concluded about triangle *ABC* if it is given that (a)$\Delta ABC \cong \Delta BCA$? **(b)** $\Delta ABC \cong \Delta ACB$? (two separate situations)

6. Plot points $K = (-4, -3)$, $L = (-3, 4)$, $M = (-6, 3)$, $X = (0, -5)$, $Y = (6, -3)$, and $Z = (5, 0)$. Show that triangle *KLM* is congruent to triangle *XZY*. Describe a transformation that transforms *KLM* onto *XZY* with respect to the line y=2x.

7. Write a proof that the two acute angles in a right triangle are complementary.

8. In Geogebra, draw a triangle and using the measure tool, measure the lengths of the three sides. Make a conjecture about what must be true about the two sides in comparison to the third side in order for the triangle to exist?

9. Let $A = (0, 0)$, $B = (1, 2)$, $C = (6, 2)$, $D = (2, -1)$, and $E = (1, -3)$. Show that angle *CAB* is the same size as angle *EAD*. You may want to use Geogebra to help you solve this problem.

GeoGebra Lab #7

<u>Altitudes</u>

In this lab you will construct the altitudes (or heights) of a triangle and investigate their properties.

A. Open a new GeoGebra file on your computer. Turn on the axes in the Graphics View.
B. With the polygon tool, draw an arbitrary triangle ABC (nothing special about it).
C. With the angle tool, measure all three angles by clicking on the triangle. Make sure that ABC is an acute triangle. If it is not, press escape and move the vertices until all three angles are acute.

D. An *altitude* of a triangle is a segment that joins one of the three vertices *perpendicularly* to a point on the line that contains the opposite side. To construct the line that contains the altitude to AB, select the perpendicular line tool (fourth from the left) and select point C and segment AB.
E. Do the same for the other two altitude lines to their respective sides. Press escape when finished.
F. Notice that the equations of three lines have appeared in the Algebra View to the left. As you click on each equation, the lines in the Graphic View should turn bold to denote which line the equation is representing. If you would rather have the altitude equations in slope/intercept form, right click on the equations and choose Equation y = mx+b. This will be a helpful way for you to check your answers in other problems.
G. What do you notice about the three altitude lines? Construct the point of intersection of these lines with the Intersect Two Objects tool in the Point Toolbox.
H. Change the name of the point (it should be automatically named D) by right clicking on the point of intersection and selecting Object Properties from the drop-down menu. In the field entitled "name" change D to Orthocenter (which is the name for the intersection of the altitudes of a triangle).
I. Save this sketch as GeoGebra Altitude Lab on your computer.

J. **Answer the following questions in a textbox on your graphics view:**
   1. Make a conjecture about the three altitudes of a triangle. What do you think is always true?
   2. Test your conjecture by dragging one or more vertices around the sketch screen. What do you observe? Does this support your conjecture?
   3. What do you observe when the triangle is obtuse?
   4. What do you observe when the triangle is right?
   5. What do you observe when the triangle is acute?

1. Let $A = (0, 0)$, $B = (8, 1)$, $C = (5, -5)$, $P = (0, 3)$, $Q = (7, 7)$, and $R = (1, 10)$. Prove that angles *ABC* and *PQR* have the same size.

2. (Continuation) Let *D* be the point on segment *AB* that is exactly 3 units from *B*, and let *T* be the point on segment *PQ* that is exactly 3 units from *Q*. What evidence can you give for the congruence of triangles *BCD* and *QRT*?

3. What is true about all of the points that lie on the perpendicular bisector of a segment?

4. *Triangle Inequality Theorem:* What must be true about the three sides of a triangle for it to exist?

5. The diagonals *AC* and *BD* of quadrilateral *ABCD* intersect at *O*. Given the information $AO = BO$ and $CO = DO$, what can you deduce about the lengths of the sides of the quadrilateral? Prove your response.

1. An *altitude* of a triangle is a segment that joins one of the three vertices to a point on the line that contains the opposite side, the intersection being *perpendicular*. For example, consider the triangle whose vertices are $A = (0, 0)$, $B = (8, 0)$, and $C = (4, 12)$.
   (a) Find the length of the altitude from C to side AB.
   (b) Find an equation for the line that contains the altitude from A to side BC.
   (c) Find an equation for the line BC.
   (d) Find coordinates for the point where the altitude from A meets side BC.
   (e) Find the length of the altitude from A to side BC.
   (f) As a check on your work, calculate BC and multiply it by $\frac{1}{2}$ times your answer to part (e). You should be able to predict the result.
   (g) It is possible to deduce the length of the altitude from B to side AC from what you have already calculated. Show how.

2. Find a point on the line $x + 2y = 8$ that is equidistant from the points (3, 8) and (9, 6).

3. What do you think of this statement – if a quadrilateral is equilateral, then its diagonals must be perpendicular. If you think it's true, draw a diagram and try to prove that it's false. If you think it's false, draw a diagram and try to come up with a reason why it must be true.

4. Let $A = (-2, 3)$, $B = (6, 7)$, and $C = (-1, 6)$.
   a. Find an equation for the perpendicular bisector of AB.
   b. Find an equation for the perpendicular bisector of BC.
   c. Find coordinates for a point K that is equidistant from A, B, and C.

5. What are the lengths of the base and the altitude of the triangle defined by the coordinates A(3,2), B(6,2) and C(4,6)? Find an alternative coordinate for C so that ABC has the same area but is now obtuse.

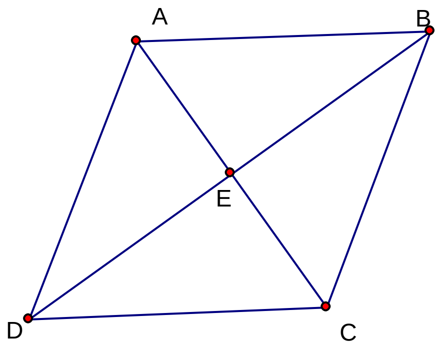
6. In quadrilateral $ABCD$, it is given that $AB = CD$ and $BC = DA$. Prove that angles ACD and CAB are the same size. Note: If a polygon has more than three vertices, the *labeling convention* is to place the letters around the polygon in the order that they are listed. Thus, AC should be one of the diagonals of $ABCD$.

7. What is true about the diagonals of a rhombus? Justify your answer.

8. Find the area of the triangle defined by $E(-2, 8)$, $W(11, 2)$, and $S(-2, -4)$. Now, find the area of triangle WLS where L is (-2, 0).

Fill in the blanks to complete the proof logically:

1. Prove that in a rhombus, the diagonals create four congruent triangles.



We know that $AB \cong BC \cong CD \cong DA$ because

____.

Since B is equidistant from A and C, point B must

____.

Since B lies on the perpendicular bisector of AC,

____________ is the midpoint of AC, and therefore

segment________ $\cong$ segment ________.

Similarly, since $DC \cong CB$ meaning that C lies on the perpendicular bisector of DB, this means that segment________ $\cong$ segment ________.

So, we can say that $\Delta AED \cong \Delta AEB \cong \Delta CEB \cong \Delta CED$

By ____________________.

Therefore, four congruent triangles are formed.

2. Given the following picture, and that HF and JG bisect each other at point E, prove $\angle H \cong \angle F$ **.**

Since HF and JG bisect each other at point E, we can say that

_______________ $\cong$ _________________ and

_______________ $\cong$ _________________.

We can also say that the pair of angles

$\angle$ _____________ and $\angle$ ________________

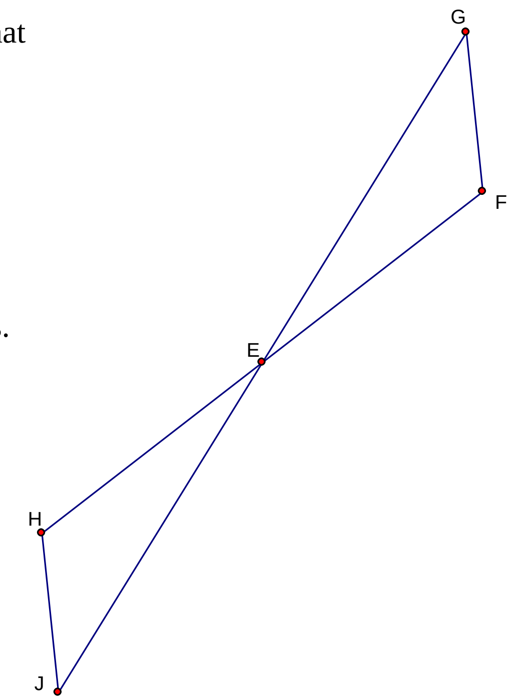
are congruent to each other because they are vertical angles.

Therefore, by ______________

we can say that the triangles ____________ and

____________ are congruent.

So, $\angle H \cong \angle F$ because _________________________.



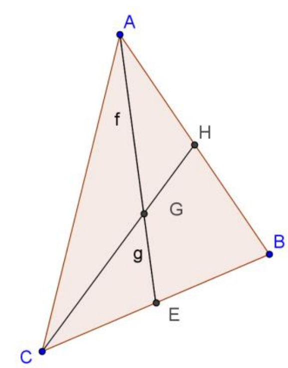
GeoGebra Lab #8

Medians

A. Open a new Geogebra file and draw an arbitrary triangle.
B. Using the Midpoint tool from the Point toolbox, construct the midpoint of each side of the triangle. Right-click one of the midpoints, and in Object Properties, change the color of each midpoint.
C. A *median* of a triangle is a segment that connects a vertex to the midpoint of the side opposite to it. Construct the medians of this triangle with the segment tool.
D. Construct the intersection of the medians. Remember to use the Intersect Two Objects tool. This point is called the *centroid* of the triangle. Right-click to rename this point G (if it is not already named this).

Answer the following questions:

1. What can be said about the three medians of a triangle?
2. Do the properties that you observed for the orthocenter hold true for this point? Test your conjecture by making the triangle right, obtuse and acute.
3. This point, the centroid, has another special property:
   a. If necessary, turn on the Algebra View. You can do this in the View menu. From your diagram identify two points that are the endpoints of a median.
   b. Using the Input Bar at the bottom of the screen type "ratio1=CG/GH" and press Enter. This will calculate for you the ratio of the longer portion of the median (from vertex to centroid) to the shorter portion of the median (from centroid to midpoint)
   c. Do the same for the other two medians, giving them appropriate names and vertex letters (like ratio2 and AG/GE)
   d. Find these values on the Algebra View. They should be near the top of the list.



4. What value do you get for the ratios? What do these ratios tell you about the segments that are on the median? What is the ratio of the vertex to the centroid to the whole median?
5. Save this file as "GeoGebra Medians Centroid Theorem"

1. A segment from one of the vertices of a triangle to the midpoint of the opposite side is called a *median*. Consider the triangle defined by $A = (-2, 0)$, $B = (6, 0)$, and $C = (4, 6)$.
   (a) Find an equation for the line that contains the median drawn from A to BC.
   (b) Find an equation for the line that contains the median drawn from B to AC.
   (c) Find coordinates for G, the intersection of the medians from A and B.
   (d) Let M be the midpoint of AB. Determine whether or not M, G, and C are collinear.

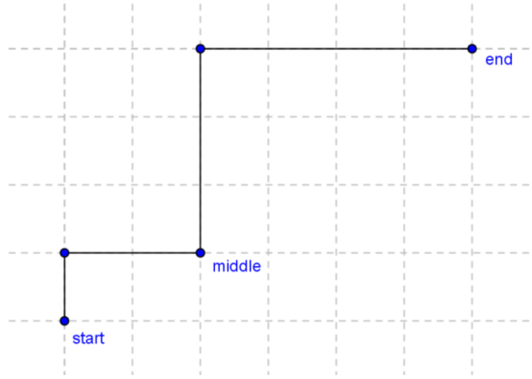
2. The line $3x + 2y = 16$ is the perpendicular bisector of the segment AB. Find coordinates of point B, given that $A = (-1, 3)$. What would you call this geometric transformation?

1. A rhombus has 25-cm sides, and one diagonal is 14 cm long. How long is the other diagonal?

2. Prove that one of the diagonals of a kite is bisected by the other.

3. Let $A = (1, 4)$, $B = (8, 0)$, and $C = (7, 8)$. Find the area of triangle ABC.

4. Triangle ABC is isosceles, with $AB = BC$, and angle ***BAC*** is 56 degrees. Find the remaining two angles of this triangle.



5. Terry walked one mile due north, two miles due east, then three miles due north again and then once more east for 4 miles. How far is Terry from the starting point? Which distance is farther – Terry's distance from the starting point or the sum of the two direct distances walked?

6. Find the area of the triangle whose vertices are $A = (-2, 3)$, $B = (6, 7)$, and $C = (0, 6)$.

7. Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
   (a) Find equations for the three lines that contain the altitudes of triangle ABC.
   (b) Show that the three altitudes are *concurrent*, by finding coordinates for their common point. The point of concurrence is called the *orthocenter* of triangle ABC.

8. Triangle ABC is isosceles, with $AB = BC$, and angle ***ABC*** is 56 degrees. Find the remaining two angles of this triangle.

9. If ABC is a right triangle with B the right angle, A=(-3,2) and B=(2,5), find possible coordinates for C.

10. Prove that if triangle ABC is isosceles, with $AB = AC$, then the medians drawn from vertices B and C must have the same length.

11. Find k so that the vectors [4,-3] and [k, -6]
    (a) point in the same direction; **(b)** are perpendicular.

12. Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
    (a) Find equations for the three medians of triangle ABC.
    (b) Show that the three medians are concurrent, by finding coordinates for their common point. The point of concurrence is called the *centroid* of triangle ABC.

13. Given points $A = (0, 0)$ and $B = (-2, 7)$, find coordinates for C and D so that $ABCD$ is a square.

1. Let A = (0, 12) and B = (25, 12). Find coordinates for a point P(x,0) on the x-axis that makes angle APB a right angle.

2. The lines $3x + 4y = 12$ and $3x + 4y = 72$ are parallel. Explain why, and then find the distance that separates these lines. You will have to decide what "distance" means in this context.

3. Give an example of an equiangular polygon that is not equilateral.

4. On a separate sheet of paper, draw parallelogram $PQRS$ with vertices at P(0, 0), Q(1, 3), R(6, 2), and S(5, -1). Cut out your parallelogram (REALLY). By making one cut through the parallelogram, show how a rectangle can be formed by the two pieces that you now have. What can you conclude about how to find the area of a parallelogram?

5. Prove that a diagonal of a square divides it into two congruent triangles.

6. Given the points A = (0, 0), B = (7, 1), and D = (3, 4), find coordinates for the point C that makes quadrilateral $ABCD$ a parallelogram. What if the question had requested $ABDC$ instead?

GeoGebra Lab #9

Perpendicular Bisectors

1. Open a new Geogebra file and draw an arbitrary triangle.
2. Select the Perpendicular Bisector Tool from the construction toolbox (4^{th} from the left), and click on each side of the triangle.
3. Construct the intersection point of all of the perpendicular bisectors. Change the name of the point to "Circumcenter."
4. Save this sketch as GeoGebra Perpendicular Bisector Lab on your computer.

Answer the following questions in a textbox on your graphics view:

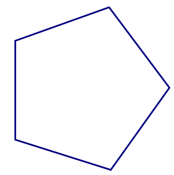
1. Move your triangle around observe what happens to the circumcenter. What happens to this point when the triangle is right?
2. What happens to this point when the triangle is obtuse?
3. What happens to this point when the triangle is acute?
4. Why might the circumcenter and the orthocenter behave in the same ways with regards to the position of the triangle?
5. The circumcenter also has another interesting property. Recall the property of perpendicular bisectors discussed in class. The intersection of the perpendicular bisectors then has that property for both segments. So what do you think is true of the circumcenter?
6. Check your conjecture by selecting the circle tool. With the cursor click on the circumcenter and drag the mouse to one of the vertices of the triangle (it doesn't matter which one – why not?). Describe the circle in relation to the triangle.

1. Find points on the line $3x + 5y = 15$ that are equidistant from the coordinate axes.

2. What do you need to write the equation of the perpendicular bisector of the points D (3, -1) and A (5, 3)? Do so.

3. Plot all points that are 3 units away from the *x*-axis. Describe the configuration algebraically and in words. What definition of distance are you using in this problem?

4. In triangle *ABC*, it is given that *CA* = *CB*. Points *P* and *Q* are marked on segments *CA* and *CB*, respectively, so that angles *CBP* and *CAQ* are the same size. Prove that *CP* = *CQ*.



5. A polygon that is both equilateral and equiangular is called *regular*. Prove that all diagonals of a regular *pentagon* (five sides) have the same length.

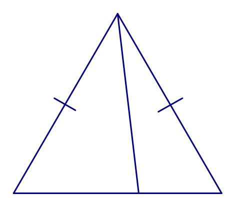
6. Plot all of the points that are 5 units away from the line x=3. Describe these points algebraically and in words.

7. Considering a Pythagorean definition of distance in the plane, plot all of the points 3 units from (5, 4) and describe their configuration algebraically and in words.

8. Let *A* = (3, 4), *B* = (0,-5), and *C* = (4,-3).
   a. Find equations for the perpendicular bisectors of segments *AB* and *BC*.
   b. Find the coordinates for their intersection point *K*.
   c. Calculate lengths *KA*, *KB*, and *KC*.
   d. Why is *K* also on the perpendicular bisector of segment *CA*?

9. (Continuation) A *circle* centered at *K* can be drawn so that it goes through all three vertices of triangle *ABC*. Explain why. This is why *K* is called the *circumcenter* of the triangle. In general, how do you locate the circumcenter of a triangle?

10. Find coordinates for the point equidistant from (-1, 5), (8, 2), and (6, -2).



11. Use the diagram at right to help you explain why SSA evidence is not by itself sufficient to justify the congruence of triangles. The tick marks designate segments that have the same length.

12. Find an equation for the line through point (7, 9) that is perpendicular to a line with slope of $\frac{2}{5}$.

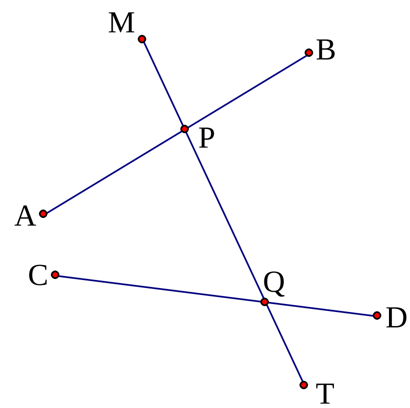
13. A triangle that has a 13-inch side, a 14-inch side, and a 15-inch side has an area of 84 square inches. Accepting this fact, find the lengths of all three altitudes of this triangle.

1. Find the lengths of *all* the altitudes of the triangle whose vertices are (0, 0), (3, 0), and (1, 4).

2. The *converse* of a statement of the form "If *A* then *B*" is the statement "If *B* then *A*." Write the converse of the statement "If it is Tuesday, we have sit down lunch."

3. (Continuation) "If point *P* is equidistant from the coordinate axes, then point *P* is on the line $y = x$". Is this a true statement?
   (a) Write the converse of the given statement. Is it true?
   (b) Give an example of a true statement whose converse is false.
   (c) Give an example of a true statement whose converse is also true.

4. The diagram at right shows lines *APB* and *CQD* intersected by line *MPQT*, which is called a *transversal*. There are two groups of angles: one group of four angles with vertex at *P*, and another group with vertex at *Q*. There is special terminology to describe pairs of angles – one from each group. If the angles are on different sides of the transversal, they are called *alternate*, for example, angles *APM* and *PQD*. Angle *BPQ* is an *interior* angle because it is between the lines *AB* and *CD*. Thus, angles *APQ* and *PQD* are called *alternate interior*, while angles *QPB* and *PQD* are called *Same Side Interior*. On the other hand, the pair of angles *MPB* and *PQD* – which are non-alternate angles, one interior, and the other exterior – is called *corresponding*. Refer to the diagram and name
   (a) the other pair of alternate interior angles;
   (b) the other pair of same side interior angles;
   (c) the angles that correspond to *CQT* and to *TQD*.

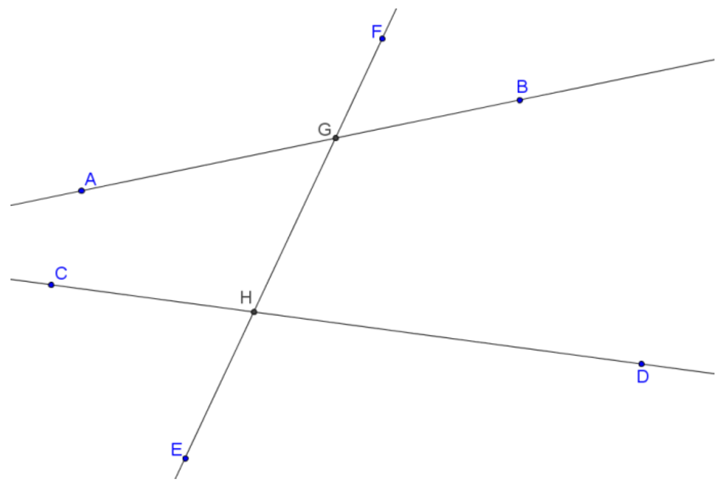


5. Let $P = (2, 7)$, $B = (6, 11)$, and $M = (5, 2)$. Find a point *D* that makes vectors $\overrightarrow{PB} = \overrightarrow{DM}$. What can you say about quadrilateral *PBMD*?

6. The diagonals of quadrilateral *ABCD* intersect perpendicularly at *O*. What can be said about quadrilateral *ABCD*?

7. What do you call **(a)** an *equiangular quadrilateral*? **(b)** an *equilateral quadrilateral*? **(c)** a *regular quadrilateral?*

GeoGebra Lab #11



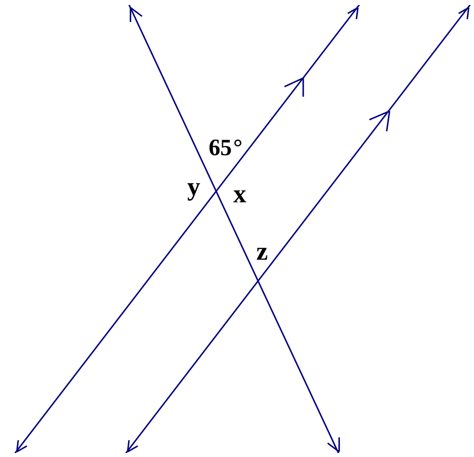
In this lab, we will discover properties of angles formed by two lines (or segments) cut by a transversal.

A. Open a GeoGebra file and turn off the axes in the Graphics View.
B. Draw two **lines**, not necessarily parallel, and a transversal, as in the diagram. Construct the intersection points and label as shown in the diagram
C. Measure all eight of the angles using the Angle tool. Recall that in order to measure an angle, you need to select the three points that define that angle in a clockwise order.
D. Move all angle measurement labels so that they are readable.
E. Press escape and select a point on either line (not the intersection). All angle measure labels should remain in view, but the measure of the angles should change.
F. In the Algebra View, make sure the equations of the lines AB and CD are in slope/intercept form and compare the slopes.
G. Drag one of the lines so that they are parallel to each other.
H. Fill in the chart below. Name a pair of each type of angle given and state the angle measurement that you observe in your Graphics View when the lines are parallel. Finally, state the relationship that exists (if any) between the angles in that pair.

Angle Type	**First Pair name and angle measurement**	**Second Pair name and angle measurement**	**Relationship?**
Corresponding			
Alternate Interior			
Same Side Interior			

I. For each type of angles, make a conjecture about the relationship of the lines. What is the requirement for your conjectures to be true?
J. Are the converses of these statements true?

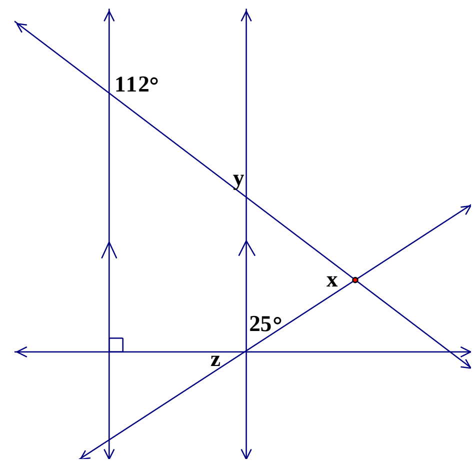
1. You have recently seen that there is no generally reliable SSA criterion for congruence. If the angle part of such a correspondence is a *right* angle, however, the criterion *is* reliable. Justify this so-called *hypotenuse-leg* criterion (which is abbreviated HL).



2. For the diagram at the right, find the measure of the angles indicated. Notice the custom of marking arrows on lines to indicate that they are known to be parallel.

3. It is a postulate (assumed without proof) that given two parallel lines cut by a transversal, corresponding angles are congruent. Given two parallel lines cut by a transversal, prove that a pair of alternate interior angles is congruent.

4. Given quadrilateral *ABCD* with $\angle BDC \cong \angle DBA$ and $AB \cong DC$, what kind of quadrilateral is *ABCD*? Prove your conjecture.



5. For the diagram at the right, find the measure of the angles indicated.

6. Given isosceles triangle *ABC* where *AB* = *BC* =10 and the altitude from *B* has length 4. Find the length of the base. Leave your answer in simplest radical form.

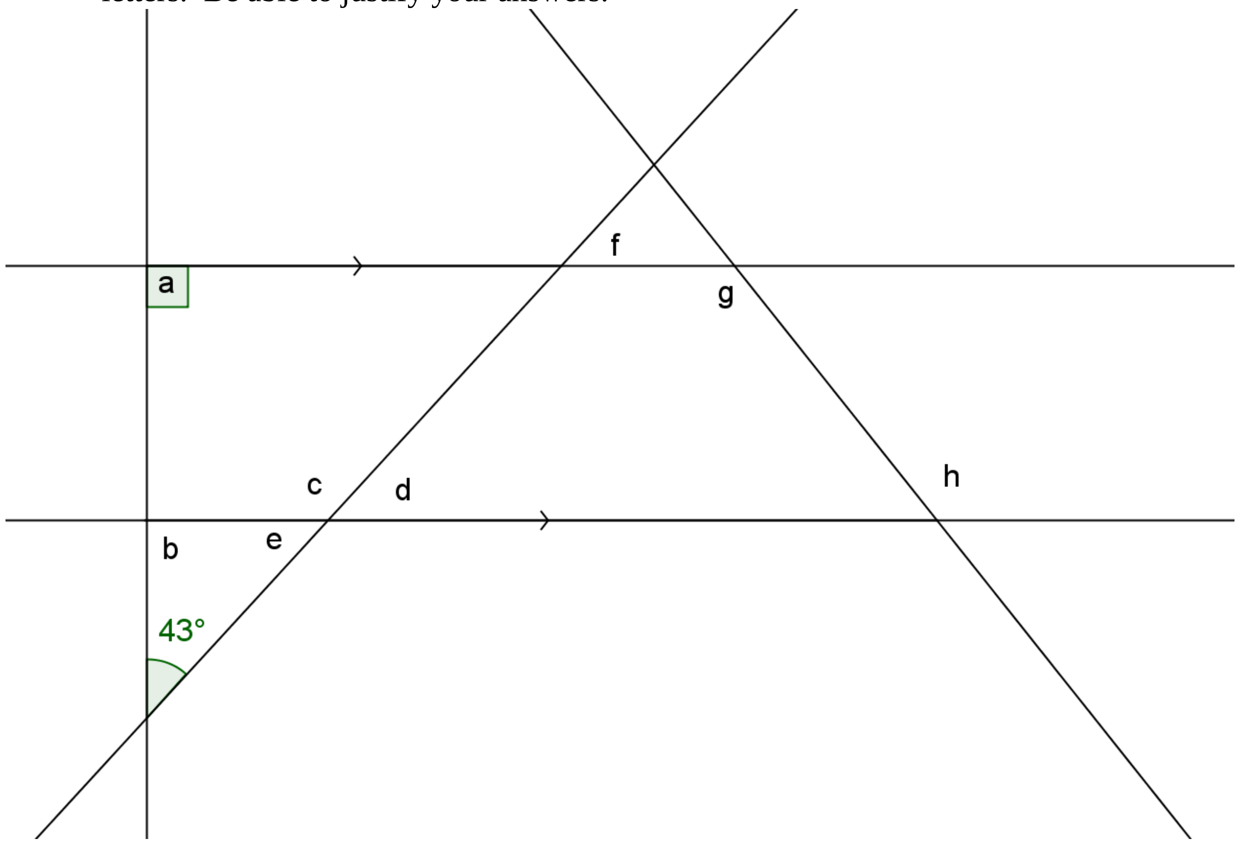
7. You probably know that the sum of the angles of a triangle is the same as the measure of a straight angle. One way to confirm this is to draw a line through one of the vertices, parallel to the opposite side. This creates some alternate interior angles. Complete the proof and include a visual representation.

8. If it is known that one pair of alternate interior angles are congruent, what can be said about **(a)** the other pair of alternate interior angles? **(b)** any pair of corresponding angles? **(c)** either pair of same side interior angles?

9. Suppose that *ABCD* is a square and that *CDP* is an equilateral triangle, with *P outside* the square. What is the size of angle *PAD*?

10. Recall that a quadrilateral that has two pairs of parallel opposite sides is called a *parallelogram*. What relationships exist between the interior angles of a parallelogram?

1. In the diagram below, find the measurements of all angles labeled with lowercase letters. Be able to justify your answers.

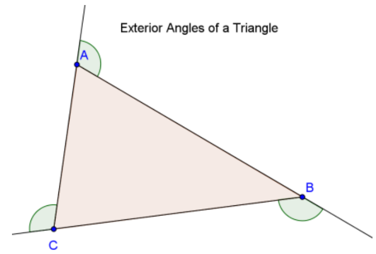


GeoGebra Lab #12

The Exterior Angle Theorem

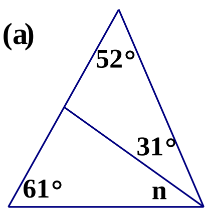
A. Open GeoGebra and draw an arbitrary triangle ABC. It's helpful to turn off the Axes and Grid so that your drawing space is cleaner.
B. Using the ray tool in the Segment toolbox, extend side AB past B.
C. Place a point, D, on ray AB past B so that B is between A and D.
D. Measure angle DBC, this is called an *exterior angle* of this triangle. Also measure angles BAC, BCA which are called the *remote interior angles* to angle DBC. Why do you think these angles are named this way?
E. You will use the Input bar create a measurement that is the sum of these two angle measures. First, find the first remote interior angle on the Algebra view. Right click it and rename it p (so that you do not have to deal with the Greek letter assigned it). Rename the other remote interior angle as q and the exterior angle as e. Then, in the input bar, type: sum = p + q.
F. Create a text box. In the box, type Exterior Angle = , then notice that under where you are typing are some drop down menus. Select the category Objects, and select "e", the name of your exterior angle. The preview of your text will show up in the bottom of the window. Finally, click OK to close the text box window.

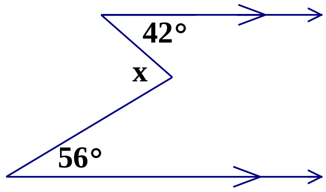
G. Create another text box and type Sum of Remote Interior Angles = and then select sum from the drop down list of Objects.
H. Drag one of the vertices of the triangle. What do you observe?
I. What do you think the relationship between the exterior angle and one of its remote interior angles would be if this was an isosceles triangle? Why?

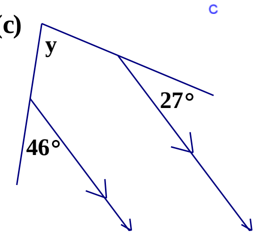


1. Given an arbitrary triangle, what can you say about the *sum* of the three exterior angles, one for each vertex of the triangle?

2. In the diagrams below, the goal is to find the sizes of the angles marked with letters, using the given numerical information.

(a)


(b)


(c)


3. Prove that the sum of the angles of any quadrilateral is 360 degrees.

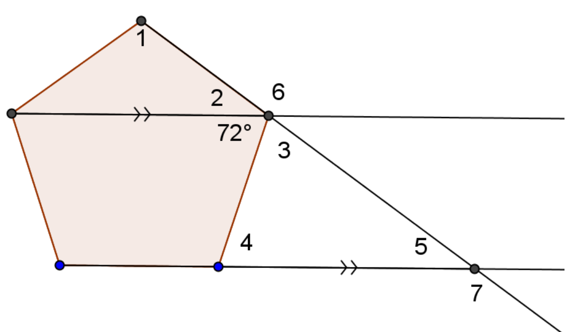
4. Write the Pythagorean Theorem in if…then form. State the converse of the Pythagorean Theorem.

5. Fill in the following table

Number of sides of polygon	3	4	5	6	7	8	…	*n*
Number of non-overlapping triangles	1	2					…	
Total sum of the angles	180°	360°					…	
One Angle on a regular *n*-sided polygon							…	

6. Given parallelogram *PQRS*, let *T* be the intersection of the bisectors of angles *P* and *Q*. Without knowing the sizes of the angles of *PQRS*, calculate the size of angle *PTQ*. Recall that the diagonals of a parallelogram are not necessarily the angle bisectors.

7. In the figure at right, find the sizes of the angles indicated by numbers if the pentagon is regular and the lines indicated are parallel.



1. Mark the point P inside square $ABCD$ that makes triangle CDP equilateral. Calculate the size of angle PAD.

2. If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent. What is the converse of this statement? Is the converse true?

3. Consider triangle MAC with vertices M(7,-1), A(-5,5) and C(5,-5).
   a. Find the circumcenter of MAC.
   b. How far are all the vertices from the circumcenter?
   c. Make a conjecture about the midpoint of the hypotenuse of a right triangle. Explain.

4. Find the measure of an interior angle of a regular decagon.

5. If ABC is any triangle, and TAC is one of its exterior angles, then what can be said about the size of angle TAC, in relation to the other angles of the figure? Draw your own triangle and assign reasonable angle measures to the interior angles and confirm that the exterior angle TAC is the sum of the two remote interior angles.

6. In regular pentagon $ABCDE$, draw diagonal AC. What are the sizes of the angles of triangle ABC? Prove that segments AC and DE are parallel.

7. Given square $ABCD$, let P and Q be the points outside the square that make triangles CDP and BCQ equilateral. Be sure to draw an accurate diagram. Prove that triangle APQ is also equilateral.

8. The sides of an equilateral triangle are 12 cm long. How long is an altitude of this triangle? The altitude divides the triangle into two right triangles. What are the measures of the angles in these right triangles? How does the short side of this right triangle compare with the other two sides? Please leave your lengths in simplest radical form.

9. If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent. Explain. What is the converse of this statement? Is it true?

10. In triangle ABC, it is given that angle A is 59 degrees and angle B is 53 degrees. Draw the altitude from B to side AC. Draw a line through A that is parallel to side BC. Extend the altitude from B to AC until it intersects that line through A that is parallel to segment BC. Call that intersection point K. Calculate the size of angle AKB.

11. Given square $ABCD$, let P and Q be the points outside the square that make triangles CDP and BCQ equilateral. Segments AQ and BP intersect at T. Find angle ATP.

12. Friday runs along the boundary of a four-sided plot of land, writing down the number of degrees turned at each corner. (Picture yourself doing this and the angle you would turn through.) What is the sum of these four numbers?

1. If the diagonals of a quadrilateral bisect each other, then the figure is a parallelogram. Prove that this is so. What about the converse statement?

2. Jackie walks along the boundary of a five-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these five numbers?

3. Marty walks along the boundary of a seventy-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these seventy numbers?

4. The preceding two questions illustrate the *Sentry Theorem*. What does this theorem say, and why has it been given this name?

5. A rectangle with area 540 has one side of length 15. Find the length of the other side and the diagonals.

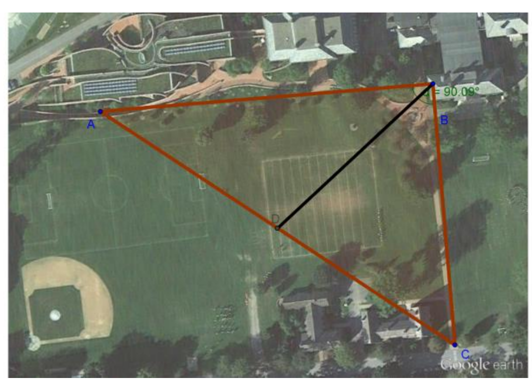
6. Can two of the angle bisectors of a triangle intersect perpendicularly? Try arguing with what's called a *Proof by Contradiction*. This is when you assume the statement is true and come up with a contradiction to an already known fact.

7. Suppose that quadrilateral *ABCD* has the property that *AB* and *CD* are congruent and parallel. Is this enough information to prove that *ABCD* is a parallelogram? Explain.

8. The *midsegment* of a triangle is a segment that connects the midpoints of two sides of the triangle. Given a triangle with coordinates *A*(1, 7), *B*(5, 3) and *C*(-1, 1) find the coordinates of the midpoints of sides *AB* and *AC*, label the midpoints *M* and *N*, respectively. Draw the midsegment MN.
   (a) Find the length of the midsegment *MN* and compare it to the length of *BC*.
   (b) What can be said about the lines containing segments *BC* and *MN*?

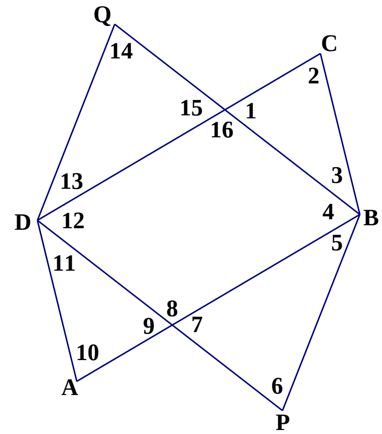
9. Given rectangle *ABCD*, let *P* be the point outside *ABCD* that makes triangle *CDP* equilateral, and let *Q* be the point outside *ABCD* that makes triangle *BCQ* equilateral. Prove that triangle *APQ* is also equilateral.

10. Ber is walking directly from the Koch Center to the crosswalk across Albany Rd. Lee plans to leave the Mem and intercept Ber exactly halfway along the straight path which happens to be the hypotenuse of a right triangle. They continue walking together to the crosswalk by Hitchcock to go on to lunch. If the Koch is 160 m away from the Mem and the Mem is 120 m away from the crosswalk, how far does each person walk in total?



11. A regular, *n*-sided polygon has 18-degree exterior angles. Find the integer *n*.

1. Draw a triangle *ABC*, and let *AM* and *BN* be two of its medians, which intersect at *G*. Extend AM to the point P that makes GM = MP. Prove that PBGC is a parallelogram.



2. Triangle FLB has a perimeter of 26 and BF = $\frac{1}{2}$ LB. The midsegment parallel to LB = 4. Find the lengths of the three sides of this triangle.

3. In the figure at right, it is given that *ABCD* and *PBQD* are parallelograms. Which of the numbered angles must be the same size as the angle numbered 1? Give a reason for each angle.

4. Triangle *PQR* has a right angle at *P*. Let *M* be the midpoint of *QR*. Draw the altitude from P to QR and let *F* be the point where that altitude meets *QR*. Given that angle *FPM* is 18 degrees, find the sizes of angles *Q* and *R*.

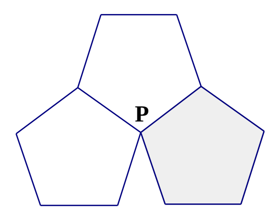
5. Given that *ABCDEFG. . .* is a regular *n*-sided polygon, with angle *CAB* = 12 degrees, find *n*.

6. *Midsegment (Midline) Theorem:* State the properties of the segment that connects the midpoints of two sides of a triangle.

7. Draw triangle *ABC* so that angles *A* and *B* are both 42 degrees. Why should *AB* be longer than *BC*? Why do you think so?

8. (Continuation) Extend *CB* to *E*, so that *CB* = *BE*. Mark *D* between *A* and *B* so that *DB* = *BC*, then draw the line *ED*, which intersects *AC* at *F*. Find the size of angle *CFD*.

9. The diagram at right shows three congruent regular pentagons that share a common vertex *P*. The three polygons do not quite surround *P*. Find the size of the uncovered acute angle at *P*.



10. (Continuation) If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the value of *n* that makes the three polygons fit perfectly.

11. How can one tell whether a given quadrilateral is a parallelogram? In other words, how much evidence is needed to be sure of such a conclusion? Hint: There are six ways to definitively show that a quadrilateral is a parallelogram.

12. You are given a square *ABCD* and midpoints *M* and *N* are marked on *BC* and *CD*, respectively. Draw *AM* and *BN*, which meet at *Q*. Find the size of angle *AQB*.

13. In triangle ABC, $\angle A = 100°$ and $\angle B = 57°$. List the sides of the triangle in order of increasing length. Justify your answer.

1. In mathematics, a *counterexample* is used to show that a statement is false. Can you find a counterexample to show that the following statement is false? If $ab = 5$ then either $a = 1$ and $b = 5$ OR $b = 1$ and $a = 5$.

2. Mark *Y* inside regular pentagon *PQRST*, so that *PQY* is equilateral. Is *RYT* straight? Explain.

3. Suppose that triangle *ABC* has a right angle at *B*, that *BF* is the altitude drawn from *B* to *AC*, and that *BN* is the median drawn from *B* to *AC*. Find angles *ANB* and *NBF*, given that angle *C* is 42 degrees.

4. The midpoints of the sides of a triangle are M(3, -1), N(4, 3), and P(0, 5). Find coordinates for the vertices of the triangle.

5. We have discussed medians, perpendicular bisectors, altitudes, midsegments and angle bisectors of triangles.
   (a) Which of these *must* go through the vertices of the triangle?
   (b) Is it possible for a median to also be an altitude? Explain.
   (c) Is it possible for an altitude to also be an angle bisector? Explain.
   (d) Is it possible for a midsegment to be a median? Explain.
   (e) Is it possible for a perpendicular bisector to be an altitude?

6. The diagonals of a rhombus have lengths 18 and 24. How long are the sides of the rhombus?

7. A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is *isosceles*. Make a diagram of an isosceles trapezoid whose sides have lengths 7 in, 10 in, 19 in, and 10 in. Find the *altitude* of this trapezoid (the distance that separates the parallel sides), then find the enclosed area.

8. If a quadrilateral is a rectangle, then its diagonals have the same length. What is the converse of this true statement? Is the converse true? If you claim it is true, attempt to prove your claim. If you believe it is not true, find a counterexample.

9. The diagonals of a parallelogram always bisect each other. Is it possible for the diagonals of a trapezoid to bisect each other? You may use *Proof by Contradiction* here.

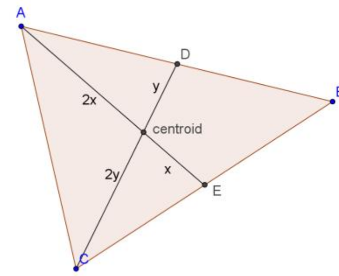
10. A trapezoid has a 60-degree angle and a 45-degree angle. What are the other angles?

11. An *n*-sided polygon has the property that the sum of the measures of its exterior angles is equal to the sum of the measures of its interior angles. Find *n*.

12. Trapezoid *ABCD* has parallel sides *AB* and *CD*, a right angle at *D*, and the lengths *AB* = 15, *BC* = 10, and *CD* = 7. Find the length *DA*.

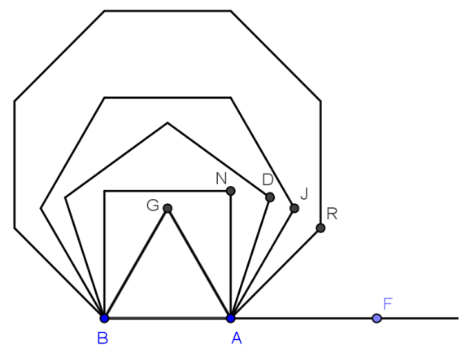
1. A trapezoid has a 60-degree angle and a 120-degree angle. What are the other angles?

2. An equilateral triangle has sides of length 6.
(a) Find the lengths of the medians of the triangle. Leave your answers in simplest radical form.
(b) The medians intersect at the centroid of the triangle. A special property that the centroid holds is that it splits each median into 2:1 ratios (see diagram). How far is the centroid from each of the vertices of the triangle in this problem (the diagram is just an explanation of this property)?



3. (Continuation) The sides of a triangle have lengths 9, 12, and 15. (This is a special triangle!). Find the lengths from each vertex to the centroid.

4. In the diagram at right, AGB is an equilateral triangle, $\overline{AN}$ is the side of a square. $\overline{AD}$ is the side of a regular pentagon, $\overline{AJ}$ is the side of a regular hexagon, and $\overline{AR}$ is the side of a regular octagon. $\overline{AB}$ is a side shared by all of the regular polygons. Find
(a) $\angle GAF$ **(b)** $\angle NAR$ **(c)** $\angle JAF$ **(d)** $\angle GAJ$



5. A line of positive slope is drawn so that it makes a 60-degree angle where it intersects the *x*-axis. What is the slope of this line?

6. What can be said about quadrilateral $ABCD$ if it has supplementary consecutive angles?

7. If $MNPQRSTUV$ is a regular polygon, then how large is each of its interior angles? If lines through the sides MN and QP are extended to meet at A then how large is angle PAN?

8. Is it possible for the sides of a triangle to be 23, 19, and 44? Explain.

9. Consider the following "trapezoids" and midlines (keeping in mind the definition of a trapezoid):
   a. A "trapezoid" with top base of length 0 and bottom base of length 10. What is the length of its midline? (In actually what is this "trapezoid" and how would find the length of its "midline"?)
   b. A "trapezoid" with top base of length 10 and bottom base of length 10. What is the length of its "midline"?
   c. A trapezoid with top base of length 6 and bottom base of length 10. From the following two examples, how might you conjecture to find the length of the midline? Try to justify your answer.

1. Let *ABCD* be a parallelogram, with *M* the midpoint of *DA*, and diagonal *AC* of length 36. Let *G* be the intersection of *MB* and *AC* and draw in diagonal *DB*. What is the length of *AG*?

2. Draw the lines $y = 0$, $y = \frac{1}{2}x$, and $y = 3x$. Use your protractor, GeoGebra to measure the angle that the line $y=\frac{1}{2}x$ makes with the x-axis. Using your intuition, make a guess what the angle is that the line y=3x makes with the x-axis. Now measure it. Explain your conclusions.

3. The parallel sides of trapezoid *ABCD* are *AD* and *BC*. Given that sides *AB*, *BC*, and *CD* are each half as long as side *AD*, find the size of angle *D*.

4. Dana buys a piece of carpet that measures 20 square yards. Will Dana be able to completely cover a rectangular floor that measures 12 ft. 4 in. by 16 ft. 8 in.?

5. The altitudes of an equilateral triangle all have length 12 cm. How long are its sides?

6. Given a triangle, you know the following result: *The point where two medians intersect (the centroid) is twice as far* ***from one end*** *of a median as it is* ***from the other*** *end of the same median*. Improve this statement so that the reader knows which end of the median is which.

7. Suppose that *ABCD* is a square with *AB* = 6. Let *N* be the midpoint of *CD* and *F* be the intersection of *AN* and *BD*. What is the length of *AF*? Hint: Look at triangle ADC.

8. The sides of a square have length 10. How long are the diagonal of the square? Keep your answer in simplest radical form. What would your answer be if the side had been 6?

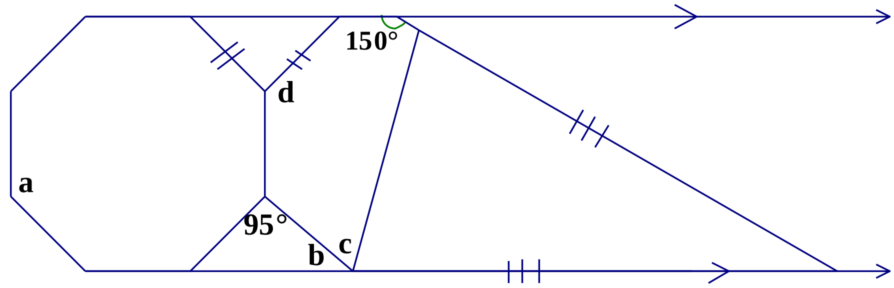
9. Triangle *PQR* is isosceles, with *PQ* = 13 = *PR* and *QR* = 10. Find the distance from *P* to the centroid of *PQR*. Find the distance from *Q* to the centroid of *PQR*.

10. In triangle *ABC*, let *M* be the midpoint of *AB* and *N* be the midpoint of *AC*. Suppose that you measure *MN* and find it to be 7.3 cm long. How long would *BC* be, if you measured it? What should be true about angles *AMN* and *ABC*?

11. A triangle with sides of 5, 12, and 13 must be a right triangle. Keeping the lengths of the legs constant, how would the triangle change if the hypotenuse was lengthened to 15? 17? 19?

12. (Continuation) Under what conditions would a triangle with two sides of 5 and 12 be obtuse?

1. In the diagram at the right, the octagon is regular. Find the measures of the angles labeled a, b, c, and d.



2. Mark A = (0, 0) and B = (10, 0) on your graph paper or in GeoGebra and use your protractor to draw the line of positive slope through A that makes a 25-degree angle with AB. Calculate (approximately) the slope of this line by making suitable measurements.

3. (Continuation) Turn on your calculator, press the MODE button, and select the *Degree* option for angles. Return to the home screen, and press the TAN button to enter the expression TAN(25), then press ENTER. You should see that the display agrees with your answer to the preceding item.

4. What would happen to a 5-12-13 triangle if the hypotenuse was shortened to 12 while the sides of 5 and 12 stayed constant? What if the hypotenuse were shortened to 7? What about 5?

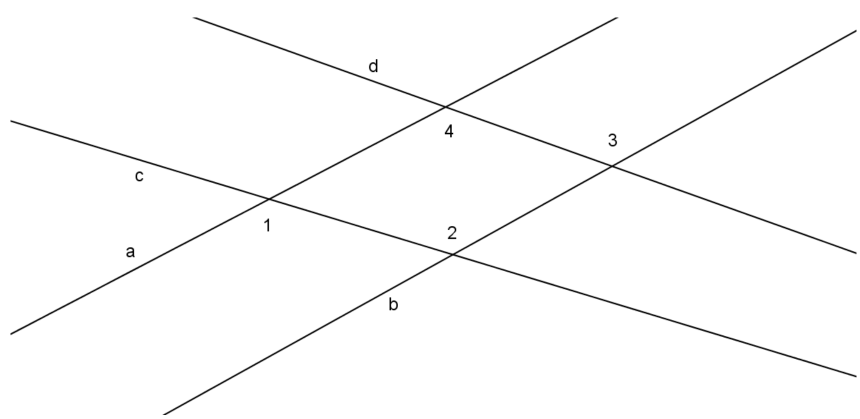
5. (Continuation) Under what conditions would a triangle with two sides of 5 and 12 be acute?

6. Given A = (0, 6), B = (-8, 0), and C = (8, 0), find coordinates for the circumcenter of triangle ABC.

7. Rearrange the letters of *doctrine* to spell a familiar mathematical word.

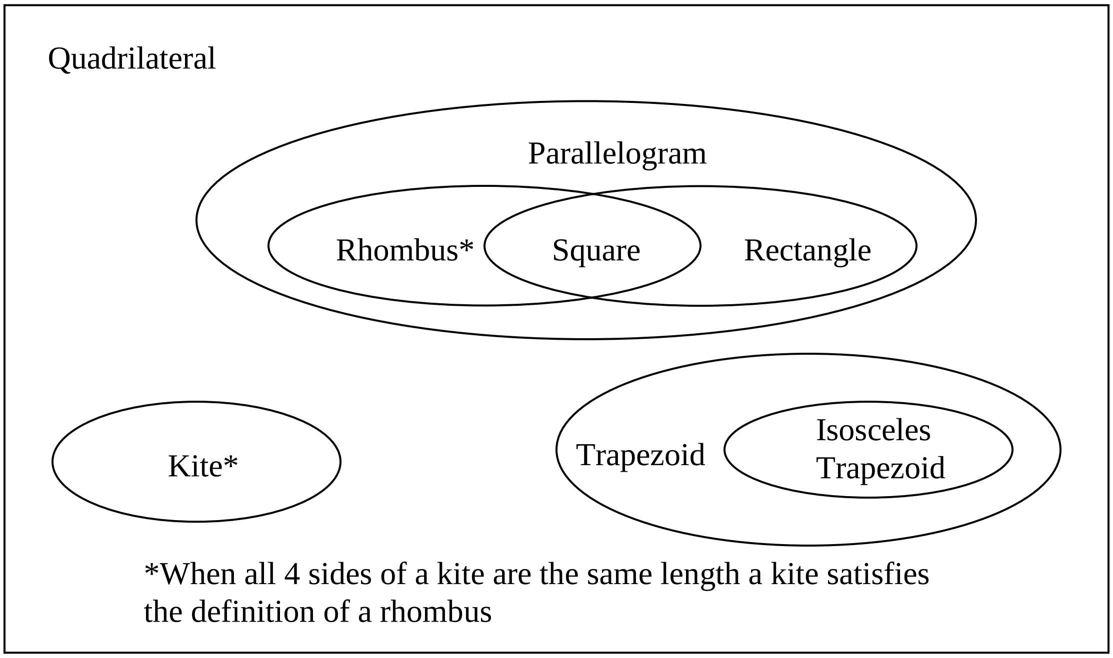
8. In the following diagram, if you know the given angles are congruent, which lines can you say are parallel?

   i. If $\angle 1 \cong \angle 2$?
   ii. If $\angle 2 \cong \angle 3$?
   iii. If $\angle 4 \cong \angle 2$?



Be sure to justify your conjectures.

Justify the following Venn diagram and check the properties that each type of quadrilateral holds.



Property	Parallel-ogram	Rectangle	Rhombus	Square	Kite	Trapezoid	Isosceles Trapezoid
Opposite sides are parallel							
Opposite sides are congruent							
Exactly one pair of opposite sides is parallel							
Opposite angles are congruent							
Exactly one pair of angles is congruent							
Consecutive angles are supplementary							
Base angles are congruent							
Diagonals bisect each other							
Diagonals are congruent							
Diagonals are perpendicular							
Diagonals bisect opposite angles							
Exactly one diagonal is the perpendicular bisector of the other							

GeoGebra Lab #15 The Three Parallels Theorem

A. Using GeoGebra, draw a horizontal **line** AB and two points, *C* and *E* below AB. Make sure that C is above E (closer to AB).
B. Create lines through C and E that are parallel to AB using the Parallel Lines tool.
C. Draw a transversal (make sure that it's a line, not a segment) by clicking above the parallel lines and finish by clicking below the parallel lines so that the transversal cuts through the three parallel lines. Construct the intersection points, using the Intersect Two Objects tool, of the transversal and the three parallel lines, from bottom to top so that the intersection on the line through E, is G, the point on the line through C, is H, and the intersection of AB and the transversal is I.
D. With the Distance tool, in the Measurement toolbox, measure the distance between points G and H, and H and I. Move the transversal by dragging a point on it that is not an intersection. What happens to these distances when you drag the transversal horizontally thereby changing its slope?
E. In the input bar, type ratio = distance[I,H]/distance[H,G], to create a number that is the ratio of these two segments. Then, in a text box, type ratio = and select ratio from the Objects drop-down list. What happens to the ratio of these distances when you drag the transversal horizontally in the sketch?
F. Draw another transversal, with a different slope than the first, and construct the intersection points with AB, *CD* and EF, called L, M, and N respectively.
G. With the Distance Tool, find the length of segments LM and MN. Then, in the input bar, type in ratio2 = distance[L,M]/distance[M,N]. What conclusions can you draw from this information?

1. A line drawn parallel to the side *BC* of triangle *ABC* intersects side *AB* at *P* and side *AC* at *Q*. The measurements *AP* = 3.8 in, *PB* = 7.6 in, and *AQ* = 5.6 in are made. If segment *QC* were now measured, how long would it be?

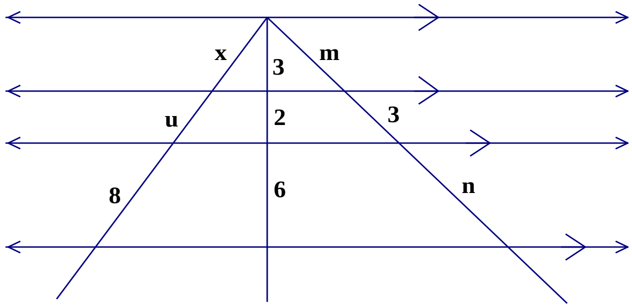
2. Given regular hexagon *BAGELS*, show that *SEA* is an equilateral triangle.

3. The parallel bases of a trapezoid have lengths 12 and 18 cm. Find the lengths of the two segments into which the *midline* of the trapezoid is divided by one diagonal. (Hint: look for some triangles!)

4. How does the value of the tangent of an angle change as an angle increases from 0 to 90 degrees? Is there a direct relationship between the slope and the angle measure?

5. Standing 50 meters from the base of a fir tree, Rory measured an *angle of elevation* of 33° to the top of the tree with a protractor. The angle of elevation is the angle formed by the horizontal ground and an ant's line-of-sight ray to the top of the tree. How tall was the tree?

1. The *Three Parallels* Theorem: If a transversal cuts three parallel lines in a given ratio, then any transversal cuts off segments of the same ratio. Use this to solve for x, u, m, and n in the following diagram.



2. You are building a tent for a sleepover. You have a 36 ft tarp that you are going to use to hang it over a clothesline symmetrically. Looking at it from the side, what are the possible base lengths of the cross-section of your tent? What realistic assumptions are you making?

3. Standing on a cliff 380 meters above the sea, Pocahontas sees an approaching ship and measures its *angle of depression*, obtaining 9 degrees. How far from shore is the ship?

4. (Continuation) Now Pocahontas sights a second ship beyond the first. The angle of depression of the second ship is 5 degrees. How far apart are the ships?

5. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Measure angle *CAB* with your protractor (you must use a protractor for this). What is the slope of *AC*? Use your calculator to compare the tangent of the angle you measured with the slope. By trial-and-error, find an angle that is a better approximation of the measure of angle *CAB*.

6. (Continuation) On your calculator, ENTER the expression $\text{TAN}^{-1}(0.75)$. Compare this answer with the approximation you obtained for the measure of angle *CAB*. What does the TAN^{-1} button do? (TAN^{-1} is said as "inverse tangent.")

7. When the Sun has risen 32 degrees above the horizon, Sandeep casts a shadow that is 9 feet 2 inches long. How tall is Sandeep, to the nearest inch?

8. A five-foot Deerfield student casts an eight-foot shadow. How high is the Sun in the sky? This is another way of asking for the angle of elevation of the Sun.

9. An isosceles trapezoid has sides of lengths 9, 10, 21, and 10. Find the distance that separates the parallel sides then find the length of the diagonals. Finally, find the angles of the trapezoid, to the nearest tenth of a degree.

10. One day at the beach, Kelly flies a kite, whose string makes a 37-degree elevation angle with the ground. Kelly is 130 feet from the point directly below the kite. How high above the ground is the kite, to the nearest foot?

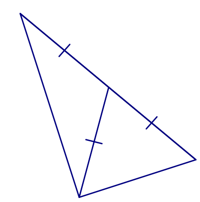
In the following list of true statements, find **(a)** the four pairs of statements whose converses are also in the list; **(b)** the statement that is a definition; **(c)** the statement whose converse is false; **(d)** the Sentry Theorem;**(e)** the Midsegment Theorem;**(f)** The Three Parallels Theorem; **(g)** The Centroid Theorem. Note: Not all statements are used.

If a quadrilateral has two pairs of parallel sides, then its diagonals bisect each other.

A. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral must be a parallelogram.

B. If a quadrilateral is equilateral, then it is a rhombus.

C. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



D. If a quadrilateral has two pairs of equal adjacent sides, then its diagonals are perpendicular.

E. If one of the medians of a triangle is half the length of the side to which it is drawn, then the triangle is a right triangle.

F. If a segment joins two of the midpoints of the sides of a triangle, then is parallel to the third side, and is half the length of the third side.

G. Both pairs of opposite sides of a parallelogram are congruent.

H. The sum of the exterior angles of any polygon – one at each vertex – is 360 degrees.

I. The median drawn to the hypotenuse of a right triangle is half the length of the hypotenuse.

J. If two lines are intersected by a transversal so that alternate interior angles are equal, then the lines must be parallel.

K. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is in fact a parallelogram.

L. If two opposite sides of a quadrilateral are both parallel and equal in length, then the quadrilateral is a parallelogram.

M. If three parallel lines intercept equal segments on one transversal, then they intercept equal segments on every transversal.

N. Both pairs of opposite angles of a parallelogram are congruent.

O. The medians of any triangle are concurrent at a point that is two thirds of the way from any vertex to the midpoint of the opposite side.

P. An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

1. Hexagon *ABCDEF* is regular. Prove that segments *AE* and *ED* are perpendicular.

2. What angle does the line $y = \frac{2}{5}x$ make with the *x*-axis?

3. Suppose that *PQRS* is a rhombus, with *PQ* = 12 and a 60-degree angle at *Q*. How long are the diagonals *PR* and *QS*?

4. Given a rectangular card that is 5 inches long and 3 inches wide, what does it mean for another rectangular card to have the *same shape*? Describe a couple of examples.

5. *The Varignon quadrilateral*. A quadrilateral has diagonals of lengths 8 and 10. The midpoints of the sides of this figure are joined to form a new quadrilateral. What is the perimeter of the new quadrilateral? What is special about it?

6. The hypotenuse of a right triangle is twice as long as one of the legs. How long is the other leg? What is the size of the smallest angle?

7. What are the angle sizes in a trapezoid whose sides have lengths 6, 20, 6, and 26?

8. Atiba wants to measure the width of the Deerfield River. Standing under a tree *T* on the river bank, Atiba sights a rock at the nearest point *R* on the opposite bank. Then Atiba walks to a point *P* on the river bank that is 50.0 meters from *T*, and makes *RTP* a right angle. Atiba then measures *RPT* and obtains 76.8 degrees. How wide is the river?

9. The legs of an isosceles right triangle have a length of *s*. What is the length of the hypotenuse with respect to *s*?

10. How tall is an isosceles triangle, given that its base is 30 cm long and that both of its base angles are 72 degrees?

11. A triangle has sides in the ratio $1:2:\sqrt{3}$. Draw a triangle with this scale. What can you say about this triangle?

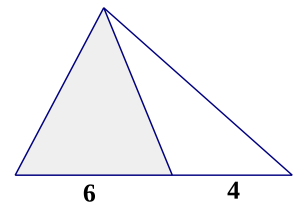
12. In Hawley State Forest near Buckland, MA there is a very old structure that was once used as a kiln for heating wood into coal. Here is a picture of the triangular structure (with Mr. Day, Mr. Thiel, Dr. Hagamen and Mr. Friends in front with their mountain bikes). If the outline of this structure is approximately an equilateral triangle with base of 18 feet across, to the nearest hundredth, how tall is the structure?

13. In a Deerfield Freshman class there are 105 students, and the day**:** boarder ratio is approximately 1:5.
    a. How many students in that class are boarders?
    b. How many day students would you expect to find in a freshman English class of fifteen students? Explain. What realistic assumptions are you making?

1. Find the equation of a line passing through the origin that makes an angle of 52 degrees with the *x*-axis.

2. *What are all of the special right triangles?* In geometry it is very useful to know many right triangles that have special properties. Which right triangles so far this year have been helpful in our work? Explain as many as you can think of and why they have been "special" in their own way. Be as specific as possible.

3. In the figure at right, the shaded triangle has area 15. Find the area of the unshaded triangle.



4. To the nearest tenth of a degree, how large are the congruent angles of an isosceles triangle that is exactly as tall as it is wide? (There is more than one interpretation).

5. Rectangle *ABCD* has *AB* = 16 and *BC* = 6. Let *M* be the midpoint of side *AD* and *N* be the midpoint of side *CD*. Segments *CM* and *AN* intersect at *G*. Find the length *AG*. (Hint: can G be considered a centroid of any triangle?)

6. An estate of $362880 is to be divided among three heirs, Alden, Blair, and Cary. According to the will, Alden is to get two parts, Blair three parts, and Cary four parts. How much money in dollars and cents does each heir receive?

7. What is the relationship between the length of the hypotenuse and the length of the legs in a 45-45-90 triangle?

8. The area of a parallelogram can be found by multiplying the distance between two parallel sides (the height, or altitude) by the length of either of those sides. Explain why this formula works. Draw a picture and explain why the phrase "two parallel sides" was used in this problem instead of "base."

9. Using GeoGebra, plot the points *A* = (0, 0), *B* = (4, -3), *C* = (6, 3), *P* = (-2, 7), *Q* = (9, 5), and *R* = (7, 19). Measure the angles of triangles *ABC* and *PQR*. Create ratios of the lengths of the corresponding sides. Find justification for any conclusions you make. (If you choose not to use technology, leave all answers in simplest radical form.)

10. The perimeter of a square is 36, what is the length of a diagonal of the square? How many squares can you come up with that have a perimeter of 36?

11. Draw a diagram of a right triangle ABC, with B the right angle and BK the altitude drawn to the hypotenuse. Give an argument as to why all three triangles, AKB, ABC and BKC would have the same angle measurements. Try to be as precise as possible.

1. A drawbridge has the shape of an isosceles trapezoid. The entire length of the bridge is 100 feet while the height is 25 feet. If the angle at which the bridge meets the land is approximately 60 degrees, how long is the part of the bridge that opens?

2. One figure is *similar* to another figure if the points of the first figure can be matched with the points of the second figure in such a way that corresponding distances are proportional. In other words, there is a *ratio of similarity, k*, such that every distance on the second figure is *k* times the corresponding distances on the first figure.
a. Open GeoGebra and plot the points K = (1, -3), L = (4, 1), M = (2, 3), P = (6, 5), Q = (2, 5) and R = (7, -2).
b. Is triangle KLM similar to triangle RPQ? Justify with measurements from GeoGebra.
c. Would it be correct to say that triangle MKL is similar to triangle RQP?

3. Draw a right triangle that has a 15-cm hypotenuse and a 27-degree angle. To the nearest tenth of a cm, measure the side opposite the 27-degree angle, and then express your answer as a percentage of the length of the hypotenuse. Compare your answer with the value obtained from your calculator when you enter SIN 27 in degree mode.

4. (Continuation) Repeat the process on a right triangle that has a 10-cm hypotenuse and a 65-degree angle. Try an example of your choosing. Write a summary of your findings.

5. In triangle *ABC*, points *M* and *N* are marked on sides *AB* and *AC*, respectively, so that $AM:AB = 1:3 = AN:AC$. Why are segments *MN* and *BC* parallel?

6. One way to find the area of a trapezoid is by multiplying its altitude (the distance between the parallel sides) by the average of the bases ($A = h \times \frac{1}{2}(b_1 + b_2)$). What is a geometric way to justify this formula? In other words, how could you redraw the trapezoid so that the $\frac{1}{2}(b_1 + b_2)$ represents something with respect to the trapezoid?

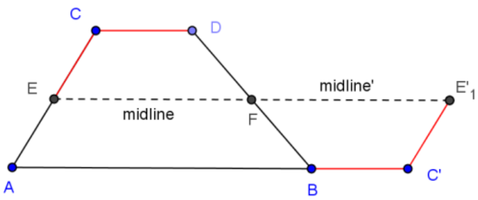
7. What is the length of an altitude of an equilateral triangle with perimeter 36?

8. *The Right Triangle Similarity Theorem*. Draw an arbitrary right triangle. Why does the altitude drawn from the right angle vertex to the hypotenuse create three right triangles? Try to justify your answer in a few sentences and a diagram.

9. To actually draw a right triangle that has a 1-degree angle and measure its sides accurately is difficult. To get the sine ratio for a 1-degree angle, however, there is an easy way – just use your calculator. Is the ratio a small or large number? How large can a sine ratio be?

10. If two sides of a triangle are 5 and 10, what is the range of values for the third side?

1. We have discussed the formula for the area of a trapezoid as $A = h \times \frac{1}{2}(b_1 + b_2)$ where you can multiply the height by the average of the bases. Find another way to represent the area of a trapezoids by looking at this diagram:



2. What is the size of the acute angle formed by the x-axis and the line $3x + 2y = 12$?

3. If triangle ABC has a right angle at C, the ratio $AC : AB$ is called the *sine ratio of angle B*, or simply the *sine of B*, and is usually written *sin B*. What should the ratio $BC : AB$ be called? Without using your calculator, can you predict what the value of the sine ratio for a 30-degree angle is? How about the sine ratio for a 60-degree angle?

4. Write an equation using the distance formula that says that $P = (x, y)$ is 5 units from (0, 0). Plot several such points. What is the configuration of all such points called? How many are lattice points?

5. (Continuation) Explain how you could use the Pythagorean Theorem to obtain the same result.

6. What is the length of a side of an equilateral triangle whose altitude is 16? How do you describe the length of the side in terms of the altitude?

7. When you take the sine of 30 degrees using your calculator you get 0.5. What do you think $\text{SIN}^{-1}(0.5)$ is? Use your calculator to test your conjecture. Find SIN^{-1} (0.3) and $\text{SIN}^{-1}\left(\frac{3}{5}\right)$.

   What do these values represent?

8. When triangle ABC is similar to triangle PQR, with A, B, and C corresponding to P, Q, and R, respectively, it is customary to write $ABC \sim PQR$. Suppose *that* $AB = 4$, $BC = 5$, $CA = 6$, and $RP = 9$. Find PQ and QR.

9. One triangle has sides that are 5 cm, 7 cm, and 8 cm long; the longest side of a similar triangle is 6 cm long. How long are the other two sides?

10. Jasper is driving along a highway that is climbing a steady 9-degree angle. After driving for two miles up this road, how much altitude has Jasper gained?

11. (Continuation) How far must Judy travel in order to gain a mile of altitude?

12. The floor plan of a house is drawn with a ratio of 1/8 inch = 1 foot. On the plan, the kitchen measures 2 in. by 2 1/4 in. What are the dimensions of the kitchen?

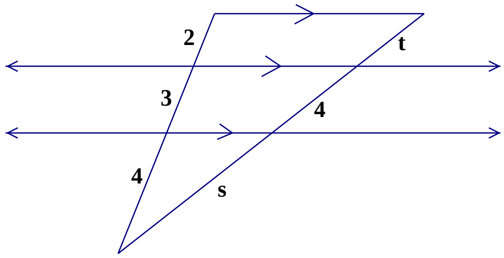
13. If an altitude is also the side of a triangle, what do you know about the triangle?

1. If two polygons are similar, explain why the corresponding angles are the same size. What is the converse of this statement? Is it true?

2. To the nearest tenth of a degree, find the sizes of the acute angles in a 5-12-13 triangle and in a 9-12-15 triangle. Put these two triangles together (side-by-side) by matching up the sides that are 12 long. By doing this, answer the question: what are the angles in a 13-14-15 triangle?

3. *AA Similarity Postulate:* If two corresponding angles of a triangle are equal in size to the angles of another triangle, then the triangles are similar. Justify this statement. State the converse of this statement. Is it true?

4. In the diagram at the right, find *t* and *s*.



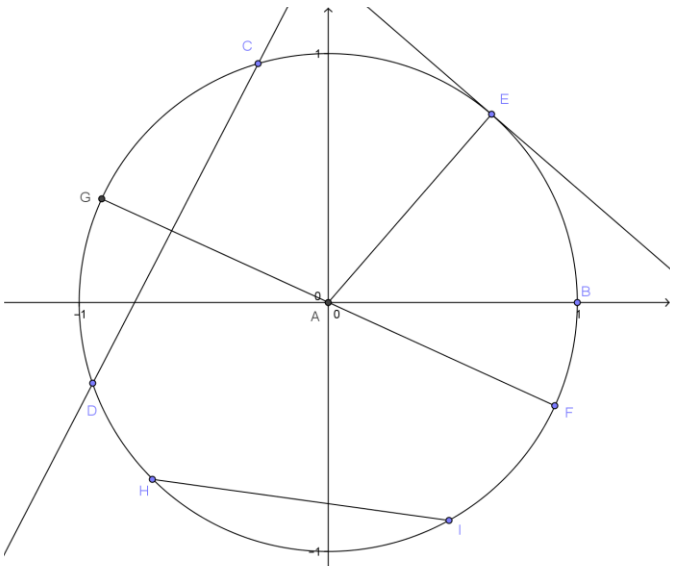
5. Is it possible to draw a triangle with the given sides? If it is possible, state whether it is acute, right, or obtuse. If it is not possible, say no and sketch why.
   (a) 9, 6, 5 **(b)** $3\sqrt{3}$, 9, $6\sqrt{3}$ **(c)** 8.6, 2.4, 6.2

GeoGebra Lab #17

Discovery of π *:* The Greek scholar Archimedes discovered a constant relationship between the circumference of a circle and its *diameter*. He called this constant π.
Describe the circumference with respect to its diameter. With respect to its radius. Create a sketch to validate this relationship. If you have time, confirm the relationship of the circle's area to its radius with respect to π as well.

6. The area of an equilateral triangle with *m*-inch sides is 8 square inches. What is the area of a regular hexagon that has *m*-inch sides?

7. In the following diagram choose which you think fulfills the definitions of the terms that relate to a circle from your previous experience with circles, common sense or resources that you have - *radius, diameter, secant line, chord, tangent line, point of tangency, minor arc, major arc, central angle*



1. (referring to previous diagram) It is interesting that the line that intersects a circle at one point is named a *tangent line* and that is also the name of the function that mathematicians created to relate an angle to the slope created with the positive x-axis. Consider the diagram above and the triangle APO. Can you give an argument for why triangle APO must be a right triangle if AP is a tangent line? What is the relationship between $\angle AOP$ and the length of AP (to make the question easier, assume the radius of the circle is 1, as in the picture). Why do you think they named AP a tangent line?

2. A parallelogram has 10-inch and 18-inch sides and an area of 144 square inches.
   (a) Find the altitude between the 18-inch sides?
   (b) Find the altitude between the 10-inch sides? (notice that it is not necessarily the same)
   (c) Find the angles of the parallelogram?

3. Write an equation that describes all the points on the circle whose *center* is at the origin and whose *radius* is **(a)** 13; **(b)** 6; **(c)** r.

4. If the lengths of the midsegments of a triangle are 3, 4, and 5, what is the perimeter of the triangle?

5. When moving a lot of plates, the dining hall packs our round plates in square boxes with a perimeter of 36 inches. If the plates fit snugly in one stack in the box, one plate per layer, what is the circumference of each plate?

6. (Continuation) Each dining hall saucer has a circumference of 12.57 in. Can four saucers fit on a single layer in the same square box? Justify your answer.

7. Graph the circle whose equation is $x^2 + y^2 = 64$. What is its radius? What do the equations $x^2 + y^2 = 1$, $x^2 + y^2 = 40$, and $x^2 + y^2 = k$ all have in common? How do they differ?

8. Taylor lets out 120 meters of kite string then wonders how high the kite has risen. Taylor is able to calculate the answer after using a protractor to measure the 63-degree angle of elevation that the string makes with the ground. How high is the kite, to the nearest meter? What (unrealistic) assumptions did you make in answering this question?

9. Find the sine of a 45-degree angle. Use your calculator *only to check your answer*.

10. (Continuation) Calculate the areas of triangles *ABC* and *A'B'C'*. Do you notice any pattern in your answers?

11. If the central angle of a slice of pizza is 36 degrees, how many pieces are in the pizza?

12. (Continuation) A 12 inch pizza is evenly divided into 8 pieces. What is the length of the crust of one piece?

1. The vertices of triangle *ABC* are $A = (-5, -12)$, $B = (5, -12)$, and $C = (5, 12)$. Confirm that the circumcenter of *ABC* lies at the origin. What is the equation for the *circumscribed circle*?

2. If the sides of a triangle are 13, 14, and 15 cm long, then the altitude drawn to the 14-cm side is 12 cm long. How long are the other two altitudes? Which side has the longest altitude?

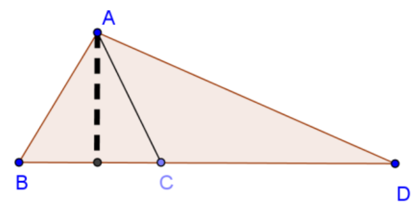
3. (Continuation) How long are the altitudes of the triangle if you double the lengths of its sides?

4. Let $A = (6, 0)$, $B = (0, 8)$, $C = (0, 0)$. In triangle *ABC*, let *F* be the point of intersection of the altitude drawn from *C* to side *AB*.
   a. Explain why the angles of triangles *ABC*, *CBF*, and *ACF* are the same.
   b. Find coordinates for *F* and use them to calculate the exact lengths *FA*, *FB*, and *FC*.
   c. Compare the sides of triangle *ABC* with the sides of triangle *ACF*. What do you notice?

5. What happens to the area of a triangle if its dimensions are doubled?

6. A rectangular sheet of paper is 20.5 cm wide. When it is folded in half, with the crease running parallel to the 20.5-cm sides, the resulting rectangle is the same shape as the unfolded sheet. Find the length of the sheet, to the nearest tenth of a cm. (In Europe, the shape of notebook paper is determined by this similarity property).

7. Write an equation that describes all the points *P*(x,y) that are 5 units away from the point *C*(1,-4). What set of points does this describe?



8. What is the ratio of the areas of ΔABC and ΔADC in the following diagram if BC=5 and CD=8?

9. A regular polygon is *inscribed* in a circle – all vertices of the polygon lie on the circle. (We can also say that the circle is *circumscribed* around the polygon). Go to the website http://www.mathopenref.com/polygoncircumcircle.html and increase the number of sides of the regular polygon. What happens to the polygon as the number of sides increases?

10. Sketch the circle whose equation is $x^2 + y^2 = 100$. Using the same system of coordinate axes, graph the line $x + 3y = 10$, which should intersect the circle twice – at $A = (10, 0)$ and at another point *B* in the second quadrant. Estimate the coordinates of *B*. Now use technology to find them exactly. Segment *AB* is called a *chord* of the circle.

11. (Continuation) Find coordinates for a point *C* on the circle that makes chords *AB* and *AC* have equal length.

1. A 25 foot ladder is leaning against a wall and makes a 57.32° angle with the floor. How far from the wall is the base of the ladder? Without using trigonometry try to predict how far from the floor a step on the ladder would be that is half way up the ladder? 8 feet up the ladder? Now use trigonometry to confirm your answers.

2. What is the radius of the smallest circle that surrounds a 5-by-12 rectangle?

3. Baking Powder is made up of Baking Soda, Cream of Tartar and sometimes Corn Starch in a ratio of 1:2:1. If you need 2 Tablespoons of Baking Powder, how much of each ingredient do you need?

4. Without doing any calculation, what can you say about the tangent of a k-degree angle, when k is a number between 90 and 180? Explain your response, then check with your calculator.

5. Ask your calculator for the sine of a 56-degree angle, then for the cosine of a 34-degree angle. Ask your calculator for the sine of a 23-degree angle, then for the cosine of a 67-degree angle. The word *cosine* is an abbreviation of *sine of the complement*. Explain the terminology.

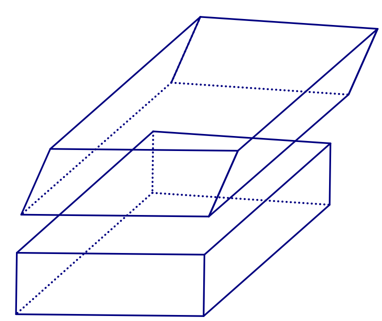
6. (Continuation) How can you represent the cosine of an angle in terms of a ratio?

7. A right triangle has a 123-foot hypotenuse and a 38-foot leg. To the nearest tenth of a degree, what are the sizes of its acute angles?

8. The line $y = x + 2$ intersects the circle $x^2 + y^2 = 10$ in two points. Call the third quadrant point R and the first-quadrant point E, and find their coordinates on GeoGebra. Let D be the point where the line through R and the center of the circle intersects the circle again. The chord DR is an example of a *diameter*. Show that triangle RED is a right triangle.

9. To the nearest tenth of a degree, find the angles of the triangle with vertices (0, 0), (6, 3), and (1, 8). Use your protractor to *check* your calculations, and explain your method.

10. Playing cards measure 2.25 inches by 3.5 inches. A full deck of fifty-two cards is 0.75 inches high. What is the volume of a deck of cards? If the cards were uniformly shifted (turning the bottom illustration into the top illustration), would this volume be affected?



11. In a right triangle, the 58-cm hypotenuse makes a 51-degree angle with one of the legs. To the nearest tenth of a cm, how long is that leg? Once you have the answer, find some other ways to calculate the length of the other leg. They should all give the same answer, of course.

1. An equilateral triangle ABC is inscribed in a circle centered at *O*. The portion of the circle that lies above chord *AB* is called an *arc*. In geometry, arcs have to types of measurement, arclength (which is length measurement) and arc angle which is an angular size in degrees. If $AB = BC = AC$, what is the angular size of AB? AB is called a *minor arc* and ACB is called a *major arc*. Why do you think they are called this? How are AB and ACB related?

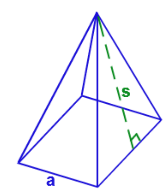
2. (Continuation) *Some Terminology:* A *central angle* is an angle whose vertex is at the center of a circle and whose sides are radii. What is the measure of angle *AOB*? What is the relationship between a central angle and the arc it intercepts?

3. What is the angular size of an arc that a diameter intercepts? This arc is called a *semicircle.*

4. If the ratio of the areas of two triangles is 18: 8, what is the ratio of similarity?

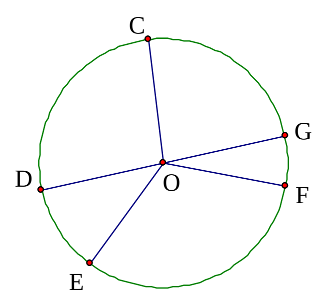
5. Draw a circle and label one of its diameters *AB*. Choose any other point on the circle and call it *C*. What can you say about the size of angle *ACB*? Does it depend on which *C* you chose? Justify your response.

6. A square *pyramid* is a pyramid with a square base and four triangular lateral faces. The slant height, s, is the distance from the vertex of the pyramid along a *lateral face* to the midpoint of a base edge. If the slant height is 10 and an edge of the square is 12, what is the altitude of this pyramid?



Square Pyramid

7. A regular pentagon can be dissected into 5 isosceles triangles whose vertex angle is at the center of the pentagon. The height of the triangles is 10 cm. Find the area of this pentagon.



8. Circle *O* has diameter *DG* and central angles $COG = 86^\circ$, $DOE = 25^\circ$, and $FOG = 15^\circ$. Find the angular size of minor arcs CG, CF, EF and major arc DGF.

9. If two chords in the same circle have the same length, then their minor arcs have the same length, too. True or false? Explain. What about the converse statement? Is it true? Why?

10. The circle $x^2 + y^2 = 25$ goes through $A = (5, 0)$ and $B = (3, 4)$. To the nearest tenth of a degree, find the angular size of the minor arc *AB*. Can you find another arc with the same angular size?

11. (Continuation) Let point O be (0, 0), now find the measure of angle OBA.

12. The sides of a triangle are found to be 10 cm, 14 cm, and 16 cm long, while the sides of another triangle are found to be 15 in, 21 in, and 24 in long. On the basis of this information, what can you say about the angles of these triangles?

1. Many people think that pyramids only exist in Egypt, but there are pyramids in China as well. The Great White Pyramid is standing in Shaanxi, China and has an approximate square base with edges of 357 meters. Its height was 76 meters but has fallen since it was built. (For our purposes we will assume this is a pyramid with a vertex, currently this pyramid has a flat top!) Calculate the slant height of The Great White Pyramid.

2. In mathematical discussion, a *right prism* is defined to be a solid figure that has two parallel, congruent polygonal bases, and rectangular *lateral faces*. How would you find the volume of such a figure? Explain your method.

3. In triangle *ABC*, it is given that angle *BCA* is right. Let $a = BC$, $b = CA$, and $c = AB$. Using *a*, *b*, and *c*, express the sine, cosine, and tangent ratios of acute angles *A* and *B*.

4. The sine of a 38-degree angle is some number *r*. Without using your calculator, you should be able to identify the angle size whose cosine is the same number *r* from what you know about right triangles.

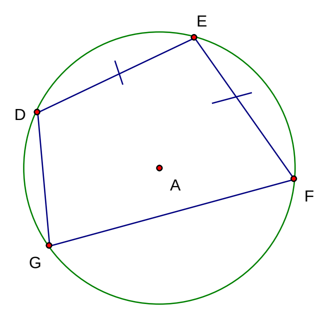
5. On a circle whose center is *O*, using your protractor or GeoGebra, mark points *P* and *A* so that minor arc *PA* is a 46-degree arc. What does this tell you about angle *POA*? Extend *PO* to meet the circle again at *T*. What is the size of angle *PTA*? This angle is *inscribed* in the circle, because its vertex is on the circle. The arc *PA* is *intercepted* by the angle *PTA*. Make a conjecture about arcs intercepted by inscribed angles.

6. (Continuation) Confirm your conjecture about inscribed angles and the arcs they intercept using GeoGebra. To measure the arc, select one endpoint, then the center of the circle, then the final endpoint, making sure to go clockwise with respect to the vertices.

7. Given lengths of three sides of an isosceles triangle, describe the process you would use to calculate the sizes of its angles. How does your method compare to your classmates'?

8. If *P* and *Q* are points on a circle, then the center of the circle must be on the perpendicular bisector of chord *PQ*. Explain. Which point on the chord is closest to the center? Why?

9. Suppose that *MP* is a diameter of a circle centered at *O*, and *Q* is any other point on the circle. Draw the line through *O* that is parallel to *MQ*, and let *R* be the point where it meets minor arc *PQ*. Prove that *R* is the midpoint of minor arc *PQ*.



10. Quadrilateral DEFG in inscribed in circle A. $ED \cong EF$, $\angle E = 100°$ and $\angle F = 70°$. Find the measures of the four minor arcs.

11. Why are there many isosceles triangles in a circle? What do they have in common? How are they possibly different?

12. Given that triangle *ABC* is similar to triangle *PQR*, write the three-term proportion that describes how the six sides of these figures are related.

1. A circle of radius 5 is **circumscribed** about a regular hexagon. By any method, find the area of the hexagon.

2. Draw a circle with a 2-inch radius, mark four points randomly (not evenly spaced) on it, and label them consecutively *G*, *E*, *O*, and *M*. Measure angles *GEO* and *GMO*. Could you have predicted the result? Name another pair of angles that would have produced the same result.

3. A regular hexagon has an **inscribed** circle of radius 4. Find the area of the hexagon.

4. A circular park 80 meters in diameter has a straight path cutting across it. It is 24 meters from the center of the park to the closest point on this path. How long is the path?

5. Triangle *ABC* is inscribed in a circle. Given that *AB* is a 40-degree arc and $\angle ABC$ is a 50-degree angle, find the sizes of the other arcs and angles in the figure.

6. Using GeoGebra, find the intersections of the line $y = 2x - 5$ with the circle $x^2 + y^2 = 25$. Then use technology to find the intersections of the line $-2x + 11y = 25$ with the same circle. Show that these lines create chords of equal length when they intersect the circle. With your protractor, measure the inscribed angle formed by these chords.

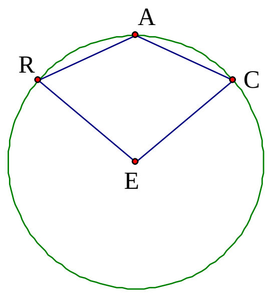
7. (Continuation) Using trigonometry, calculate the angle between the chords to the nearest 0.1 degree. What is the angular size of the arc that is intercepted by this inscribed angle?

8. A triangle has a 3-inch side, a 4-inch side, and a 5-inch side. The altitude drawn to the 5-inch side cuts this side into segments of what lengths?

9. A chord 6 cm long is 2 cm from the center of a circle. How long is a chord that is 1 cm from the center of the same circle?

10. By using the triangle whose sides have lengths 1, $\sqrt{3}$ and 2, you should be able to write non-calculator expressions for the sine, cosine, and tangent of a 30-degree angle. Do so. You can use your calculator to check your answers, of course.

11. The figure at right shows points *C*, *A*, and *R* marked on a circle centered at *E*, so that chords *CA* and *AR* have the same length, and so that major arc *CR* is a 260-degree arc. Find the angles of quadrilateral *CARE*. What is special about the sizes of angles *CAR* and *ACE*?



12. Find all the angles in a 5-12-13 triangle.

13. A trapezoid has two 65-degree angles and 8-inch and 12-inch parallel sides. How long are the non-parallel sides? What is the area enclosed by this figure?

1. Two circles of radius 10 cm are drawn so that their centers are 12 cm apart. The two points of intersection determine a *common chord*. Find the length of this chord.

2. You are at the scenic overlook at Mt. Sugarloaf in South Deerfield looking through the panoramic viewer that is looking straight ahead. By what degree measure must you rotate the viewer directly downward to see the Mullins Center at UMass, which you know to be 8 miles away if the scenic overlook is 600 feet high?

3. Can the diagonals of a kite bisect each other? Why or why not?

4. Draw trapezoid ABCD so that AB is parallel to CD and the diagonals of the trapezoid. Label the intersection as E. Show that triangles ABE and CDE are similar.

5. The area of an equilateral triangle is $100\sqrt{3}$ square inches. How long are its sides?

6. Points *E*, W, and *S* are marked on a circle whose center is *N*. In quadrilateral *NEWS*, angles *S* and *W* are found to be 54° and 113°, respectively. What are the other two angles?

7. The points $A = (0, 13)$ and $B = (12, 5)$ lie on a circle whose center is at the origin. Show that the perpendicular bisector of *AB* goes through the origin.

8. Draw a 30-60-90 triangle of any size and include lengths of all three sides. Using the ratios that you know, write expressions for $\sin(30)$ and $\cos(60)$. What do you notice? Write a sentence or two to justify what you notice.

9. The areas of two similar triangles are 24 square cm and 54 square cm. The smaller triangle has a 6-cm side. How long is the corresponding side of the larger triangle?

10. Find the perimeter of a regular 36-sided polygon inscribed in a circle of radius 20 cm.

11. When two circles have a common chord, their centers and the endpoints of the chord form a quadrilateral. What kind of quadrilateral? What special property do its diagonals have?

12. Find the sine and cosine of two angles that are complements of each other. Do you see any patterns?

13. If the ratio of similarity between two triangles is 3: 5, what is the ratio of the areas of these triangles?

14. Let $P = (-25, 0)$, $Q = (25, 0)$, and $R = (-24, 7)$.

a. Find an equation for the circle that goes through *P*, *Q*, and *R*.
b. Find at least two ways of showing that angle *PRQ* is right.
c. Find coordinates for another point *R* that would have made angle *PRQ* right.

15. How much evidence is needed to be sure that two triangles are similar?

1. Trapezoid *ABCD* has parallel sides *AB* and *CD*, of lengths 8 and 24, respectively. Diagonals *AC* and *BD* intersect at *E*, and the length of *AC* is 15. Find the lengths of *AE* and *EC*.

2. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Mark point *D* so that *ACD* is a right angle and *DAC* is a 45-degree angle. Find coordinates for *D*. Find the tangent of angle *DAB*.

3. A regular octagon has a perimeter of 64. Find its area.

4. Two circles have a 24-cm common chord, their centers are 14 cm apart, and the radius of one of the circles is 13 cm. Make an accurate drawing, and find the radius for the second circle in your diagram. Are there other possible answers?

5. Triangle *ABC* has *P* on *AC*, *Q* on *AB*, and angle *APQ* equal to angle *B*. The lengths $AP = 3$, $AQ = 4$, and $PC = 5$ are given. Find the length of *AB*.

6. A *cyclic* quadrilateral is a quadrilateral whose vertices are points on a circle. Draw a cyclic quadrilateral *SPAM* in which the size of angle *SPA* is 110 degrees. What is the size of angle *AMS*? Would your answer change if *M* were replaced by a different point on major arc *SA*?

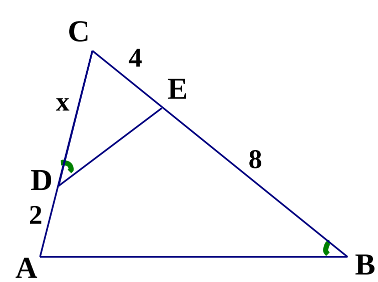
7. If A = (3, 1), B = (3, 4), and C = (8, 1) find the measure of angle B.

8. A 20-inch chord is drawn in a circle with a 12-inch radius. What is the *angular size* of the minor arc of the chord? What is the *length* of the arc, to the nearest tenth of an inch?

9. In a certain building, there are two large regular octagonal pillars. The edges are 6.5 in and they are 9 feet tall. How much granite was needed to build these pillars?

10. Quadrilateral *WISH* is *cyclic*. Diagonals *WS* and *HI* intersect at *K*. Given that arc *WI* is 100 degrees and arc *SH* is 80 degrees, find the sizes of as many angles in the figure as you can. Note: *K* is not the center of the circle.

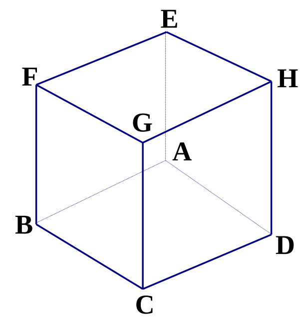
11. Refer to the figure, in which angles *ABE* and *CDE* are equal in size and various segments have been marked with their lengths. Find *x*.



12. Quadrilateral *BAKE* is cyclic. Extend *BA* to a point *T* outside the circle, thus producing the exterior angle *KAT*. Why do angles *KAT* and *KEB* have the same size?

1. Use Desmos to graph $y = 2x - 5$ and the circle $x^2 + y^2 = 5$. Show that these graphs touch at only one point. It is customary to say that a line and a circle are *tangent* if they have exactly one point in common.

2. (Continuation) Find the slope of the segment that joins the point of tangency to the center of the circle and compare your answer with the slope of the line $y = 2x - 5$. What do you notice?

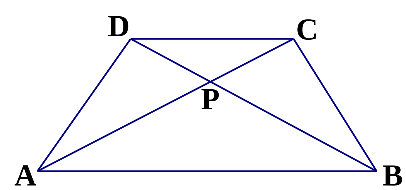


3. Given that *ABCDEFGH* is a cube (shown at right), what is the relationship between the volumes of the cube and the three square pyramids *ADHEG*, *ABCDG*, and *ABFEG*? (note: the square pyramids are not necessarily congruent).

4. The parallel sides of a trapezoid have lengths 9 cm and 12 cm. Draw *one* diagonal, dividing the trapezoid into two triangles. What is the ratio of their areas? If the other diagonal had been drawn instead, would this have affected your answer?

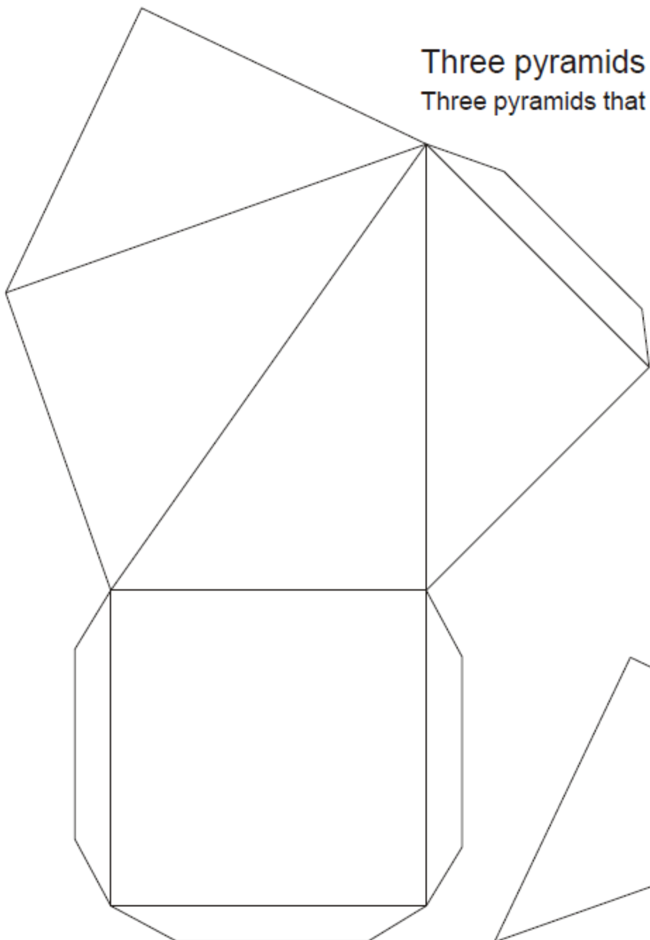
5. On the next page there are two dimensional networks that you should print, cut out and attempt to fold up and tape into three oblique pyramids (you will need to print one extra) that will help you to visualize pyramids *ADHEG*, *ABCDG* and *ABFEG*. Attempt to put them together and form the cube *ABCDEFGH* once again and justify the volume formula of a pyramid. Why do you conjecture is volume of a pyramid that has the same base area and height as a cube? Bring your cubes to class.

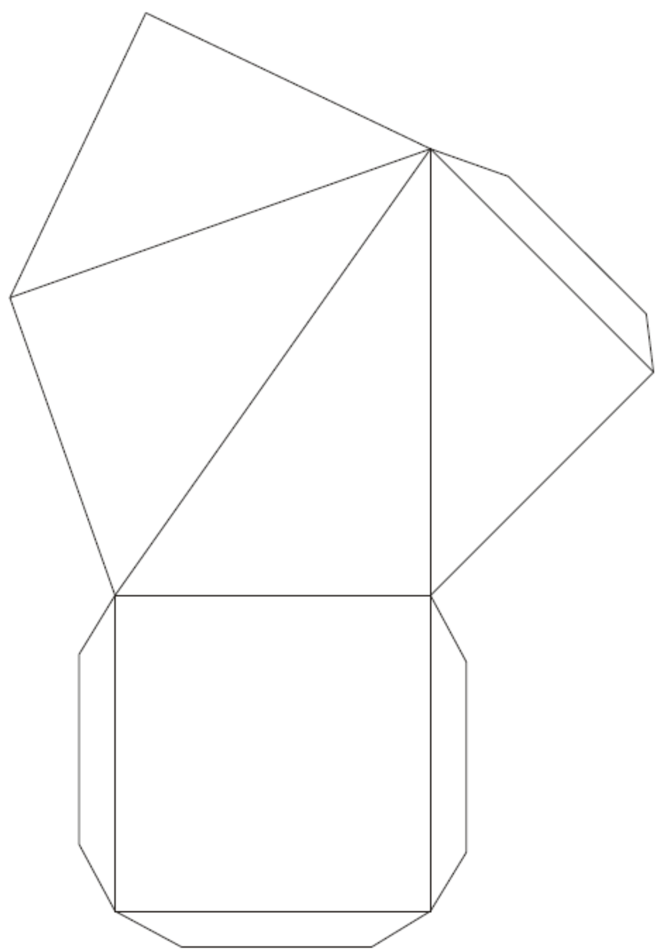
6. Suppose that *ABCD* is a trapezoid with *AB* parallel to *CD* and diagonals *AC* and *BD* intersecting at *P*. Explain why
   a. triangles *CDA* and *CDB* have the same area;
   b. triangles *BCP* and *DAP* have the same area;
   c. triangles *ABP* and *CDP* are similar;
   d. triangles *BCP* and *DAP* need not be similar.



Three pyramids

Three pyramids that fit in one cube





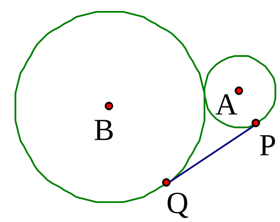
1. Drawn in a circle whose radius is 12 cm, chord *AB* is 16 cm long. Calculate the angular size of minor arc *AB*.

2. Show that the line $y = 10 - 3x$ is tangent to the circle $x^2 + y^2 = 10$. Find an equation for the line perpendicular to the tangent line at the point of tangency. Show that this line goes through the center of the circle.

3. Let $A = (4, 6)$, $B = (6, 0)$, and $C = (9, 9)$. Find the size of angle *BAC*.

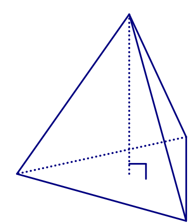
4. Segment *AB*, which is 25 inches long, is the diameter of a circle. Chord *PQ* meets *AB* perpendicularly at *C*, where *AC* = 16 in. Find the length of *PQ*.

5. Prove that the arcs between any two parallel chords in a circle must be the same size.



6. A circle with a 4-inch radius is centered at *A* and a circle with a 9-inch radius is centered at *B*, where *A* and *B* are 13 inches apart. There is a segment that is tangent to the small circle at *P* and to the large circle at *Q*. It is a common external tangent of the two circles. What kind of quadrilateral is *PABQ*? What are the lengths of its sides?

7. *Two Tangents Theorem*. From any point *P* outside a given circle, there are two lines through *P* that are tangent to the circle. Explain why the distance from *P* to one of the points of tangency is the same as the distance from *P* to the other point of tangency. What special quadrilateral is formed by the center of the circle, the points of tangency, and *P*?



8. The altitude of a regular triangular pyramid is the segment connecting a vertex to the centroid of the opposite face. A regular triangular pyramid has edges of length 6 in. How tall is such a pyramid, to the nearest hundredth of an inch?

9. A 72-degree arc *AB* is drawn in a circle of radius 8 cm. How long is chord *AB*?

10. It is not hard to believe that a 360-sided polygon is very close to looking like a circle. Find the perimeter of a regular 360-sided polygon that is inscribed in a circle of radius 5 inches. If someone did not remember the formula for the circumference of a circle, how could that person use a calculator's trigonometric functions to find the circumference of a circle with a 5-inch radius?

11. The segments *GA* and *GB* are tangent to a circle with center *O* at *A* and *B*, and *AGB* is a 60-degree angle. Given that $GA = 12\sqrt{3}$ cm, find the distance *GO*. Find the distance from *G* to the nearest point on the circle.

12. A circle T has two tangents that intersect at 54° at point *M*. The points of tangency are *A* and *H*. What is the angular size of AH?

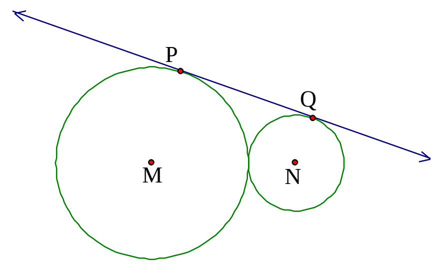
1. The Great Pyramid at Giza was originally 483 feet tall, and it had a square base that was 756 feet on a side. It was built from rectangular stone blocks measuring 7 feet by 7 feet by 15 feet. Such a block weighs seventy tons. Approximately how many tons of stone were used to build the Great Pyramid? The volume of a pyramid is one third the base area times the height.

2. In a group of 12 students, only 4 of them like olives on their pizza. If they are sharing a 16-in pizza what is the area of the part the pizza covered with olives?

3. A triangle that has a 50-degree angle and a 60-degree angle is inscribed in a circle of radius 25 inches. The circle is divided into three arcs by the vertices of the triangle. To the nearest tenth of an inch, find the lengths of these three arcs. An accurate diagram is useful here.

4. Stacy wants to decorate the side of a cylindrical can by using a rectangular piece of paper and wrapping it around the can. The paper is 21.3 cm by 27.5 cm. Find the two possible diameters of the cans that Stacy could use. (Assume the paper fits exactly).

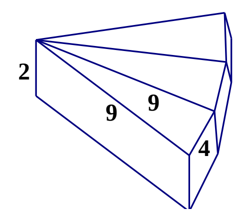
5. The area of a trapezoidal cornfield *IOWA* is 18000 sq m. The 100-meter side *IO* is parallel to the 150-meter side *WA*. This field is divided into four sections by diagonal roads *IW* and *OA*. Find the areas of the triangular sections.



6. *PQ* is tangent to circles $\odot M$ and $\odot N$. $\odot M$ and $\odot N$ are externally tangent.

(a) If $\odot M$ and $\odot N$ are not congruent; what kind of quadrilateral is *MNQP*?

(b) If $\odot M$ and $\odot N$ are congruent, what kind of quadrilateral is *MNQP*.



7. A wedge of cheese is 2 inches tall. The triangular base of this right prism has two 9-inch edges and a 4-inch edge. Several congruent wedges are arranged around a common 2-inch segment, as shown. How many wedges does it take to complete this wheel? What is the volume of the wheel, to the nearest cubic inch?

8. The segments *GA* and *GB* are tangent to a circle at *A* and *B*, and *AGB* is a 48-degree angle. Given that *GA* = 12 cm, find the distance from *G* to the nearest point on the circle.

9. A 16.0-inch chord is drawn in a circle whose radius is 10.0 inches. What is the angular size of the minor arc of this chord? What is the length of the arc, to the nearest tenth of an inch?

10. The line $x + 2y = 5$ divides the circle $x^2 + y^2 = 25$ into two arcs. Find the lengths of the arcs.

11. (Continuation) A *sector* is a region formed by two radii and an arc of a circle. Find the area of the smaller sector. Justify your method.

1. The equation of a circle is $x^2 + y^2 = 50$, find the area of the circle.

2. If the area of a circle centered at the origin is 40π, write the equation for this circle.

3. Write the equation of the circle that passes through the vertices of the triangle defined by (-1, -7), (5, 5), (7, 1).

4. All triangles have circumscribed circles. Why? What property must a given quadrilateral hold in order to have a circumscribed circle? Explain.

5. Pyramid *TABCD* has a square base *ABCD* with 20-cm base edges. The lateral edges that meet at *T* are 27 cm long. Make a diagram of *TABCD*, showing *F*, the point of *ABCD* closest to *T*. To the nearest 0.1 cm, find the height *TF*. Find the volume of *TABCD*, to the nearest cm^3.

6. (Continuation) Find the slant height of pyramid TABCD. The slant height is the height of the *lateral face*.

7. (Continuation) Let *K*, *L*, *M*, and *N* be the points on *TA*, *TB*, *TC*, and *TD*, respectively, which are 18 cm from *T*. What can be said about polygon *KLMN*? Explain.

8. Two of the tangents to a circle meet at *Q*, which is 25 cm from the center. The circle has a 7-cm radius. To the nearest tenth of a degree, find the angle formed at *Q* by the tangents.

9. Which is the better (tighter) fit: A round peg in a square hole or a square peg in a round hole?

10. From the top of Mt Washington, which is 6288 feet above sea level, how far is it to the horizon? Assume that the Earth has a 3962-mile radius, and give your answer to the nearest mile.

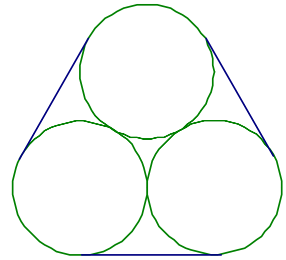
11. What is the minimum amount of wrapping paper needed to wrap a box with dimensions 20 cm by 10 cm by 30 cm?

12. Which polygons can have circumscribed circles? Explain.

13. A paper towel tube has a diameter of 1.7 inches and a height of 11 inches. If the tube were cut and unfolded to form a rectangle, what would be the area of the rectangle?

14. Find the area of a kite whose longer diagonal is divided into two parts that are 4 and 12 and whose shorter side is 5.

15. The area of a sector of a circle with radius 12 is 16π cm^2. What is the central angle of this sector?



1. The figure shows three circular pipes, all with 12-inch diameters, that are strapped together by a metal band. How long is the band?

2. (Continuation) Suppose that four pipes are strapped together with a snugly-fitting metal band. How long is the band?

3. The lateral edges of a regular hexagonal pyramid are all 20 cm long, and the base edges are all 16 cm long. To the nearest cm^3, what is the volume of this pyramid? To the nearest square cm, what is the combined area of the base and six lateral faces?

4. *Surface area of a Sphere*: The surface area of a sphere is found using the formula $4\pi r^2$. Find the surface area of the Earth, given that its diameter is 7924 miles.

5. For any pyramid, the volume is $\frac{1}{3}\bullet$base area$\bullet$height. A cone is a pyramid with a circular base. Find the volume of a cone with a slant height of 13 and a diameter of 10.

6. The radius of the Sun is 109 times the radius of the Earth. Find the surface area of the Sun.

7. The radius of a circular sector is r. The central angle of the sector is θ. Write formulas for the arc length and the perimeter of the sector.

8. Suppose that the lateral faces *VAB*, *VBC*, and *VCA* of triangular pyramid *VABC* all have the same height drawn from *V*. Let *F* be the point in base *ABC* that is closest to *V*, so that *VF* is the altitude of the pyramid. Does this imply that $AB \cong BC \cong CA$?

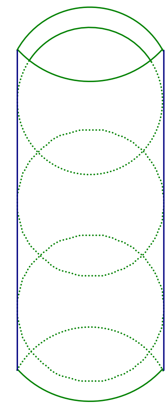
9. Schuyler has made some glass prisms to be sold as window decorations. Each prism is four inches tall, and has a regular hexagonal base with half-inch sides. They are to be shipped in cylindrical tubes that are 4 inches tall. What radius should Schuyler use for the tubes? Once a prism is inserted into its tube, what volume remains for packing material?

10. A conical cup has a 10-cm diameter and is 12 cm deep. How much can this cup hold?

11. (Continuation) Water in the cup is 6 cm deep. What percentage of the cup is filled?

12. A sphere of ice cream is placed on an ice cream cone. You can assume the volume of a sphere is $V = \frac{4}{3}\pi r^3$ and the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Both have a diameter of 8 cm. The height of the cone is 12 cm. Will all the ice cream, if pushed down into the cone, fit?

13. Dana takes a paper cone with a 10-cm diameter and is 12 cm deep, cuts it along a straight line from the rim to the vertex, then flattens the paper out on a table. Find the radius, the arc length, and the central angle of the resulting circular sector.



1. The base radius of a cone is 6 inches, and the cone is 8 inches tall. If the cone is cut along its slant height and unrolled flat, what shape is created? What questions can you answer about this shape?

2. Three tennis balls fit snugly inside a cylindrical can. What percent of the available space inside the can is occupied by the balls?

3. Fill in the following proportion relating the sectors of circles to the whole circle. Why does this proportion hold true?

$$\frac{sector\ area}{?} = \frac{central\angle}{?} = \frac{?}{2\pi r}$$

4. The areas of two circles are in the ratio of 50: 32. If the radius of the larger circle is 10, what is the radius of the smaller circle?

5. Find the perimeter of the semicircle with radius 10.

6. A squash ball fits snugly inside a cubical box whose edges are 4 cm long. Guess the percentage of the box's volume that the ball occupies, then calculate that percentage. (This is an example of a *sphere inscribed in a cube*.)

7. There is a park, 27 feet wide, that is between two buildings whose heights are 123 ft and 111 ft. Two Deerfield teachers, Mr. Barnes and Ms.Schettino are standing on top of the shorter building looking at a rare bird perched on top of the taller building. If Mr. Barnes is 77 inches tall and Ms. Schettino is 62 inches tall who has the smaller angle of elevation while looking at the bird? Explain your answer.

8. As a spherical gob of ice cream that once had a 2-inch radius melts, it drips into a cone of the same radius. The melted ice cream exactly fills the cone. What is the height of the cone?

9. A conical cup is 64/125 full of liquid. What is the ratio of the depth of the liquid to the depth of the cup? Conical cups appear fuller than cylindrical cups – explain why.

10. Two similar triangles have medians in a ratio of 5: 6, what is the ratio of their areas?

11. A 10 cm tall cylindrical glass 8 cm in diameter is filled to 1 cm from the top with water. If a gold ball 4 cm in diameter is dropped into the glass, will the water overflow?

12. An *annulus* is defined as the region lying between two *concentric* circles. If the diameter of the larger circle is 20 in and the radius of the smaller circle is 8, find the area of the annulus.

1. A Reese's Big Cup has a diameter of two inches and a height of 0.8 inches. A Reese's bar has dimensions 4 inches by 0.8 inches by 0.5 inches. Using approximations, which candy has more peanut butter?

2. (Continuation) Assuming a uniform chocolate thickness, which candy has more chocolate?

3. Ice cream scoops are often 6 cm in diameter. How many scoops should you get from a half-gallon of ice cream? A half-gallon container can be approximated by a cylinder with a diameter of 12 cm and a height of 14 cm.

4. A spherical globe, 12 inches in diameter, is filled with spherical gumballs, each having a 1-inch diameter. Estimate the number of gumballs in the globe, and explain your reasoning.

5. The altitudes of two similar triangles are 6 cm and 9 cm. If the area of the larger triangle is 36 cm^2, what is the area of the smaller triangle?

6. Seventy percent of the Earth's surface is covered in water. Find the approximate surface area of the Earth that is dry land.

7. Given triangle *EWS* defined by *E*(5, 5), *W*(4, -8), and *S*(-6, 6), write the equation of the median from point *E* to *WS*. How far is it from point *E* to the centroid?

8. The ratio of similarity of two triangular prisms is 3: 5. What is the ratio of their surface areas? What is the ratio of their volumes?

9. The sum of the lengths of the two bases of a trapezoid is 22 cm and its area is 946 cm^2. Find the height of this trapezoid.

10. The volumes of two similar hexagonal prisms are in the ratio of 8: 125. What is the ratio of their heights? If the surface area of the larger prism is 100, what is the surface area of the smaller prism?

11. Find the point that is equidistant from the points (0, 4), (2, 3), and (5, 9). (Hint: this is a special point)

12. The base of a pyramid is the regular polygon *ABCDEFGH*, which has 14-inch sides. All eight of the pyramid's lateral edges, *VA*, *VB*, etc, are 25 inches long. To the nearest tenth of an inch, calculate the height of pyramid *VABCDEFGH*.

13. An equilateral triangle is inscribed in the circle of radius 1 centered at the origin (the *unit circle*). If one of the vertices is (1, 0), what are the coordinates of the other two? The three points divide the circle into three arcs; what are the angular sizes of these arcs?

14. The surface areas of two cubes are in the ratio of 49: 81. If the volume of the smaller cube is 20, what is the volume of the larger cube?

1. Charlie built a treasure box. Lucy built a treasure box with dimensions twice as large as Charlie's. If it takes one-half gallon of paint to cover the surface of Charlie's box, how many gallons of paint would it take to paint Lucy's box? How many times more volume will Lucy's box hold than Charlie's?

2. Find the area of the regular polygon whose exterior angle is 45 and whose sides are 3.5 inches.

3. Suppose that *DRONE* is a regular pentagon and that *DRUM*, *ROCK*, *ONLY*, *NEAP*, and *EDIT* are squares attached to the outside of the pentagon. Show that decagon *ITAPLYCKUM* is equiangular. Is this decagon equilateral?

4. A 20-inch chord is drawn in a circle with a 12-inch radius. What is the *angular size* of the minor arc of the chord?

5. Find the lengths of both altitudes in the parallelogram determined by [2, 3] and [−5, 7].

6. Suppose that square *PQRS* has 15-cm sides and that *G* and *H* are on *QR* and *PQ*, respectively, so that *PH* and *QG* are both 8 cm long. Let *T* be the point where *PG* meets *SH*. Find the size of angle *STG*, with justification.

7. (Continuation) Find the lengths of *PG* and *PT*.

8. It is given that the sides of an isosceles trapezoid have lengths 3 in, 15 in, 21 in, and 15 in. Make a diagram. Show that the diagonals intersect perpendicularly.

9. Triangle *ABC* has *AB* = *AC*. The bisector of angle *B* meets *AC* at *D*. Extend side *BC* to *E* so that *CE* = *CD*. Triangle *BDE* should look isosceles. Is it? Explain.

10. Can a circle always be drawn through three given points? If so, describe a procedure for finding the center of the circle. If not, explain why not.

11. Find a triangle two of whose angles have sizes $\mathrm{TAN}^{-1}(1.5)$ and $\mathrm{TAN}^{-1}(3)$. Answer this question either by giving coordinates for the three vertices, or by giving the lengths of the three sides. To the nearest 0.1 degree, find the size of the third angle in your triangle.

12. Let *RICK* be a parallelogram, with *M* the midpoint of *RI*. Draw the line through *R* that is parallel to *MC*; it meets the extension of *IC* at *P*. Prove that *CP* = *KR*.

1. Suppose that *PEANUT* is a regular hexagon, and that *PEGS, EACH, ANKL, NUMB, UTRY,* and *TPOD* are squares attached to the outside of the hexagon. Decide whether or not dodecagon *GSODRYMBKLCH* is regular and give your reasons.

2. A kite has an 8-inch side and a 15-inch side, which form a right angle. Find the length of the diagonals of the kite.

3. Point P is marked inside regular pentagon *TRUDY* so that triangle *TRP* is equilateral. Decide whether or not quadrilateral *TRUP* is a parallelogram and give your reasons.

4. Find the equation of the line that contains all of the points equidistant from the points A(-2, 7) and B(3, 6).

5. A triangle with sides 6, 8, and 10 and a circle with radius is r are drawn so that no part of the triangle lies outside the circle. How small can r be?

6. Diagonals AC and BD of regular pentagon $ABCDE$ intersect at H. Decide whether or not $AHDE$ is a rhombus, and give your reasons.

7. Let $A = (3, 1)$, $B = (9, 5)$, and $C = (4, 6)$. Your protractor should tell you that angle CAB is about 45 degrees. Explain why angle CAB is in fact exactly 45 degrees.

8. The sides of a polygon are cyclically extended to form *rays*, creating one exterior angle at each vertex. Viewed from a great distance, what theorem does this figure illustrate?

9. In trapezoid $ABCD$, AB is parallel to CD, and $AB = 10$, $BC = 9$, $CD = 22$, and $DA = 15$. Points P and Q are marked on BC so that $BP = PQ = QC = 3$, and points R and S are marked on DA so that $DR = RS = SA = 5$. Find the lengths PS and QR.

10. Triangle ABC has $AB = 12 = AC$ and angle A is 120 degrees. Let F and D be the midpoints of sides AC and BC, respectively, and G be the intersection of segments AD and BF. Find the lengths FD, AD, AG, BG, and BF.

11. The midpoints of the sides of a quadrilateral are joined to form a new quadrilateral. For the new quadrilateral to be a rectangle, what must be true of the original quadrilateral?

12. The vectors [8, 0] and [3, 4] form a parallelogram. Find the lengths of its altitudes.

13. One leg of a right triangle is twice as long as the other and the perimeter of the triangle is 40. Find the lengths of all three sides.

14. Suppose that *PEANUT* is a regular hexagon, and that *PEGS, EACH, ANKL, NUMB, UTRY,* and *TPOD* are squares attached to the outside of the hexagon. Decide whether or not dodecagon *GSODRYMBKLCH* is regular and give your reasons.

1. Square *ABCD* has 8-inch sides, *M* is the midpoint of *BC*, and *N* is the intersection of *AM* and diagonal *BD*. Find the lengths of *NB*, *NM*, *NA*, and *ND*.

2. Parallelogram *PQRS* has *PQ* = 8 cm, *QR* = 9 cm, and diagonal *QS* = 10 cm. Mark *F* on *RS*, exactly 5 cm from *S*. Let *T* be the intersection of *PF* and *QS*. Find the lengths *TS* and *TQ*.

3. The parallel sides of a trapezoid are 12 inches and 18 inches long. The non-parallel sides meet when one is extended 9 inches and the other is extended 16 inches. How long are the non-parallel sides of this trapezoid?

4. Show that the area of a square is half the product of its diagonals. Then consider the possibility that there might be other quadrilaterals with the same property.

5. The dimensions of rectangle *ABCD* are *AB* = 12 and *BC* = 16. Point *P* is marked on side *BC* so that *BP* = 5 and the intersection of *AP* and *BD* is called *T*. Find the lengths of the four segments *TA*, *TP*, *TB*, and *TD*.

6. The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into two segments, whose lengths are 8 inches and 18 inches. How long is the altitude?

7. A triangle has two 13-cm sides and a 10-cm side. The largest circle that fits inside this triangle meets each side at a point of tangency. These points of tangency divide the sides of the triangle into segments of what lengths?

8. (Continuation) What is the radius of this circle?

9. In the middle of the nineteenth century, octagonal barns and sheds (and even some houses) became popular. How many cubic feet of grain would an octagonal barn hold if it were 12 feet tall and had a regular base with 10-foot edges?

10. The diagonal of a rectangle is 15 cm, and the perimeter is 38 cm. What is the area? It is possible to find the answer without finding the dimensions of the rectangle – try it.

11. A triangle that has a 5-inch and a 6-inch side can be similar to a triangle that has a 4-inch and an 8-inch side. Find all possible examples. Check that your examples really *are* triangles.

12. What is the radius of the circumscribed circle for a triangle whose sides are 15, 15, and 24 cm long?

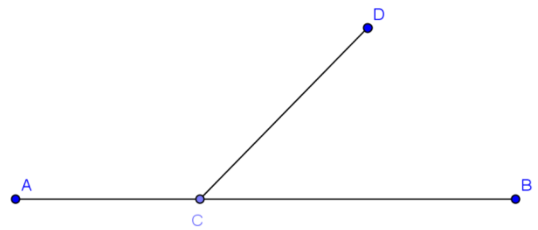
13. A kite has a 5-inch side and a 7-inch side. One of the diagonals is bisected by the other. The bisecting diagonal has length 8 inches. Find the length of the bisected diagonal.

AA similarity: Two *triangles* are sure to be similar if at least two of their angles are equal in size.

adjacent angles: Two angles with a common vertex that share a side but have no common interior points.

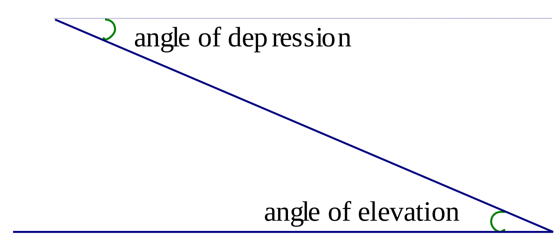
altitude: In a triangle, an altitude is a line through one of the vertices, perpendicular to the opposite side. In obtuse triangles, it may be necessary to extend a side to meet the altitude. The *distance* from the vertex to the point of intersection with the line containing the opposite side is also called an altitude, as is the distance that separates the parallel sides of a trapezoid.

angles can often be identified by a single letter, but sometimes three letters are necessary. The angles shown can be referenced as *C*, *ACD*, or *BCD*.



angle of depression: Angle formed by a horizontal ray and a line-of-sight ray that is below the horizontal. See the diagram at right.

angle of elevation: Angle formed by a horizontal ray and a line-of-sight ray that is above the horizontal. See the diagram at right.



Angle-Angle-Side (corresponding): When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that one pair of corresponding sides has the same length, then the triangles are congruent. This rule of evidence is abbreviated to AAS.

angle bisector: Given an angle, this ray divides the angle into two equal parts.

Angle Bisector Theorem: The bisector of any angle of a triangle cuts the opposite side into segments whose lengths are proportional to the sides that form the angle.

Angle-Side-Angle: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that the (corresponding) shared sides have the same length, then the triangles are congruent. This rule of evidence is abbreviated to ASA.

angular size of an arc: This is the size of the central angle formed by the radii that meet the endpoints of the arc. Also called the measure of the arc.

arc: A section of the perimeter of a circle. Can be a *minor arc* (measure less than 180 degrees) or *major arc* (measure greater than 180 degrees). The portion of a circle that lies to one side of a chord is also called an *arc*.

arc length: The linear distance along the circle from one endpoint of the arc to the other. Given a circle, the length of any arc is proportional to the size of its central angle.

areas of similar figures: If two figures are similar, then the ratio of their areas equals the *square* of the ratio of similarity.

bisect: Divide into two equal parts.

central angle: An angle formed by two radii of a circle.

centroid: The medians of a triangle are concurrent at this point, which is the balance point (also known as the *center of gravity*) of the triangle.

chord: A segment that joins two points on a circle is called a *chord* of the circle.

circle: The set of all points equidistant from a given point, called the *center*. The common distance is the *radius* of the circle. A segment joining the center to a point on the circle is also called a *radius*.

circumcenter: The perpendicular bisectors of the sides of a triangle are concurrent at this point, which is equidistant from the vertices of the triangle.

circumscribed circle: When possible, the circle that goes through all the vertices of a polygon.

collinear: Three (or more) points that all lie on a single line are *collinear*.

common chord: A segment that joins the points where two circles intersect.

complementary: Two angles that fit together to form a right angle are called complementary. Each angle is the *complement* of the other.

components the two parts of a vector that describe how to move from one point to another. Vectors [x,y] describe the motion with the x component (describing the horizontal shift) and the y component (the vertical shift). They are obtained by *subtracting* coordinates, being mindful of the direction of the vector.

concentric: Two figures that have the same center are called *concentric*.

concurrent: Three (or more) lines that go through a common point are *concurrent*.

concyclic: Points that all lie on a single circle are called *concyclic*.

congruent: When the points of one figure can be matched with the points of another figure, so that corresponding parts have the same size, then the figures are called *congruent*, which means that they are considered to be equivalent.

converse: The converse of a statement of the form "if [something] then [something else]" is the statement "if [something else] then [something]."

convex: A polygon is called *convex* if every segment joining a pair of points within it lies entirely within the polygon.

coordinates: Numbers that describe the position of a point in relation to the origin of a coordinate system.

coplanar: objects that are within the same plane

corresponding: They are parts of polygons that are in the same position relative to each other. Corresponding describes parts of figures (such as angles or segments) that could be matched by means of a transformation. In congruent polygons, if the polygons were superimposed, the corresponding parts would be right on top of one another.

cosine ratio: Given a right triangle, the cosine of one of the acute angles is the ratio of the length of the side *adjacent* to the angle to the length of the hypotenuse. The word cosine is a combination of *complement* and *sine*, so named because the cosine of an angle is the same as the sine of the complementary angle.

counterexample: an example used to show that a statement is false.

CPCTC: *Corresponding Parts of Congruent Triangles are Congruent*.

cyclic: A polygon, all of whose vertices lie on the same circle, is called *cyclic*. Also called an *inscribed polygon*.

decagon: A polygon that has ten sides.

diagonal: A segment that connects two nonadjacent vertices of a polygon.

diameter: A chord that goes through the center of a circle is called a *diameter*.

distance formula: The distance from (x_1, y_1) to (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This formula is a consequence of the *Pythagorean Theorem*.

dodecagon: A polygon that has twelve sides.

dynamic: A figure is dynamic if it is not fixed in the plane.

equiangular: A polygon all of whose angles are the same size.

equidistant: A shortened form of *equally distant*.
equilateral: A polygon all of whose sides have the same length.

Euclidean geometry (also known as plane geometry) is characterized by its parallel postulate, which states that, *given a line, exactly one line can be drawn parallel to it through a point not on the given line*. A more familiar version of this assumption states that *the sum of the angles of a triangle is a straight angle*.

Euler line: The centroid, the circumcenter, and the orthocenter of any triangle are collinear.

exterior angle: An angle that is formed by a side of a polygon and the extension of an adjacent side. It is supplementary to the adjacent interior angle.

Exterior Angle Theorem: An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

function: A function is a rule that describes how the value of one thing is determined uniquely by the value of another thing.

glide-reflection: An transformation created by a vector translation and reflection. The mirror line contains the transformation vector.

Greek letters appear often in mathematics. Some of the common ones are α (alpha), β (beta), Δ or δ (delta), θ (theta), Λ and λ (lambda), μ (mu), π (pi), and Ω or ω (omega).

head: Vector terminology for the second vertex of a directed segment.

hexagon: a polygon that has six sides.

Hypotenuse-Leg: When the hypotenuses of two right triangles have the same length, and a leg of one triangle has the same length as a leg of the other, then the triangles are congruent. This rule of evidence is abbreviated to HL.

image: The result of applying a transformation to a point P is called the *image of P*, often denoted P'. One occasionally refers to an *image segment* or an *image triangle*.

incenter: The angle bisectors of a triangle are concurrent at this point, which is equidistant from the sides of the triangle.

included angle: The angle formed by two designated segments.

inscribed angle: An angle formed when two chords meet at a point on the circle. An inscribed angle is *half* the angular size of the arc it intercepts. In particular, an inscribed angle that intercepts a semicircle is a *right* angle.

inscribed polygon: A polygon whose vertices all lie on the same circle; also called a *cyclic polygon*.

integer: Any whole number, whether it be positive, negative, or zero.

intercepted arc: The part of an arc that is found inside a given angle.

isometry: A geometric transformation that preserves distances. The best-known examples of isometries are *translations*, *rotations*, and *reflections*.

isosceles triangle: A triangle that has two sides of the same length. The word is derived from the Greek *iso* + *skelos* (equal + leg)

Isosceles Triangle Theorem: If a triangle has at least two sides of equal length, then the angles opposite the congruent sides are also the same size.

isosceles trapezoid: A trapezoid whose nonparallel sides have the same length.

kite: A quadrilateral that has two pairs of congruent adjacent sides.

labeling convention: Given a polygon that has more than three vertices, place the letters around the figure in the order that they are listed.

lateral face: Any face of a pyramid or prism that is not a base.

lattice point: A point whose coordinates are both integers.

lattice rectangle: A rectangle whose vertices are all lattice points.

leg: The perpendicular sides of a right triangle are called its legs.

length of a vector: This is the length of any segment that represents the vector. Notation: length of $\vec{v}$ is labeled as $|\vec{v}|$. $\vec{v} = [x, y]$, then $|\vec{v}| = \sqrt{x^2 + y^2}$.

linear equation: Any straight line can be described by an equation in the form $Ax + By = C$.

linear pair: Two adjacent angles whose sum is 180 degrees; Two angles that form a straight angle.

major/minor arc: A non-diameter chord of a circle divides a circle into two parts. Of the two arcs, the smaller one is called *minor* (less than 180 degrees), and the larger one is called *major*

(more than 180 degrees). Often, a major arc is described with a label that has 3 letters and a minor arc is described with 2 letters.
median of a triangle: A segment that joins a vertex of a triangle to the midpoint of the opposite side.

midline of a trapezoid: This segment joins the midpoints of the non-parallel sides. Its length is the average of the lengths of the parallel sides, to which it is also parallel. Also known as the *median* in some books.

Midsegment Theorem: A segment that joins the midpoints of two sides of a triangle is parallel to the third side, and is half as long.

midpoint: The point on a segment that is equidistant from the endpoints of the segment. If the endpoints are (a, b) and (c, d), the midpoint is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

Mirror line: In a reflection, the mirror line is the perpendicular bisector of the segments that connect the initial point (pre-image) and its reflected point (image).

Negative (opposite) reciprocal: One number is the negative reciprocal of another if the product of the two numbers is -1.

octagon: a polygon that has eight sides.

opposite: Two numbers or vectors are opposite if they differ in sign. For example, 17.5 is the opposite of −17.5, and [2,−11] is the opposite of [−2, 11].

orthocenter: The altitudes of a triangle are concurrent at this point.

parallel: Coplanar lines that do not intersect. When drawn in a coordinate plane, they are found to have the same slope, or else no slope at all. The shorthand // is often used.

parallelogram: A quadrilateral that has two pairs of parallel sides.

pentagon: a polygon that has five sides.

perpendicular: Coplanar lines that intersect to form a right angle.

perpendicular bisector: Given a line segment, this is the line that is perpendicular to the segment and that goes through its *midpoint*. The points on this line are all *equidistant* from the endpoints of the segment.

point-slope form: A non-vertical straight line can be described by $y - y_0 = m(x - x_0)$ or by $y = m(x - x_0) + y_0$. One of the points on the line is (x_0, y_0) and the slope is m.

postulate: A statement that is accepted as true, without proof.

prism: A three-dimensional figure that has two congruent and parallel *bases*, and parallelograms for its remaining *lateral faces*. If the lateral faces are all rectangles, the prism is a *right prism*. If the base is a regular polygon, the prism is also called *regular*.

Proof by Contradiction (Indirect Proof): method of mathematical proof in which the mathematician assumes the opposite of what they are attempting to proof in the hope of coming up with a contradiction of already known fact. This contradiction thereby proves that the assumption that the statement must have been false.

proportion: An equation that expresses the equality of two *ratios*.

pyramid: A three-dimensional figure that is obtained by joining all the points of a polygonal *base* to a *vertex*. Thus all the lateral faces of a pyramid are triangles. If the base polygon is regular, and the lateral edges are all congruent, then the pyramid is called *regular*.

Pythagorean Theorem: The area of the square with a side equal to the hypotenuse of a right triangle equals the sum of the areas of the squares whose sides are the lengths of the legs of the right triangle. If a and b are the lengths of the legs of a right triangle, and if c is the length of the hypotenuse, then these lengths fit the Pythagorean equation
$a^2 + b^2 = c^2$.

quadrant: one of the four regions formed by the coordinate axes. Quadrant I is where both coordinates are positive, and the other quadrants are numbered (using Roman numerals) in a counterclockwise fashion.

quadratic formula: $x = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $x = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$ are the two solutions to $ax^2 + bx + c = 0$.

quadrilateral: a four-sided polygon.

ratio of similarity: The ratio of the lengths of any two corresponding segments of similar figures.

ray: A ray is a line bounded at one end and infinite at the other.

rectangle: An equiangular quadrilateral.

reflection: A reflection maps points on one side of the *mirror line* to the other side. If the point is on the mirror line, then it maps onto itself.

regular: A polygon that is both equilateral and equiangular.

rhombus: An equilateral quadrilateral.

right angle: An angle that is its own supplement, in other words, an angle that is 90 degrees.

rotation: A transformation in a plane that moves a figure about a single fixed point. The fixed point is called the *center of rotation*.

SAS similarity: Two triangles are certain to be similar if two sides of one triangle are proportional to two sides of the other, and if the included angles are equal in size.

Same Side interior angles – angles formed by two parallel lines and a transversal which are non-adjacent, interior angles (i.e. they do not share a vertex and are both interior to the parallel lines).

scalar: In the context of vectors, this is just another name for a number that can change the magnitude and/or direction of the vector.

scalene: A triangle that has 3 different side lengths.

segment: That part of a line that lies between two designated points.

Sentry Theorem: The sum of the exterior angles (one per vertex) of any polygon is 360 degrees.

Shared Altitude Theorem: If two triangles share an altitude, then the ratio of their areas is proportional to the ratio of the corresponding bases.

Shared Base Theorem: If two triangles share a base, then the ratio of their areas is proportional to the ratio of the corresponding altitudes.

Side-Angle-Side: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding sides have the same lengths, and so that the (corresponding) angles they form are also the same size, then the triangles are congruent. This rule of evidence is abbreviated to just SAS.

Side-Side-Angle: Insufficient grounds for congruence. See *Hypotenuse-Leg*, however.

Side-Side-Side: When the parts of one triangle can be matched with the parts of another triangle, so that all three pairs of corresponding sides have the same lengths, then the triangles are congruent. This rule of evidence is abbreviated to just SSS.

similar: Two figures are similar if their points can be matched in such a way that all ratios of corresponding lengths are proportional to a fixed *ratio of similarity*. Corresponding angles of similar figures must be equal in size.

sine ratio: Given a right triangle, the sine of one of the acute angles is the ratio of the length of the side *opposite* the angle to the length of the hypotenuse.

skew lines: Non-coplanar lines that do not intersect.

slope: The slope of the segment that joins the points (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$.

slope-intercept form: Any non-vertical straight line can be described by an equation that takes the form $y = mx + b$. The slope of the line is m, and the y-intercept is b.

SSS similarity: Two triangles are *similar* if their corresponding sides are proportional.

square: A regular quadrilateral.

supplementary: Two angles whose measures add up to 180 degrees and could be fit together to form a straight line are called *supplementary*. Each angle is the *supplement* of the other.

tail: Vector terminology for the first vertex of a directed segment.

tail-to-tail: Vector terminology for directed segments with a common first vertex.

tangent ratio: Given a right triangle, the tangent of one of the acute angles is the ratio of the side opposite the angle to the side adjacent to the angle.

tangent and slope: When an angle is formed by the x-axis and a ray through the origin, the *tangent* of the angle is the *slope* of the ray. Angles are measured in a counterclockwise sense, so that rays in the second and fourth quadrants determine negative tangent values.

tangent to a circle: A line that has one and only one intersection with a circle. This intersection is called the *point of tangency.* Such a line is perpendicular to the radius drawn to the point of tangency.

tesselate: To fit non-overlapping tiles together to cover a planar region.

Three Parallels Theorem: Given three parallel lines, the segments they intercept on one transversal are proportional to the segments they intercept on any transversal.

transformation: A *function* that maps points to points.

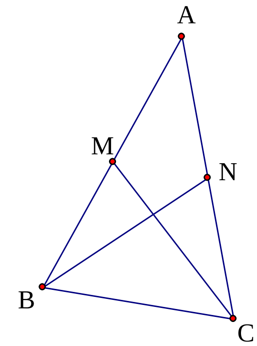
translate: To slide a figure by applying a vector to each of its points.

transversal: A line that intersects two other lines in a diagram.

trapezoid: A quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is called *isosceles*.

triangle inequality: The sum of the lengths of two sides of a triangle is greater than the length of the third side.

two-column proof: A way of outlining a geometric deduction. Steps are in the left column, and supporting reasons are in the right column. For example, here is how one might show that an isosceles triangle *ABC* has two medians of the same length. It is given that *AB* = *AC* and that *M* and *N* are the midpoints of sides *AB* and *AC*, respectively. The desired conclusion is that medians *CM* and *BN* have the same length.



$AB = AC$	given
$AM = AN$	*M* and *N* are midpoints
$\angle MAC = \angle NAB$	shared angle
$\Delta MAC \cong \Delta NAB$	SAS
$CM = BN$	CPCTC

Two Tangent Theorem: From a point outside a circle, there are two segments that can be drawn tangent to the circle. These segments have the same length.

unit circle: This circle consists of all points that are 1 unit from the origin, O, of the *xy*-plane. Given a point *P* on this circle, the coordinates of *P* are the *cosine* and the *sine* of the counterclockwise angle formed by segment *OP* and the positive *x*-axis.

unit square: Its vertices are (0, 0), (1, 0), (0, 1), and (1, 1).

Varignon parallelogram: Given any quadrilateral, this is the figure formed by connecting the midpoints of consecutive sides.

vectors have *magnitude* (size), *direction*, and slope. Visualize them as directed segments (arrows). Vectors are described by *components*, just as points are described by coordinates. The vector from point *A* to point *B* is often denoted $\overrightarrow{AB}$ or abbreviated by a boldface letter such as **u**, and its magnitude is often denoted $\left|\overrightarrow{AB}\right|$ or |**u**|. ***See Components.***

vertex: A labeled point in a figure. The plural is *vertices*, but "vertice" is not a word. The point on a parabola that is closest to the focus is also called the vertex.

vertical angles: Two non-adjacent angles that share a vertex and are formed by the intersection of two lines.

volume of a prism: This is the product of the *base area* and the *height*, which is the distance between the parallel base planes.

volume of a pyramid: This is one third of the product of the *base area* and the *height*, which is the distance from the vertex to the base plane.

volumes of similar figures: If two three-dimensional figures are similar, then the ratio of their volumes equals the *cube* of the ratio of similarity.