Mathematics 203: Honors Geometry
Deerfield Academy
2014-2015

Problem Book

To MAT203 Students

Members of the Deerfield Academy, Emma Willard School and Phillips Exeter Academy Mathematics Department have created the material in this book. As you work through it, you will discover that algebra, geometry, and trigonometry have been integrated into a mathematical whole. There is no Chapter 5, nor is there a distinct section on right triangles. The curriculum is problem-based, rather than chapter-oriented.

A major goal of this course is have you practice thinking mathematically and to learn to become a more independent and creative problem solver. Problem solving techniques, new concepts and theorems will become apparent as you work through the problems, and it is your classroom community's responsibility to make these conclusions together. Your responsibility is to keep appropriate notes for your records — there are no boxes containing important theorems. There is no index as such, but the reference section at the end of the problems should help you recall the meanings of key words that are defined in the problems (where they usually appear italicized).

I. <u>The Mathematical Thinking Process</u>

1. Stay/Think/Say/Draw
    a. Reading each question carefully and repeatedly is essential, especially since definitions, highlighted in italics, are routinely inserted into the problem texts. Check the reference section regularly.
    b. It is important to make accurate diagrams whenever appropriate.
2. Talk/Use Resources
    a. Talk out loud, speak to friends, ask questions, use Voicethread groups to get feedback on your ideas
    b. Your prior knowledge – what you know already or have forgotten that you know – is your best resource.
    c. Use your notes, the internet
3. Estimate
    a. Before you try any mathematical formulas at all, you should have some idea of what the answer should be – is really large like 3000? Or should it be something small like .05?
4. Mathematize
    a. Formulas(Pythagorean theorem, quadratic formula, equations of lines), concepts (area, linear motion, what a triangle is, the sum of the angles in a triangle) and rules of mathematics (two points determine a line, all numbers squared are positive) can be used at this point in the process.
5. Try/Refine/Revise
    a. If something does work, see why it didn't work
    b. Change the method
    c. Try something else!

II. **<u>Problem Solving as Homework</u>**: You should approach each problem as an **<u>exploration.</u>** **You are not expected to come to class every day with every problem completely solved. In fact, problem presentations are expected to be partial solutions.**

- Useful strategies to keep in mind are:
  - create an easier problem
  - guess and check
  - work backwards
  - make use of prior knowledge
  - recall a similar problem.
- It is important that you work on each problem when assigned, since the questions you may have about a problem will likely motivate class discussion the next day. In other words, doing homework to get ahead is not a good idea since class discussion will help you prepare for future problems.
- Try to justify each step you do – ask *why* not just *how*. Justification is more important than the answer on a nightly basis.
- Problem-solving requires persistence as much as it requires ingenuity. When you get stuck, or solve a problem incorrectly, back up and start over. Keep in mind that you're probably not the only one who is stuck, and that may even include your teacher.
- If you have taken the time to think about a problem, <u>you should bring to class a written record of your efforts</u>, not just a blank space in your notebook. There should be a diagram, equation, reference to similar problem, evidence of your work or questions you had on the problem. This is what will get you credit for doing your homework!!

The methods that you use to solve a problem, the corrections that you make in your approach, the means by which you test the validity of your solutions, and your ability to communicate ideas are just as important as getting the correct answer. <u>You are not to spend more than the allotted time for that night's homework on any one nightly assignment, so please manage your study hall time carefully!</u>

Most importantly, be patient with yourself – learning to problem solve independently takes time, courage and practice.

III. **<u>About technology</u>**: Many of the problems in this book require the use of technology (graphing calculators or computer software) in order to solve them. Moreover, you are encouraged to use technology to explore, and to formulate and test conjectures. Keep the following guidelines in mind:

- write before you calculate, so that you will have a clear record of what you have done
- store intermediate answers in your calculator for later use in your solution
- pay attention to the degree of accuracy requested
- be prepared to explain your method to your classmates, including bringing your laptop to class with the file on it (or emailing it to your teacher the night before) in order to project your solution to the class

IV. Keeping a Mathematics Journal:

As part of this curriculum you will be asked to write about your problem solving processes on a regular basis. This will help you to organize your thoughts around not only problem solving, but the content of the course. When you write your journal entries you should keep in mind a few things:

- Write in complete sentences as if you were explaining to yourself or to another student how to do the problem
- Justify the steps of your process and explain to yourself why you chose the methods you used in the problem
- Make connections between why you chose a certain step in the process and ideas that have been discussed in class
- Make connections between problems – see if patterns emerge in how the problems are laid out in the curriculum
- Draw diagrams that help you to understand the problem better, even if a student used that diagram in class and explain why it helped your understanding

At any time during the year, if you have questions about journal writing or want more feedback, do not hesitate to speak with your instructor, or see your instructor's grading rubric for journal entries.

V. Classroom Contribution:

Learning in a PBL classroom is very different for most students for many different reasons. What is valued in the PBL classroom and what is considered successful takes time to understand, so most importantly you should come with an open mind and be ready to openly communicate. Be sure to communicate your learning needs to your teacher throughout the year. Here are some comments from past students:

About presenting homework solutions:

"The fact that we have to get up in front of the class helped in my learning"

"The accumulative mixture of problems the book had really helped me see the connections between concepts"
"I got more comfortable with taking mathematical risks"

"This curriculum has made me a better problem solver"

"It helped challenge me and taught me even if I didn't think I was learning"

About doing journal writing:

"Keeping a journal has really helped make reviewing and preparing for tests very easy"

“Journals totally helped, although having them on the test is useless. Once you’ve done a journal you know the subject.”

“Although I never fully bought into keeping a journal, it gave me a good resource for studying.”

About communication in class:

“I loved being able to discuss issues with classmates”

“It helps when the teacher summarizes what we learn”

“I liked finding more than one way to do something”

About getting support:

“Meeting with my teacher really helped”

“Asking questions is sign of strength not weakness”

“I liked how it was focused on yourself figuring out the problem – though that was hard for me to adjust to – however it’s made me much more independent math-wise”

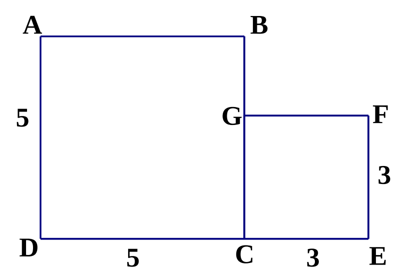
Becoming a better independent problem solver is not an easy journey, but it does need your whole-hearted curiosity and effort. The mathematics department is here to support you through this year so please make use of the support systems that are available if you feel you need them.

1. You are given the two squares and the sheets of paper (see handouts). Your task is to cut the two given squares into pieces and reform those pieces into a larger square. (Hint: Cut out the entire outline of the two squares, and cut along the dotted line shown, but do not cut the dashed line between the squares).
    a. Compare your "puzzle" to others' method. Is what you did to form the third square the same? Did you start with the same squares?
    b. Are you sure that the new shape that you formed is in fact a square? Can you justify that to your neighbor? What is enough evidence and reasoning? Write down your argument and then compare with another set of classmates.

2. *Some terminology*: In a right triangle, the *legs* are the sides adjacent to the right angle. The *hypotenuse* is the side opposite to the right angle. Given the two points A(3, 7) and B(5, 2) find C so that triangle ABC is a right triangle with the right angle at C. How long are legs? How long is the hypotenuse?

3. The length of a rectangle is $(3x - 4)$ and the width is $(2x + 1)$. Find the perimeter and area of this rectangle. What is your definition of a rectangle? Write out a sentence in your own words.

4. A 5×5 square and a 3×3 square can be cut into pieces that will fit together to form a third square.
   (a) In the diagram at right, mark *P* on segment *DC* so that *PD* = 3, then draw segments *PA* and *PF*. Calculate the lengths of these segments.
   (c) Segments *PA* and *PF* divide the squares into pieces. Arrange the pieces to form the third square.



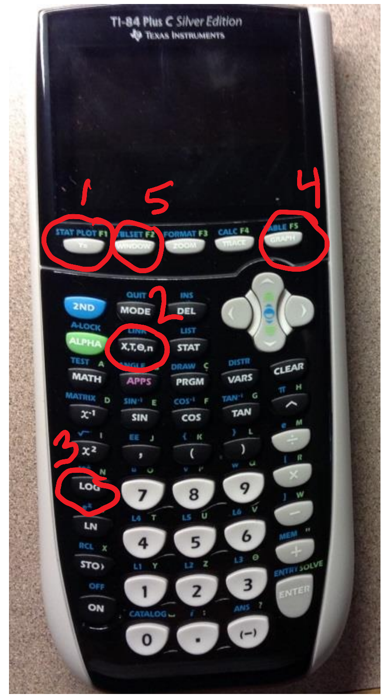
5. (Continuation) Change the sizes of the squares to *AD* = 8 and *EF* = 4, and redraw the diagram. Where should point *P* be marked this time? Form the third square again.

6. (Continuation) Will the preceding method *always* produce pieces that form a new square? If your answer is *yes*, prepare a written explanation. If your answer is *no*, provide a counterexample – two specific squares that can *not* be converted to a single square.

7. Let $A = (0, 0)$, $B = (7, 1)$, $C = (12, 6)$, and $D = (5, 5)$. Plot these points and connect the dots to form the *quadrilateral ABCD*. Verify that all four sides have the same length. Such a figure is called *equilateral*.

8. The main use of the Pythagorean Theorem is to find distances. Originally (6*th* century BC), however, it was regarded as a statement about *areas*. Explain this interpretation.

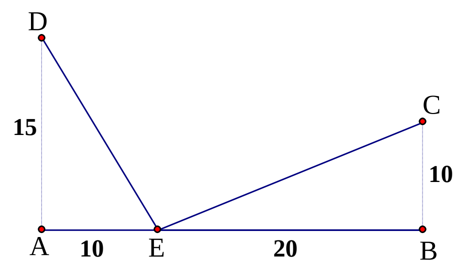
9. *Factor:* $x^2 - 5x + 6$



1. Using your graphing calculator, press the y = button and enter the parabola $y = 2x^2 - 133x + 2$. (To type x^2 press the variable button and then the x^2 button) Then Press graph.
   (a) What do you see on your screen?
   (b) Press the Window button (next to y =), and enter Xmin = -50, Xmax = 50,Ymin = -500, Ymax = 500. Press Graph. Now, what do you see?
   (c) Find a window where you can see the bottom of the curve. Which window did you pick? Present your answer as [Xmin, Xmax], [Ymin, Ymax].

2. If the hypotenuse of a right triangle is 12 and one of the legs is 4, express the length of the other leg in simplest radical form.

3. In the diagram, *AEB* is straight and angles *A* and *B* are right. Calculate the total distance *DE* + *EC*.

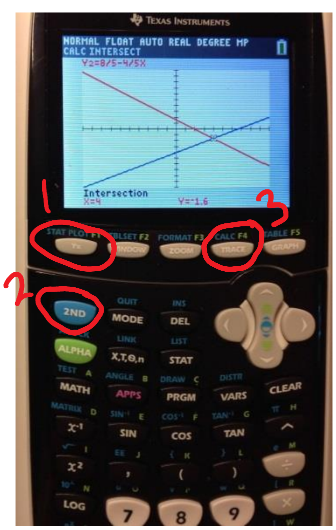


4. (Continuation) If *AE* = 15 and *EB* = 15 instead, would *DE* + *EC* be the same?

5. (Continuation) You have seen that *DE* + *EC* is dependent on the length of *AE*. Let *x* represent *AE* (and 30 − *x* for *EB*), write a formula for *DE* + *EC*. Enter this formula into Y= on your calculator, graph it, and trace to find the value of *x* that produces the *shortest* path from *D* to *C* through *E*. (The minimum of the graph).

6. Two different points on the line *y* = 2 are each exactly 13 units from the point (7, 14). Draw a picture of this situation, and then find the coordinates of these points.

7. The general notation in geometry is that points are labeled with capital letters and coordinates are defined with lowercase letters. Given the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ what do the subscripts on *x* and *y* represent? If triangle ABC is right triangle with C being the right angle, find expressions for the lengths of all three sides.



8. To find the intersection of two lines using your calculator, first graph the lines (press Y=) and make sure that you can see the intersection point in the window. If you can't, be sure to change the window accordingly. Then bring up the CALC menu (2nd trace) and choose intersect (#5). Using the arrow keys, move the cursor along one of the lines close to the point of intersection and press ENTER. The calculator will prompt you to then press ENTER on the other intersecting line. Then, press ENTER once more and read the coordinates of the intersection point from the bottom of your screen. Find the intersection of 4*x* + 5*y* = 8 and *y* = 0.6*x* – 4.

1. Give an example of a point that is the same distance from (3, 0) as it is from (7, 0). Find lots of examples. Describe the configuration of all such points. In particular, how does this configuration relate to the two given points?

2. Verify that the hexagon formed by $A = (0, 0)$, $B = (2, 1)$, $C = (3, 3)$, $D = (2, 5)$, $E = (0, 4)$, and $F = (-1, 2)$ is equilateral. Is it also *equiangular*?

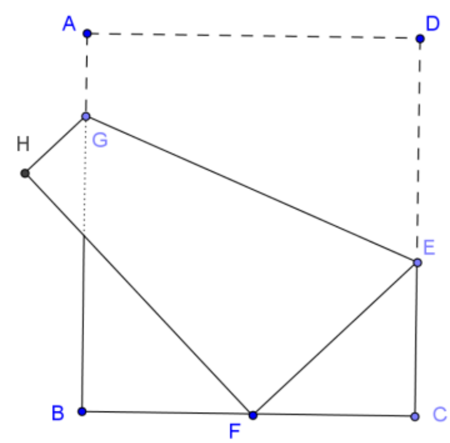
3. Draw a 20-by-20 square *ABCD*. Mark *P* on *AB* so that *AP* = 8, *Q* on *BC* so that *BQ* = 5, *R* on *CD* so that *CR* = 8, and *S* on *DA* so that *DS* = 5. Find the lengths of the sides of quadrilateral *PQRS*. Is there anything special about this quadrilateral? Explain.

4. Given the two points A(-2, 1) and B(4, 7) describe two different methods to find the distance between A and B. Which method do you prefer?

5. Una recently purchased two boxes of ten-inch candles – one box from a discount store, and the other from an expensive boutique. One evening, Una noticed that the inexpensive candles last only three hours each while the expensive candles last five hours each. The next evening, Una hosted a dinner party and lit two candles – one from each box – at 7:30 pm. During dessert, a guest noticed that one candle was twice as long as the other. At what time was this observation made?

6. You may have learned in the past that the sum of the angles in any triangle is 180°. We will prove this more rigorously later on. For now, given this, what can we say about the two non-right angles of a right triangle?

7. The dimensions of rectangular piece of paper ABCD are AB = 10 and BC = 9. It is folded so that corner D is matched with a point F on edge BC. Given that length DE = 6, find EF, EC, and F C.

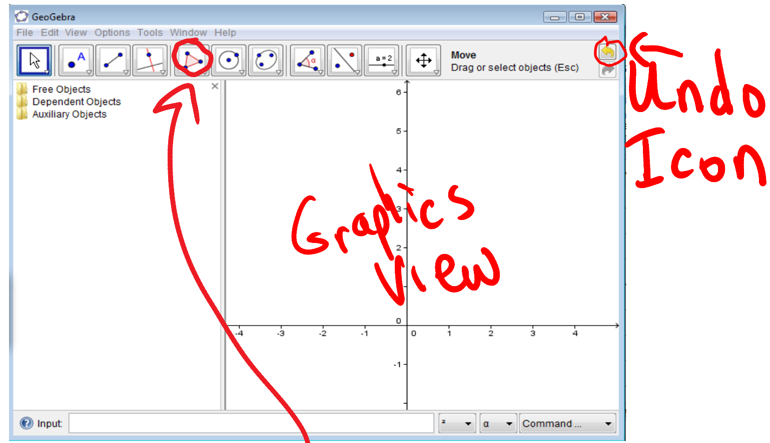


8. (Continuation) The lengths EF, EC, and FC are all functions of the length DE. The area of triangle EFC is also a function of DE. Using x to stand for DE, write formulas for these four functions.

9. (Continuation) Find the value of x that maximizes the area of triangle EFC.

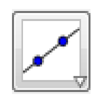
10. How would you proceed if you were asked to verify that $P = (1, -1)$ is the same distance from $A = (5, 1)$ as it is from $B = (-1, 3)$? It is customary to say that *P* is *equidistant* from *A* and *B*. Find three more points that are equidistant from *A* and *B*. By the way, to "find" a point means to find its *coordinates*. Can points equidistant from *A* and *B* be found in every *quadrant*

Geogebra Lab #1

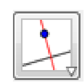


If you have never used a dynamic software program before, you should be aware of the difference between properties and objects that are "constructed" (using either a tool or a CONSTRUCT command) rather than "drawn" freehand with a tool. To observe this difference, complete the following activity

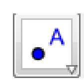
A. Under the View menu, choose Axes. This will hide the axes (or right click and select Axes)
B. Click on the polygon tool. It is 5th from the left and looks like a triangle. It is circled in the above diagram.
C. You are going to *draw* a triangle that looks right by doing the following: click somewhere on the drawing space (Graphics View) to start drawing the triangle, then click three more times (two more points) to draw a triangle that looks like a right triangle, finishing it by clicking on the first point you placed.
D. Press Escape (ESC), or click on the selection tool (the one that looks like an arrow) and click on any endpoint of a segment and move the triangle around. You will notice that the triangle changes, and is no longer a right triangle. You can undo, if you like, by pressing the yellow arrow at the top right of the screen.
E. Now you will *construct* a right triangle. In the bottom right-hand corner of the 3rd toolbox from the left (pictured at right) click on the bottom right corner of the button to expand it (the corner looks a little like a tiny white triangle, but changes to red when you can click it). Choose the tool that says "Segment between two points." Click once for each endpoint of the segment you want to create.



F. Now choose the 4th toolbox (pictured at right) which is the Perpendicular Line. Click on an endpoint of the segment and the segment itself.

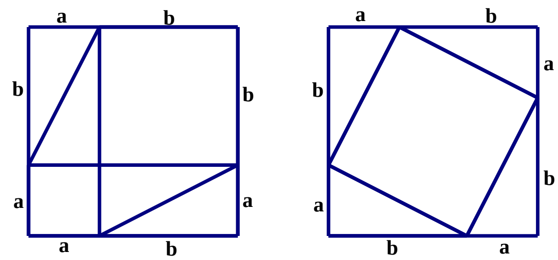


G. Place a point on the perpendicular line using the Point tool (pictured at right), 2nd from the left. When you are on the line, it is highlighted.



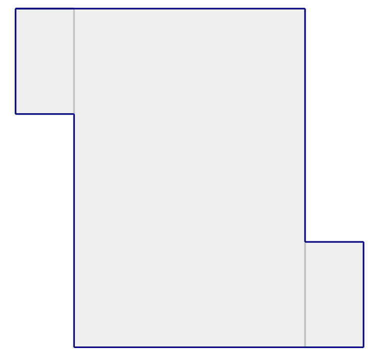
H. Choose the Polygon tool and connect your three points, clicking on each point, and making sure to again click the first point you choose at the end.
I. At the left-side of your screen you have a list of what is on your Graphics View. This section of your screen is called the Algebra View. Find the line equation – it will have *y* in it. Click on the circle to the left of the equation to hide the line. Bonus: Right-click the equation and you can turn it from Standard Form to y = mx + b form.
J. Drag the vertices of this right triangle. How is this right triangle different from the one you drew in part C? what do you think is the difference between drawing and constructing objects in GeoGebra?

1. The two-part diagram at right, which shows two different dissections of the same square, was designed to help *prove* the Pythagorean Theorem. Provide the missing details.



2. Find both points on the line $y = 3$ that are 10 units from (3, -3).

3. On a number line, where is $\frac{1}{2}(p + q)$ in relation to p and q?



4. Some terminology: Figures that have exactly the same shape and size are called *congruent*. Dissect the region shown at right into two congruent parts. How many different ways of doing this can you find?

5. Let $A = (2, 4)$, $B = (4, 5)$, $C = (6, 1)$, $T = (7, 3)$, $U = (9, 4)$, and $V = (11, 0)$. Triangles ABC and TUV are specially related to each other. Make calculations to clarify this statement, and write a few words to describe what you discover.

6. If you were writing a geometry book, and you had to define a mathematical figure called a *kite*, how would you word your definition?

7. A triangle that has at least two sides of equal length is called *isosceles*. Make up an example of an isosceles triangle, one of whose vertices is (3, 5). Give the coordinates of the other two vertices. If you can, find a triangle that does not have any horizontal or vertical sides.

8. Instead of walking along two sides of a rectangular field, Fran took a shortcut along the diagonal.
(a) Let a be the short side, b be the long side, and c her path length. Write an equation that relates these variables.
(b) By taking the shortcut along the diagonal, Fran can save a distance equal to half of the length of the longer side. Write an equation that describes this relationship.
(c) Find the length of the long side of the field, given that the length of the short side is 156 meters.

9. Let $A = (1, 5)$ and $B = (3, -1)$. Verify that $P = (8, 4)$ is equidistant from A and B. Find at least two more points that are equidistant from A and B. Describe all such points.

10. Solve for x: $\sqrt{x+1} = \sqrt{2x-3}$. Hint: you can square both sides to eliminate the radical.

11. Find two points on the y-axis that are 9 units from (7, 5).

GeoGebra Lab #2

In this lab, you will explore the various ways to create circles.

A. First, click the x in the top right corner of the Algebra View. This will close it, and give you more space to draw. You can re-open it in the View menu. Also, right click in the Graphics View window and select Axes to turn them off.

B. The tools to create circles are in the toolbox with the circle icon. It is right next to the polygon toolbox. Try out the "Circle with Center Through Point" tool by following the instructions it gives you, at the top right part of your screen. Press ESC or select the Move tool (looks like small axes) and move the two points around to see what happens. Is there any way to move the circle that doesn't change it?

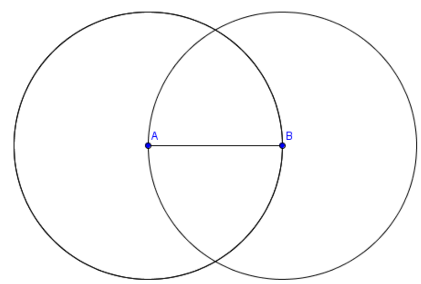
C. Right-click on the circle itself, select Object Properties, and notice that this gives you its name and how it was constructed. It also gives you several other options. Rename this circle "Circle_1." Notice that this creates a subscript.

D. Select Object Properties again. Explore some of the options in the box that pops up. Especially look through the Style tab.

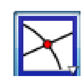
E. Now, try the Circle with Center and Radius tool. How is it different? Name this circle, "Circle_2" and change its color.

F. Right-click Circle_1 and choose Show Object. Do the same thing for Circle_2. This should hide the circles. Do this for any other objects on the screen.

G. Construct a segment. With the Compass tool, the third Circle tool, click on one endpoint of the segment, then the other endpoint to set the compass radius, then select one of the endpoints of this segment as the center of the circle. Repeat, using the other endpoint of the segment as its center. Your diagram should look like this:



H. Create the intersection points using the Intersect Two Objects tool in the Point toolbox (pictured at right). Then, connect these intersections with a line. What do you think is special about this line? Hint: How do the points on the line relate to the endpoints of the segment?



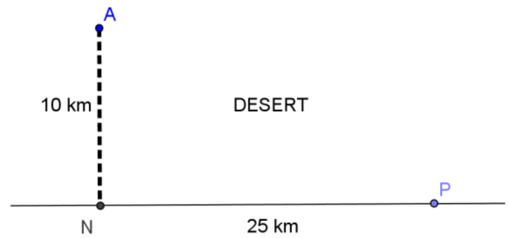
I. Find a way to check your conjecture.

1. A *lattice point* is a point whose coordinates are *integers*. For example, (2, 3) is a lattice point, but (2.5, 3) is not. Find two lattice points that are 5 units apart but do not form a horizontal or vertical line.

2. (Continuation) Find two lattice points that are exactly $\sqrt{13}$ units apart. Is it possible to find lattice points that are $\sqrt{15}$ units apart?

3. *Some terminology*: When two angles fit together to form a straight angle (a 180-degree angle, in other words), they are called *supplementary angles* and either angle is the *supplement* of the other. When an angle is the same size as its supplement (a 90-degree angle), it is called a *right angle*. When two angles fit together to form a right angle, they are called *complementary* angles and either angle is the *complement* of the other. Two lines that form a right angle are said to be *perpendicular*. Draw a diagram for each definition.

1. Alex the geologist is in the desert, 10 km from a long, straight road. On the road, Alex's jeep can do 50 kph, but in the desert sands, it can manage only 30 kph. Alex is very thirsty, and knows that there is a gas station 25 km down the road (from the nearest point N on the road) that has ice-cold Pepsi.
   (a) How many minutes will it take for Alex to drive to P through the desert?
   (b) Would it be faster if Alex first drove to N and then used the road to P?
   (c) Find an even faster route for Alex to follow. Is your route the fastest possible?



2. Two iron rails, each 50 feet long, are laid end to end with no space between them. During the summer, the heat causes each rail to increase in length by 0.04 percent. Although this is a small increase, the lack of space at the joint makes the joint buckle upward. What distance upward will the joint be forced to rise? (Assume that each rail *remains straight*, and that the other ends of the rails are anchored.) Round your answer to the nearest hundredth.

3. Graph the lines $2x - y = 5$ and $x + 2y = -10$ on a piece of graph paper on the same set of axes. Using your protractor, measure the angle of intersection.

4. Given that $2x - 3y = 17$ and $4x + 3y = 7$, and without using paper, pencil, or calculator, find the value of *x*.

5. Blair is walking along the sidewalk and sees a bird walking along the telephone pole that crosses the street she is walking along. Draw a picture of this scenario. Are their paths *parallel*? If so, why? If not, why not?

6. The point on segment *AB* that is equidistant from *A* and *B* is called the *midpoint* of *AB*. For each of the following, find coordinates for the midpoint of *AB*:
   (a) *A* = (-1, 5) and *B* = (5, -7) **(b)** *A* = (*m*, *n*) and *B* = (*k*, *l*)

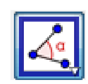
7. A unique line exists through any two points. In one form or another, this statement is a fundamental *postulate* of Euclidean geometry – accepted as true, without proof. Taking this for granted, then, what can be said about three points?

8. A river bank runs along the line $x = 3$ and a dog is tied to a post at the point D=(10,5). If the dog's leash is 25 units long (the same units as the coordinates), and if a fence were going to be placed at the edge of the river along $x = 3$, name the two coordinates along the river where it would be safe for the fence to end so that the dog could not fall in the river even though he is tethered at D. How long would the fence be?

GeoGebra Lab #3

In this lab you will explore some of the graphing and measurement features of GeoGebra for segments and angles. Answer all questions in a text box in your ggb file.

A. Open GeoGebra. Notice that a coordinate axes does not always appear in the Graphics View. Right-click and choose Axes or Select Axes from the View Menu. Currently the x and y axes are in a 1:1 ratio meaning that the scales are equal.
B. Press and hold down the shift key and click on the y axis simultaneously. A label will appear stating the scale "x:y = 1:1". Now drag the y axis up and down. What happens to that scale definition? To return the scale definition to 1:1 you can simply right click on the graphic window and choose **x axis: y axis -> 1:1**.
C. Now, choose OPTIONS→Point Capturing→Snap to Grid. This will allow drawn points to snap to lattice points.
D. With your cursor, click on the Input Command Line and simply type (4, 5) and press enter. The point (4, 5) should be plotted and labeled A.
E. Plot the points (7, 3) and (1, 0) in the same way. GeoGebra automatically labels in alphabetical order and also keeps a record of your objects in the Algebra View to the left. If the Algebra View is not there, select Algebra View from the View menu.
F. Select the Move tool (farthest to the right) and move the Graphics View down so that the first quadrant is in full view in the window.
G. Press shift and select the y axis (as described in part B). What happens to points A, B and C? Is this what you expected? Do the coordinates of A, B and C change?
H. Construct the sides of triangle ABC using the Segment between Two Points tool which is the second tool in the Line toolbox.
I. What type of triangle does ABC appear to be? Measure the angles of triangle ABC using the Angle tool (fourth from the right, pictured at right). Select the three points in a clockwise order with the vertex of the angle to be measured as the second point.



J. What information is provided in the Algebra View at this time?
K. You can easily rotate your triangle when you know a center of rotation and an angle. With the point tool, draw a point on the Graphics View. Choose the Rotate tool from the Transformation Toolbox (third from the right) and follow the directions to do the rotation using your new point D as the center. When prompted for the angle of rotation use 70 degrees and be sure to include the degree symbol in the notation. If you accidentally erase the given degree symbol, you can find it in the first drop down menu. In what direction does the rotation take your triangle?
L. Repeat step K choosing -40 degrees for your angle of rotation. Which direction does it rotate now?

1. Using GeoGebra or Geometry Pad, plot the points P(3, 5), Q(0, 0) and R(-5, 3). Measure angle PQR, being careful to select the points in a clockwise manner. Create the segments PQ and QR. Use the Slope tool in the same toolbox as the Angle tool to find the slope of segment PQ. Do the same thing for segment QR. Make a conjecture about how these slopes are related. Verify by calculating the slopes by hand.

1. Write a formula for the distance from A = (-1, 5) to P = (x, y), and another formula for the distance from P = (x, y) to B = (5, 2). Then write an equation that says that P is equidistant from A and B. Simplify your equation to linear form.

2. Alex is in the desert again, 10 km from a long straight road and 45 km from base camp, also in the desert. Base camp is also 10 km from the road on the same side of the road that Alex is. On the road, the jeep can do 50 kph, but in the desert sands, it can manage only 30 kph. Alex wants to return to base camp as quickly as possible after driving on the road to collect a dirt sample. How quickly can he manage this trip?

3. When you wrote an equation that said that the distance from A = (-1, 5) to P = (x, y) is equidistant to the distance from P = (x, y) to B = (5, 2), the line you found was called the *perpendicular bisector of AB.* Verify this by calculating two slopes and one midpoint.

4. Is it possible to form a square whose area is 18 by connecting four lattice points? Explain.

5. Find the slope of the line through
   (a) (3, 1) and (3 + 4t, 1 + 3t); **(b)** (m − 5, n) and (5 + m, n^2).

6. Is it possible for a line $ax + by = c$ to lack a y-intercept? To lack an x-intercept? Explain.

7. Factor: **(a)** x^2-16; **(b)** $x^2+8x+16$**;** **(c)** $x^2+6x-16$

8. Find the point of intersection of the lines $3x + 2y = 1$ and $-x+y=-2$.

9. The sides of a triangle are formed by the graphs of $3x + 2y = 1$, $y = x - 2$ and $-4x+9y=22$. Use GeoGebra or Geometry Pad to discover if the triangle is isosceles. How do you know?

10. Consider the linear equation $y = 3.5(x - 1.3) + 2$.
    (a) What is the slope of this line?
    (b) What is the value of y when $x = 1.3$?
    (c) This equation is written in *point-slope* form. Explain the terminology.
    (d) Use your calculator or GeoGebra to graph this line.
    (e) Find an equation for the line through (4.2, -2.5) that is parallel to this line. Leave your answer in point-slope form.
    (f) Describe how you would graph by hand a line that has slope -2 and that goes through the point (-7, 3).

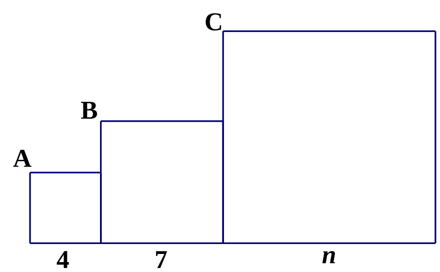
11. Given the points A = (-2, 7) and B = (3, 3), find two points P that are on the perpendicular bisector of AB. In each case, what can be said about the triangle PAB?

GeoGebra Lab #4: Investigating Point-Slope Form

A. Open the GeoGebra file Point-Slope Lab. You can find it where you find your class' assignments.
B. How did the slope slider change the line? What is the difference between negative and positive values?
C. How did the xcorr slider change the line? What is the difference between positive and negative values? How are these reflected in the equation of the line?
D. How did the ycorr slider change the line? What is the difference between positive and negative values? How are these reflected in the equation of the line?

1. For each of the following questions, fill in the blank with always true (A), never true (N), or sometimes true (S). Please write a few sentences explaining your choice.
   (a) Two parallel lines are ________ coplanar.
   (b) Two lines that are not coplanar _______ intersect.
   (c) Two lines parallel to the same plane are ________ parallel to each other.
   (d) Two lines parallel to a third line are ________ parallel to each other.
   (e) Two lines perpendicular to a third line are ________ perpendicular to each other.

2. A slope can be considered to be a *rate*. Explain this interpretation and give an example. Explain the difference between a line that has *undefined slope* and a line whose slope is zero.



3. Three squares are placed next to each other as shown. The vertices *A*, *B*, and *C* are *collinear*. Find the dimension *n*.

4. (Continuation) Replace the lengths 4 and 7 by *m* and *k*, respectively. Express *k* in terms of *m* and *n*.

5. A five-foot tall Deerfield student casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

6. (Continuation) How far from the streetlight should the student stand in order to cast a shadow that is exactly as long as the student is tall?

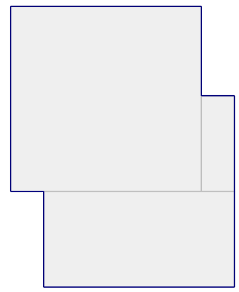
7. An airplane 27000 feet above the ground begins descending at the rate of 1501.5 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?

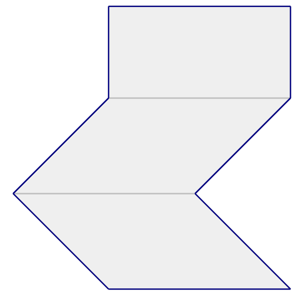
8. (Continuation) Graph the line $y = 27000 - 1501.5x$, using an appropriate window on your calculator. With the preceding problem in mind, explain the significance of the slope of this line and its two intercepts.

1. In a dream, Blair is confined to a coordinate plane, moving along a line with a constant speed. Blair's position at 4 am is (2, 5) and at 6 am it is (6, 3). What is Blair's position at 8:15 am when the alarm goes off?

2. Find a way to show that points A = (-4, -1), B = (4, 3), and C = (8, 5) are collinear.

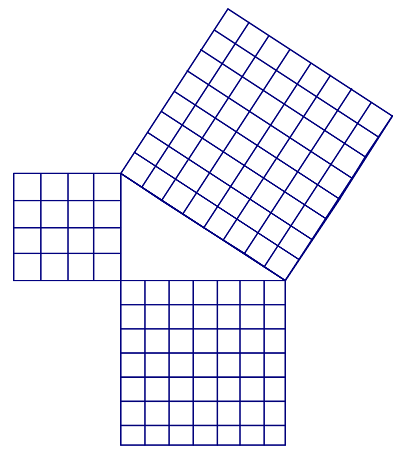
3. Find as many ways as you can to dissect each figure at right into two congruent parts.





4. An airplane is flying at 36000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.

5. Is there enough evidence in the given diagram at the right to conclude that the triangle is right? Explain why or why not?



6. Golf balls cost $0.90 each at Leonard's Club, which has an annual $25 membership fee. At Alex & Taylor's sporting goods store, the price is $1.35 per ball for the same brand. Where you buy your golf balls depends on how many you wish to buy. Explain, and illustrate your reasoning by drawing a graph.

7. Graph the following segments. What do they have in common?
   (a) From (3, -1) to (10, 3); (**b**) From (1.3, 0.8) to (8.3, 4.8);
   (**c**) From ($\sqrt{3}$, $\sqrt{2}$) to ($7+\sqrt{3}$, $4+\sqrt{2}$).

8. (Continuation) The above segments all have the *same* length and the *same* direction. Each represents the *vector* [7, 4]. The horizontal *component* of the vector is positive 7 and the vertical component is positive 4.
   (a) Find another example of two points that represent this vector. The initial point of your segment is called the *tail* of the vector, and the final point is called the *head*
   (b) Which of the following segments represents vector [7, 4]? From (-2, -3) to (5, -1); from (-3, -2) to (11, 6); from (10, 5) to (3, 1); from (-7, -4) to (0, 0).

9. Given the line $y=\frac{3}{4}(x+3)-2$ and the point (9, 2). Using point-slope form, write equations for the lines parallel and perpendicular to this line through the given point.

10. Let P = (a, b), Q = (0, 0), and R = (-b, a), where a and b are positive numbers. Prove that angle PQR is right, by introducing two congruent right triangles into your diagram by connecting points P and Q to the x-axis. Using these two triangles, verify that the slope of *segment QP* is the *opposite reciprocal* of the slope of segment QR.

1. Show that the triangle formed by the lines $y = 2x - 7$, $x + 2y = 16$, and $3x + y = 13$ is isosceles. Show also that the lengths of the sides of this triangle fit the Pythagorean equation. Can you identify the right angle just by looking at the equations?

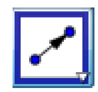
2. The perimeter of an isosceles right triangle is 24 cm. How long are its sides?

3. A clock takes 3 seconds to chime at 3 pm, how long does it take to chime at 6 pm? Hint: The answer may be based on your interpretation of the question.

GeoGebra Lab #5

Using GeoGebra, you can facilitate visualization of vector translations.

A. Open a new GeoGebra file and hide the axes in the Graphics View.
B. Select the regular polygon tool from the polygon toolbox and construct a small regular pentagon.
C. Draw a vector somewhere else in the Graphics View, using the fifth tool in the Line toolbox (pictured at right). This vector is going to serve as your vector for a translation of the regular polygon, so the length of the vector will be how far the polygon is moved and the slope of the vector is the slope it will be moved by. The direction of the vector is denoted by the arrow at its head.



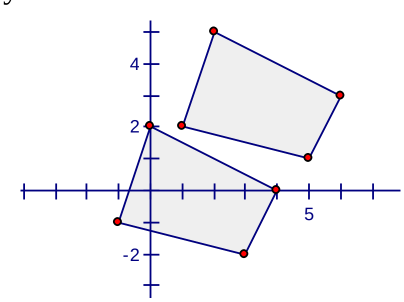
D. Now select the Translate tool for the Transformation toolbox (fourth toolbox from the right) and follow the directions to translate the regular pentagon by the vector you had drawn.
E. Now, press escape (to give you back the Move tool) and drag the original vector by the point at the head end. What happens to the translated pentagon? Does anything happen to the original pentagon? Why or why not?

4. A triangular plot of land has boundary lines of 45 meters, 60 meters, and 70 meters. The 60 meter boundary runs north-south. Is there a boundary line for the property that runs due east-west?

5. Using GeoGebra or Geometry Pad, plot the points A = (-5, 0), B = (5, 0), and C = (2, 6), K = (5, -2), L = (13, 4), and M = (7, 7). Find the lengths of each side and the measure of each angle of the triangles ABC and KLM. It is customary to call two triangles *congruent* when all corresponding sides and angles are the same.

6. (Continuation) Are the triangles related by a vector translation? Why?

7. Let $A = (1, 2)$, $B = (5, 1)$, $C = (6, 3)$, and $D = (2, 5)$. Let $P = (-1, -1)$, $Q = (3, -2)$, $R = (4, 0)$, and $S = (0, 2)$. Use a vector to describe how quadrilateral $ABCD$ is related to quadrilateral $PQRS$. What is the length of this vector?



1. Let K = (3, 8), L = (7, 5), and M = (4, 1). Find coordinates for the vertices of the triangle that is obtained by using the vector [2, -5] to slide triangle KLM. How *far* does each vertex slide?

2. The *length of a vector* is defined as the hypotenuse of the right triangle created by its *components*. The horizontal component of the vector [-1, 7] is -1 and the vertical component is 7. What is the length of the vector [-1, 7]? What is the length of vector [a, b]? Some notation: the length of a vector is written as $\left|[a,b]\right|$.

3. Let A = (2, 4), B = (4, 5), and C = (6, 1). Draw three new triangles as follows:
   (a) ΔPQR has P = (11, 1), Q = (10, -1), and R = (6, 1);
   (b) ΔKLM has K = (8, 10), L = (7, 8), and M = (11, 6);
   (c) ΔTUV has T = (-2, 6), U = (0, 5), and V = (2, 9).
   These triangles are not obtained from ABC by applying a vector translation. Instead, each of the appropriate transformations is described by one of the suggestive names *reflection*, *rotation*, or *glide-reflection*. Decide which is which, with justification.

4. The senior grass can be approximated by the triangle seen in the picture at the right. The juniors are jealous of the seniors and they want to copy the senior grass onto the lawn behind the MSB. They have a limited amount of time so they measure <u>one of the sides</u> and create a congruent segment on the lawn. If they do not do any more measurements does this guarantee that the junior triangle will be congruent to the senior triangle? Sketch a diagram of this scenario. Another group measured only one angle and created a congruent angle on the field. If they do not do any more measurements does this guarantee that the junior triangle will be congruent to the senior triangle? Sketch a diagram.

5. In baseball, the bases are placed at the corners of a square whose sides are 90 feet long. Home plate and second base are at opposite corners. How far is it from home plate to second base to two decimal places?

6. Give the components of the vector whose length is10 and that points in the opposite direction of [-4, 3].

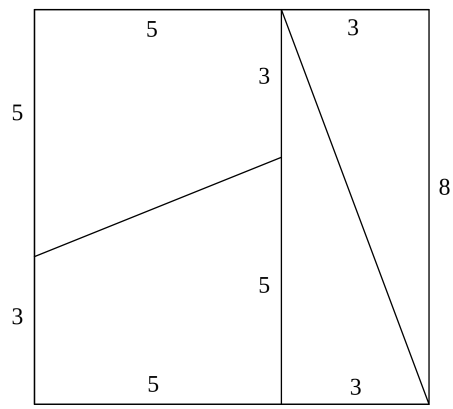
7. A 9-by-12 rectangular picture is framed by a border of uniform width. Given that the combined area of picture plus frame is 180 square units, find the width of the border.

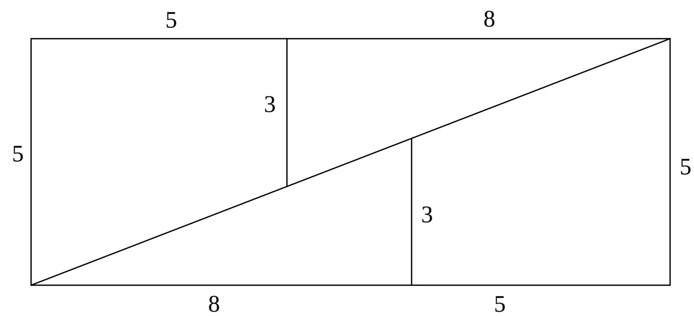
8. Let A = (0, 0), B = (2, -1), C = (-1, 3), P = (8, 2), Q = (10, 3), and R = (5, 3). Plot these points. Angles BAC and QPR should look like they are the same size. Find evidence to support this conclusion.

1. The juniors realize that copying a single measurement will not guarantee an exact copy of the senior triangle. They decide to try measuring two parts. What are the combinations of two corresponding parts that they could measure? Does the use of any of these pairs ensure congruent triangles?

2. An equilateral quadrilateral is called a *rhombus*. A square is a simple example of a rhombus. Find a non-square rhombus whose *diagonals* and sides are *not* parallel to the rulings on your graph paper. Use coordinates to describe its vertices. Write a brief description of the process you used to find your example.

3. Compare the two figures shown below. Is there anything wrong with what you see? Write a few sentences justifying your answer.





4. A bug is initially at (-3, 7). Where is the bug after being displaced by vector [-7, 8]?

5. Plot points $K = (0, 0)$, $L = (7, -1)$, $M = (9, 3)$, $P = (6, 7)$, $Q = (10, 5)$, and $R = (1, 2)$. Show that the triangles *KLM* and *RPQ* are congruent. Show also that neither triangle is a vector translation of the other. Describe how one triangle has been transformed into the other.

6. What is the slope of the line $ax + by = c$? Find an equation for the line through the origin that is perpendicular to the line $ax + by = c$.

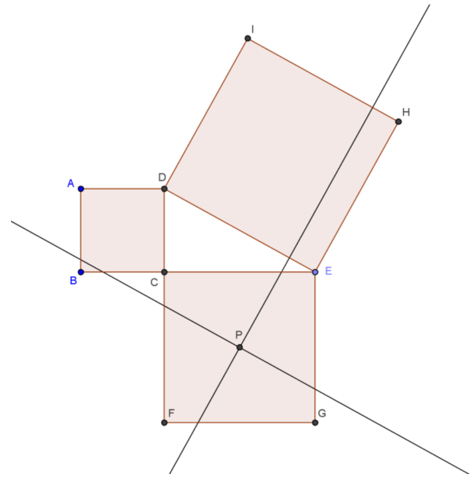
7. Let $A = (3, 2)$ and $B = (7, -10)$. What is the displacement vector that moves point A onto point B? What vector moves B onto A?

8. Realizing that one or two corresponding parts do not ensure congruent triangles the juniors conjecture that they must use three parts to create a new junior triangle. Create a table of the possibilities. Which do you think will work and why?

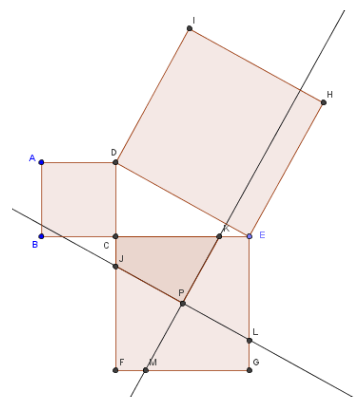
9. (Continuation) Your teacher will post some links on your moodle site entitled "Triangle Congruence Criteria Links." For each link, answer the following questions:
    a. Do you believe that this combination of three parts of a triangle are enough to create a unique triangle? If yes, why and if no, why not?
    b. How does this GeoGebra applet help justify your conjecture

GeoGebra Lab #6

In the following lab, you will prove the Pythagorean Theorem by "dissection" – explain this terminology after you have completed the lab.



A. Open the GeoGebra file called Pythagorean Puzzle. You can find it where you get your assignments.
B. The squares in the diagram are based on the sides of a right triangle. This will geometrically represent the a^2, b^2 and c^2 in the Pythagorean Theorem that most students know algebraically. Notice that the space in the middle is a right triangle.
C. Construct the diagonals of the square based on the side CE.
D. Construct the intersection of those diagonals using the Intersect Two Objects tool in the point toolbox.
E. Hide the two diagonals, but not the center point. Right-click this point, choose Rename and rename it P.
F. You will now make some pieces to put together as a puzzle. Using the Perpendicular line tool, draw a line perpendicular to the hypotenuse of the center triangle (segment DE), through P. Then, draw the line parallel to the hypotenuse through P. See the diagram at top right for what it will look like.
G. Now, Using the Intersect Two Objects tool, create the intersections between the lines and the sides of square CEGF. You will create four intersections in total.
H. Using the intersection points you just created, you will now make some "puzzle pieces." Create the "northwest" using the first polygon tool and selecting first the point below C, then C, then the point to the right of C, point P, and finally clicking back on the point below C. See the bottom diagram at right.



I. Create the other three portions of the larger square, using the polygon tool and the same procedure. When you are finished, hide the parallel and perpendicular lines.
J. Right-click a polygon in the big square and select Object Properties. A window will pop up and there will be a list of objects on the left-hand side. Poly2 will be highlighted. Uncheck the box next to "Show Object". Then, select the Quadrilateral **category heading**, which will select all of the quadrilaterals at the same time, and will allow you to control the properties of all of the quadrilaterals at once. On the Style Tab change Opacity to at least 50%. Hit OK to close this Object Properties box.
K. Right-click the Quadrilaterals one by one, and change their color in the Color tab in Object Properties. Do the same for the squares as well.
L. You will be moving the pieces you created, plus the square attached to the other leg of the right triangle, with vectors. Create 5, relatively short, horizontal vectors using the Vector between Two Points tool, in the Lines Toolbox (3rd from the left). Place them, spaced out, around your diagram.
M. In the Transformation Toolbox, 4th from the right, select the Translate Object by Vector tool. Select one of the puzzle pieces and then the vector you want to use to move that piece. It is helpful to rename your vectors with the color of the piece they are moving.

<Lab Continues on the next page>

GeoGebra Lab #6, continued

N. Move the pieces by selecting the point at the head of the vectors and changing its direction. Try to get them to fit into the square based on the hypotenuse. You may need to move the screen using the Move tool (double arrow) or move the vectors themselves, as well as change their direction.

O. Why does this prove the Pythagorean Theorem?

1. We know that $|[3, 4]| = 5$; how do we find a vector that points in the same direction and has length 1? We call a vector of length 1 a *unit vector*.

2. Let $M = (a, b)$, $N = (c, d)$, $M' = (a + h, b + k)$, and $N' = (c + h, d + k)$. Use the distance formula to show that segments MN and $M'N'$ have the same length. Explain why this could be expected.

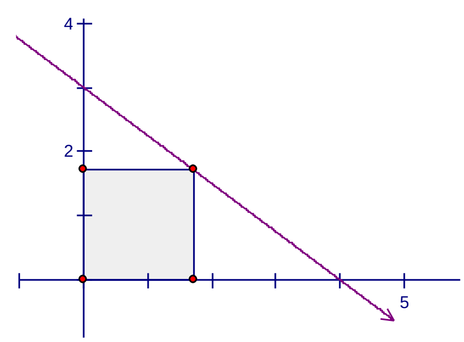
3. *Some terminology*: When the components of the vector [5, -7] are multiplied by a given number t, the result may be written either as $[5t, -7t]$ or as $t[5, -7]$. This is called the *scalar multiple* of vector [5, -7] by the *scalar t*. Find components for the following scalar multiples:
   (a) [12, -3] by scalar 5 **(b)** $[\sqrt{5}, \sqrt{10}]$ by scalar $\sqrt{5}$
   (c) $\left[-\frac{3}{4}, \frac{2}{3}\right]$ by scalar $-\frac{1}{2}+\frac{2}{6}$ **(d)** $[p, q]$ by scalar $\frac{1}{pq}$

4. Two of the sides of a right triangle have lengths $360\sqrt{2008}$ and $480\sqrt{2008}$. Find the possible lengths for the third side.

5. If two figures are congruent, then their parts *correspond*. In other words, each part of one figure has been matched with a definite part of the other figure. Given congruent triangles KLM and RPQ, in the triangle *RPQ*, which angle corresponds to angle *M*? Which side corresponds to *KL*? In general, what can be said about corresponding parts of congruent figures? How might you confirm your hunch experimentally?

6. A nice acronym to shorten the statement about corresponding parts of congruent triangles can be written as CPCTC. What do you think these letters represent?

7. The diagram at right shows the graph of $3x + 4y = 12$. The shaded figure is a square, three of whose vertices are on the coordinate axes. The fourth vertex is on the line. Find
   (a) the x- and y-intercepts of the line;
   (b) the length of a side of the square.
   (c) Show that your point is equidistant from the coordinate axes.



8. Given the vector [-5, 12], find the following vectors:
   (a) same direction, twice as long **(b)** same direction, length 1
   (c) opposite direction, length 10 **(d)** opposite direction, length c

1. One of the legs of a right triangle is twice as long as the other and the perimeter of the triangle is 28. Find the lengths of all three sides, to three decimal places.

2. Find the lengths of the following vectors:
   (a) [3, 4] **(b)** 2006 [3, 4] **(c)** $\frac{2008}{5}$ [3, 4] **(d)** t[3, 4]
   (e) In terms of *a* and *b:* t[*a*, *b*]

3. A triangle has six principal parts – three sides and three angles. The SSS criterion states that three of these items (the sides) determine the other three (the angles). What other combinations of three parts determine the remaining three? In other words, if the class is given three measurements with which to draw and cut out a triangle, which three measurements will guarantee that everyone's triangles will be congruent?

4. The vector that is defined by a segment *AB* is often denoted $\overrightarrow{AB}$. Given A(1, 1) and B(3, 5); (**a)** Use the midpoint formula to find the midpoint of $\overline{AB}$; (**b)** find the vector $\overrightarrow{AB}$ and multiply by $\frac{1}{2}$. **(c)** Translate A by $\frac{1}{2}\overrightarrow{AB}$. Did you expect this result?

5. Let $A = (1, 4)$, $B = (0, -9)$, $C = (7, 2)$, and $D = (6, 9)$. Prove that angles *DAB* and *DCB* are the same size. Can anything be said about the angles *ABC* and *ADC*?

6. Plot the three points $P = (1, 3)$, $Q = (5, 6)$, and $R = (11.4, 10.8)$. Verify that $PQ = 5$, $QR = 8$, and $PR = 13$. Can you draw any conclusions about these points?

7. Sidney calculated three distances of the collinear points *A*, *B*, and *C*. She reported them as $AB = 29$, $BC = 23$, and $AC = 54$. What do you think of Sidney's data, and why?

8. Find the number that is two thirds of the way **(a)** from -7 to 17; **(b)** from *m* to *n*.

9. After drawing the line $y = 2x - 1$ and marking the point $A = (-2, 7)$, Kendall is trying to decide which point on the line is closest to *A*. The point $P = (3, 5)$ looks promising. To check that *P* really is the point on $y = 2x - 1$ that is closest to *A*, what would help Kendall decide? Is *P* closest to *A*?

10. Let $K = (-2, 1)$ and $M = (3, 4)$. Find coordinates for the two points that divide segment *KM* into three congruent segments.

11. Let $A = (-5, 2)$ and $B = (19, 9)$. Find coordinates for the point *P* between *A* and *B* that is three fifths of the way from *A* to *B*. Find coordinates for the point *Q* between *A* and *B* that is three fifths of the way from *B* to *A*.

12. Given the points $K = (-2, 1)$ and $M = (3, 4)$, find coordinates for a point *J* that makes angle *JKM* a right angle.

1. When two lines intersect, four angles are formed. It is not hard to believe that the nonadjacent angles in this arrangement are congruent. If you had to prove this to a skeptic, what reasons would you offer? These pairs of angles are called *vertical angles*.

2. Find a point on the line $2x + y = 8$ that is equidistant from the coordinate axes. How many such points are there?

3. Let $A = (2, 9)$, $B = (6, 2)$, and $C = (10, 10)$. Verify that segments *AB* and *AC* have the same length. Measure angles *ABC* and *ACB*. On the basis of your work, propose a general statement that applies to any triangle that has two sides of equal length. Write an argument supporting your assertion, which might be called the *Isosceles Triangle Theorem*.

4. A line goes through the points (2, 5) and (6, -1). Let *P* be the point on this line that is closest to the origin. Calculate the coordinates of *P*.

5. Given that $P = (-1, -1)$, $Q = (4, 3)$, $A = (1, 2)$, and $B = (7, k)$, find the value of *k* that makes the line *AB* **(a)** parallel to *PQ*; **(b)** perpendicular to *PQ*.

6. Let $A = (-6, -4)$, $B = (1, -1)$, $C = (0, -4)$, and $D = (-7, -7)$. Show that the opposite sides of quadrilateral *ABCD* are parallel. Such a quadrilateral is called a *parallelogram*.

7. The sides of a right triangle are $x - y$, x, and $x + y$, where *x* and *y* are positive numbers and $y < x$. Find the ratio of *x* to *y*.

8. Find the components of a vector that is three fifths as long as [24, 7].

9. Let $A = (0, 0)$, $B = (4, 2)$, and $C = (1, 3)$, find the exact size of angle *CAB*. Justify your answer without your protractor.

10. Let $A = (3, 2)$, $B = (1, 5)$, and $P = (x, y)$. Find *x*- and *y*-values that make *ABP* a right angle.

11. (Continuation) Describe the configuration of all such points *P*.

12. Find coordinates for the vertices of a *lattice rectangle* that is three times as long as it is wide with none of the sides horizontal.

13. Find components for the following vectors $\overrightarrow{AB}$:
(a) $A = (1, 2)$ and $B = (3, -7)$ **(b)** $A = (2, 3)$ and $B = (2 + 3t, 3 - 4t)$

14. If $C = (-2, 5)$ and $D = (-3, 9)$, find components for the vector that points
(a) from *C* to *D* **(b)** from *D* to *C*

15. If *M* is the midpoint of segment *AB*, how are vectors $\overrightarrow{AM}$, $\overrightarrow{AB}$, $\overrightarrow{MB}$, and $\overrightarrow{BM}$ related?

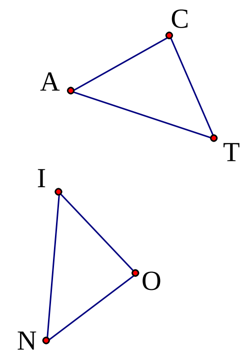
16. Show that the lines $3x - 4y = -8$, $x = 0$, $3x - 4y = 12$, and $x = 4$ form the sides of a rhombus.

1. Choose a point P on the line $2x + 3y = 7$, and draw the vector [2, 3] with its tail at P and its head at Q. Confirm that the vector is perpendicular to the line. What is the distance from Q to the line? Repeat the preceding, with a different choice for point P.

2. Given the points D, A, and Y with the property $DA = 5$, $AY = 7$, and $DY = 12$. What can be said about these three points? What would be true if DY is less than 12?

3. Describe a transformation that carries the triangle with vertices (0, 0), (13, 0), and (3, 2) onto the triangle with vertices (0, 0), (12, 5), and (2, 3). Where does your transformation send the point (6, 0)? If you cannot find the exact coordinates make your best guess.

4. Suppose that triangle ACT has been shown to be congruent to triangle ION, with vertices A, C, and T corresponding to vertices I, O, and N, respectively. It is customary to record this result by writing $\Delta ACT \cong \Delta ION$. Notice that corresponding vertices occupy corresponding positions in the equation. Let $B = (5, 2)$, $A = (-1, 3)$, $G = (-5,-2)$, $E = (1, -3)$, and $L = (0, 0)$. Using only these five labels, find as many pairs of congruent triangles as you can, and express the congruences accurately.



5. (Continuation) How many ways are there of arranging the six letters of $\Delta ACT \cong \Delta ION$ to express the two-triangle congruence?

6. What can be concluded about triangle ABC if it is given that (a)$\Delta ABC \cong \Delta BCA$? **(b)** $\Delta ABC \cong \Delta ACB$? (two separate situations)

7. Plot points $K = (-4, -3)$, $L = (-3, 4)$, $M = (-6, 3)$, $X = (0, -5)$, $Y = (6, -3)$, and $Z = (5, 0)$. Show that triangle KLM is congruent to triangle XZY. Describe a transformation that transforms KLM onto XZY. Where does this transformation send the point (-5, 0)?

8. Write a proof that the two acute angles in a right triangle are complementary.

9. Given $A = (6, 1)$, $B = (1, 3)$, and $C = (4, 3)$, find a lattice point P that makes the vectors $\overrightarrow{AB}$ and $\overrightarrow{CP}$ perpendicular.

10. (Continuation) Write an equation of a line in point-slope form that describes the set of points P for which $\overrightarrow{AB}$ and $\overrightarrow{CP}$ are perpendicular.

11. Let $A = (0, 0)$, $B = (1, 2)$, $C = (6, 2)$, $D = (2, -1)$, and $E = (1, -3)$. Show that angle CAB is the same size as angle EAD. You may want to use GeoGebra or Geometry Pad to help you solve this problem.

12. *Triangle Inequality Theorem:* What must be true about the three sides of a triangle for it to exist?

13. Geometrically, what does "closest" mean? Write a definition in your own words.

GeoGebra Lab #7

Altitudes

In this lab you will construct the altitudes (or heights) of a triangle and investigate their properties.

A. Open a new GeoGebra file on your computer. Turn on the axes in the Graphics View.
B. With the polygon tool, draw an arbitrary triangle ABC (nothing special about it).
C. With the angle tool, measure all three angles by clicking on the triangle. Make sure that ABC is an acute triangle. If it is not, press escape and move the vertices until all three angles are acute.
D. An *altitude* of a triangle is a segment that joins one of the three vertices *perpendicularly* to a point on the line that contains the opposite side. To construct the line that contains the altitude to AB, select the perpendicular line tool (fourth from the left) and select point C and segment AB.
E. Do the same for the other two altitude lines to their respective sides. Press escape when finished.
F. Notice that the equations of three lines have appeared in the Algebra View to the left. As you click on each equation, the lines in the Graphic View should turn bold to denote which line the equation is representing. If you would rather have the altitude equations in slope/intercept form, right click on the equations and choose Equation y = mx+b. This will be a helpful way for you to check your answers in other problems.
G. What do you notice about the three altitude lines? Construct the point of intersection of these lines with the Intersect Two Objects tool in the Point Toolbox.
H. Change the name of the point (it should be automatically named D) by right clicking on the point of intersection and selecting Object Properties from the drop-down menu. In the field entitled "name" change D to Orthocenter (which is the name for the intersection of the altitudes of a triangle).
I. Save this sketch as GeoGebra Altitude Lab on your computer.

Answer the following questions in a textbox on your graphics view:

1. Make a conjecture about the three altitudes of a triangle. What do you think is always true?
2. Test your conjecture by dragging one or more vertices around the sketch screen. What do you observe? Does this support your conjecture?
3. What do you observe when the triangle is obtuse?
4. What do you observe when the triangle is right?
5. What do you observe when the triangle is acute?

1. Let $A = (0, 0)$, $B = (8, 1)$, $C = (5, -5)$, $P = (0, 3)$, $Q = (7, 7)$, and $R = (1, 10)$. Prove that angles *ABC* and *PQR* have the same size.

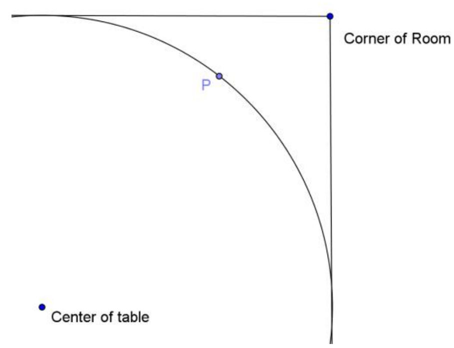
2. (Continuation) Let *D* be the point on segment *AB* that is exactly 3 units from *B*, and let *T* be the point on segment *PQ* that is exactly 3 units from *Q*. What evidence can you give for the congruence of triangles *BCD* and *QRT*?

3. What is true about all of the points that lie on the perpendicular bisector of a segment?

1. The diagonals AC and BD of quadrilateral $ABCD$ intersect at O. Given the information $AO = BO$ and $CO = DO$, what can you deduce about the lengths of the sides of the quadrilateral? Prove your response.

2. An *altitude* of a triangle is a segment that joins one of the three vertices to a point on the line that contains the opposite side, the intersection being *perpendicular*. For example, consider the triangle whose vertices are $A = (0, 0)$, $B = (8, 0)$, and $C = (4, 12)$.
   (a) Find the length of the altitude from C to side AB.
   (b) Find an equation for the line that contains the altitude from A to side BC.
   (c) Find an equation for the line BC.
   (d) Find coordinates for the point where the altitude from A meets side BC.
   (e) Find the length of the altitude from A to side BC.
   (f) As a check on your work, calculate BC and multiply it by your answer to part (e). You should be able to predict the result.
   (g) It is possible to deduce the length of the altitude from B to side AC from what you have already calculated. Show how.

3. Find a point on the line $x + 2y = 8$ that is equidistant from the points (3, 8) and (9, 6).



4. A circular table is placed in a corner of a room so that it touches both walls. A mark is made on the edge of the table, exactly 18 inches from one wall and 25 inches from the other. What is the radius of the table?

5. If a quadrilateral is equilateral, its diagonals are perpendicular. True or false? Why?

6. Make up a geometry problem to go with the equation $x + 3x + x\sqrt{10} = 42$.

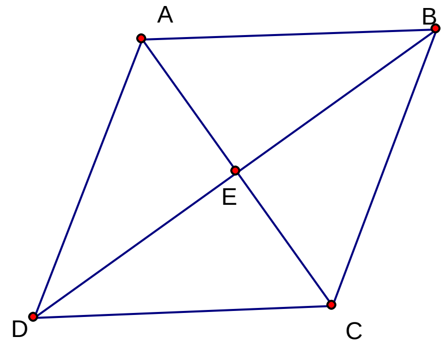
7. Let $A = (-2, 3)$, $B = (6, 7)$, and $C = (-1, 6)$.
(a) Find an equation for the perpendicular bisector of AB.
(b) Find an equation for the perpendicular bisector of BC.
(c) Find coordinates for a point K that is equidistant from A, B, and C.

8. Consider the triangle defined by $P = (1, 3)$, $Q = (2, 5)$, and $R = (6, 5)$. The *transformation* defined by $T(x, y) = (x + 2, y - 1)$ is mathematical notation for translating a point by the vector [2, -1]. The point P(1, 3) becomes T(1, 3) = (1 + 2, 3 – 1) = P'(3, 2). Find Q' and R'. Graph both the original triangle and its *image*.

9. (Continuation) The transformation $T(x, y) = (y + 2, x - 2)$ is a reflection. Verify this by calculating the effect of T on triangle PQR. Sketch triangle PQR, find coordinates for the *image points P''*, *Q''*, and *R''*, and sketch the *image triangle P''Q''R''*. Then, identify the mirror line and add it to your sketch. Notice that triangle PQR is labeled in a clockwise sense; what about the labels on triangle $P''Q''R''$?

Fill in the blanks to complete the proof logically:

1. Prove that in a rhombus, the diagonals create four congruent triangles.



We know that $AB \cong BC \cong CD \cong DA$ because

_______________________________________.

Since B is equidistant from A and C, point B must

_______________________________________.

Since B lies on the perpendicular bisector of AC,

____________ is the midpoint of AC, and therefore

segment________ $\cong$ segment ________.

Similarly, since $DC \cong CB$ meaning that C lies on the perpendicular bisector of DB, this means that segment________ $\cong$ segment ________.

So, we can say that $\Delta AED \cong \Delta AEB \cong \Delta CEB \cong \Delta CED$

By ____________________.

Therefore, four congruent triangles are formed.

2. Given the following picture, and that HF and JG bisect each other at point E, prove $\angle H \cong \angle F$ **.**

Since HF and JG bisect each other at point E, we can say that

_______________ $\cong$ _________________ and

_______________ $\cong$ _________________.

We can also say that the pair of angles

$\angle$ ______________ and $\angle$ ________________

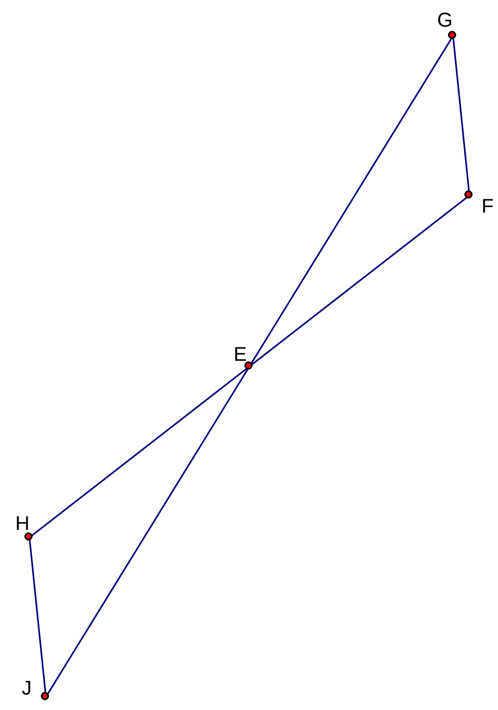
are congruent to each other because they are vertical angles.

Therefore, by ______________

we can say that the triangles ____________ and

____________ are congruent.

So, $\angle H \cong \angle F$ because _________________________.



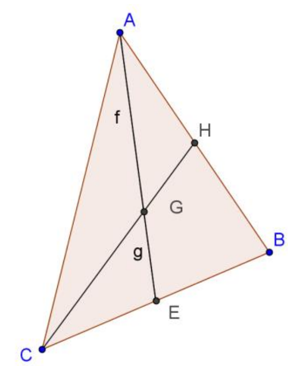
GeoGebra Lab #8

Medians

A. Open a new Geogebra file and draw an arbitrary triangle.
B. Using the Midpoint tool from the Point toolbox, construct the midpoint of each side of the triangle. Right-click one of the midpoints, and in Object Properties, change the color of each midpoint.
C. A *median* of a triangle is a segment that connects a vertex to the midpoint of the side opposite to it. Construct the medians of this triangle with the segment tool.
D. Construct the intersection of the medians. Remember to use the Intersect Two Objects tool. This point is called the *centroid* of the triangle. Right-click to rename this point G (if it is not already named this).

Answer the following questions:

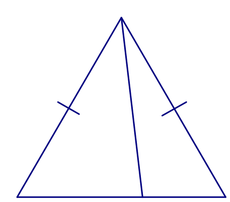
1. What can be said about the three medians of a triangle?
2. Do the properties that you observed for the orthocenter hold true for this point? Test your conjecture by making the triangle right, obtuse and acute.
3. This point, the centroid, has another special property:
   a. If necessary, turn on the Algebra View. You can do this in the View menu. From your diagram identify two points that are the endpoints of a median.
   b. Using the Input Bar at the bottom of the screen type "ratio1=CG/GH" and press Enter. This will calculate for you the ratio of the longer portion of the median (from vertex to centroid) to the shorter portion of the median (from centroid to midpoint)
   c. Do the same for the other two medians, giving them appropriate names and vertex letters (like ratio2 and AG/GE)
   d. Find these values on the Algebra View. They should be near the top of the list.



4. What value do you get for the ratios? What do these ratios tell you about the segments that are on the median? What is the ratio of the vertex to the centroid to the whole median?
5. Save this file as "GeoGebra Medians Centroid Theorem"

1. In quadrilateral *ABCD*, it is given that *AB* = *CD* and *BC* = *DA*. Prove that angles *ACD* and *CAB* are the same size. Note: If a polygon has more than three vertices, the *labeling convention* is to place the letters around the polygon in the order that they are listed. Thus, *AC* should be one of the diagonals of *ABCD*.

2. Use the diagram at right to help you explain why SSA evidence is not by itself sufficient to justify the congruence of triangles. The tick marks designate segments that have the same length.



1. Find the area of the triangle defined by $E(-2, 8)$, $W(11, 2)$, and $S(-2, -4)$. Now, find the area of triangle WLS where L is (-2, 0).

2. A segment from one of the vertices of a triangle to the midpoint of the opposite side is called a *median*. Consider the triangle defined by $A = (-2, 0)$, $B = (6, 0)$, and $C = (4, 6)$.
(a) Find an equation for the line that contains the median drawn from A to BC.
(b) Find an equation for the line that contains the median drawn from B to AC.
(c) Find coordinates for G, the intersection of the medians from A and B.
(d) Let M be the midpoint of AB. Determine whether or not M, G, and C are collinear.

3. The line $3x + 2y = 16$ is the perpendicular bisector of the segment AB. Find coordinates of point B, given that **(a)** $A = (-1, 3)$; **(b)** $A = (0, 3)$.

4. (Continuation) Point B is called the *reflection of A across the line* $3x + 2y = 16$; sometimes B is simply called the *image* of A. Explain this terminology. Using the same line, find another point C and its image C'. Explain your method for finding your pair of points.

5. A rhombus has 25-cm sides, and one diagonal is 14 cm long. How long is the other diagonal?

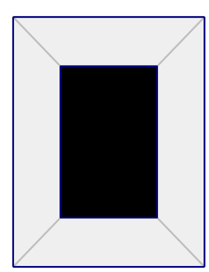
6. Let $A = (0, 0)$ and $B = (12, 5)$, and let C be the point on segment AB that is 8 units from A. Find coordinates for C.

7. Prove that one of the diagonals of a kite is bisected by the other.

8. Let $A = (1, 4)$, $B = (8, 0)$, and $C = (7, 8)$. Find the area of triangle ABC.

9. Sketch triangle PQR, where $P = (1, 1)$, $Q = (1, 2)$, and $R = (3, 1)$. For each of the following, apply the given transformation T to the vertices of triangle PQR, sketch the image triangle $P'Q'R'$. It is advisable to sketch the respective images in different colors from the original triangle PQR. Then decide which of the terms *reflection*, *rotation*, *translation*, or *glide-reflection* accurately describes the action of T. Provide appropriate detail to justify your choices.
(a) $T(x, y) = (x + 3,\ y - 2)$ **(b)** $T(x, y) = (y,\ x)$
(c) $T(x, y) = (-x + 2, -y + 4)$ **(d)** $T(x, y) = (x + 3, -y)$

10. Robin is mowing a rectangular field that measures 24 yards by 32 yards, by pushing the mower around and around the outside of the plot. This creates a widening border that surrounds the un-mowed grass in the center. During a brief rest, Robin wonders whether the job is half done yet. How wide is the uniform mowed border when Robin *is* half done?



11. Triangle ABC is isosceles, with $AB = BC$, and angle $\boldsymbol{BAC}$ is 56 degrees. Find the remaining two angles of this triangle.

1. Terry walked one mile due north, two miles due east, then three miles due north again and then once more east for 4 miles. How far is Terry from her starting point?

2. Triangle *ABC* is isosceles, with *AB* = *BC*, and angle ***ABC*** is 56 degrees. Find the remaining two angles of this triangle.

3. Find the area of the triangle whose vertices are *A* = (-2, 3), *B* = (6, 7), and *C* = (0, 6).

4. Let *A* = (-4, 0), *B* = (0, 6), and *C* = (6, 0).
   (a) Find equations for the three lines that contain the altitudes of triangle *ABC*.
   (b) Show that the three altitudes are *concurrent*, by finding coordinates for their common point. The point of concurrence is called the *orthocenter* of triangle *ABC*.

5. Pat and Chris were out in their rowboat one day, and Chris spied a water lily. Knowing that Pat liked a mathematical challenge, Chris announced that, with the help of the plant, it was possible to calculate the depth of the water under the boat. When pulled taut, directly over its root, the top of the plant was originally 10 inches above the water surface. While Pat held the top of the plant, which remained rooted to the lake bottom, Chris gently rowed the boat five feet. This forced Pat's hand to the water surface. Use this information to calculate the depth of the water.

6. Prove that if triangle *ABC* is isosceles, with *AB* = *AC*, then the medians drawn from vertices *B* and *C* must have the same length.

7. Find *k* so that the vectors [4,-3] and [*k*, -6]
   (a) point in the same direction; **(b)** are perpendicular.

8. Let *A* = (-4, 0), *B* = (0, 6), and *C* = (6, 0).
   (a) Find equations for the three medians of triangle *ABC*.
   (b) Show that the three medians are concurrent, by finding coordinates for their common point. The point of concurrence is called the *centroid* of triangle *ABC*.

9. Given points *A* = (0, 0) and *B* = (-2, 7), find coordinates for *C* and *D* so that *ABCD* is a square.

10. Let *A* = (0, 12) and *B* = (25, 12). If possible, find coordinates for a point *P* on the *x*-axis that makes angle *APB* a right angle

11. The lines $3x + 4y = 12$ and $3x + 4y = 72$ are parallel. Explain why, and then find the distance that separates these lines. You will have to decide what "distance" means in this context.

12. Give an example of an equiangular polygon that is not equilateral.

GeoGebra Lab #9

Perpendicular Bisectors

1. Open a new GeoGebra file and draw an arbitrary triangle.
2. Select the Perpendicular Line Tool, and select each midpoint and side of the triangle to construct the perpendicular bisectors of each side. (There is also a perpendicular bisector tool you can use!)
3. Notice that the equations of three lines have appeared in the Algebra View to the left.
4. Construct the intersection point of all of the perpendicular bisectors. Change the name of the point to "Circumcenter."
5. Save this sketch as GeoGebra Perpendicular Bisector Lab on your computer.

Answer the following questions in a textbox on your graphics view:

1. Move your triangle around observe what happens to the circumcenter. What happens to this point when the triangle is right?
2. What happens to this point when the triangle is obtuse?
3. What happens to this point when the triangle is acute?
4. Why might the circumcenter and the orthocenter behave in the same ways with regards to the position of the triangle?
5. The circumcenter also has another interesting property. Recall the property of perpendicular bisectors discussed in class. The intersection of the perpendicular bisectors then has that property for both segments. So what do you think is true of the circumcenter?
6. Check your conjecture by selecting the circle tool. With the cursor click on the circumcenter and drag the mouse to one of the vertices of the triangle (it doesn't matter which one – why not?). Describe the circle in relation to the triangle.

1. On a separate sheet of paper, draw parallelogram $PQRS$ with vertices at $P(0, 0)$, $Q(1, 3)$, $R(6, 2)$, and $S(5, -1)$. Cut out your parallelogram and dissect it to form a rectangle. What can you conclude about the area of a parallelogram?

2. Prove that a diagonal of a square divides it into two congruent triangles.

3. Find the area of the parallelogram whose vertices are (0, 0), (7, 2), (8, 5), and (1, 3).

4. Given the points $A = (0, 0)$, $B = (7, 1)$, and $D = (3, 4)$, find coordinates for the point C that makes quadrilateral $ABCD$ a parallelogram. What if the question had requested $ABDC$ instead?

5. Find a vector that is perpendicular to the line $3x - 4y = 6$.

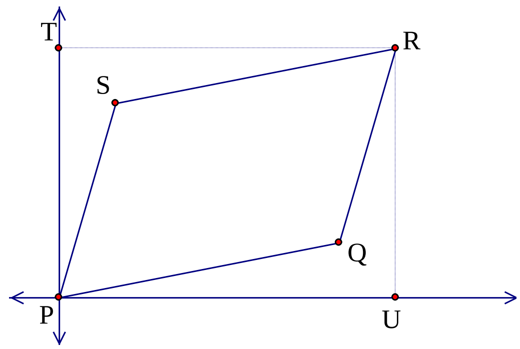
6. Let $P = (-1, 3)$. Find the point Q for which the line $2x + y = 5$ serves as the perpendicular bisector of segment PQ.

7. Find points on the line $3x + 5y = 15$ that are equidistant from the coordinate axes.

8. Plot all points that are 3 units from the x-axis. Describe the configuration. Then, plot all the points 3 units from (5, 4) and describe their configuration.

1. In triangle *ABC*, it is given that *CA* = *CB*. Points *P* and *Q* are marked on segments *CA* and *CB*, respectively, so that angles *CBP* and *CAQ* are the same size. Prove that *CP* = *CQ*.

2. The figure at right shows a parallelogram *PQRS*, three of whose vertices are *P* = (0, 0), *Q* = (*a*, *b*), and *S* = (*c*, *d*). You can also see that *TRUP* is a rectangle. All of your expressions should be in terms of *a*, *b*, *c*, and *d*.
(a) Find the coordinates of *R*.
(b) Write an expression for the area of the rectangle *TRUP*.
(c) Find expressions for the areas of triangles *TSR*, *TSP*, *PQU*, and *RQU*.
(d) Find an expression for the area of *PQRS*, and simplify your formula.
(e) There are two limitations on this formula. What do you think they are?

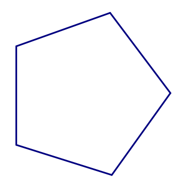


3. Let *A* = (3, 4), *B* = (0,-5), and *C* = (4,-3). Find equations for the perpendicular bisectors of segments *AB* and *BC*, and coordinates for their common point *K*. Calculate lengths *KA*, *KB*, and *KC*. Why is *K* also on the perpendicular bisector of segment *CA*?

4. (Continuation) A *circle* centered at *K* can be drawn so that it goes through all three vertices of triangle *ABC*. Explain. This is why *K* is called the *circumcenter* of the triangle. In general, how do you locate the circumcenter of a triangle?

5. *Some Terminology:* Draw a parallelogram whose *adjacent* edges are determined by vectors [2, 5] and [7, -1], placed so that they have a common initial point. This is called placing vectors *tail-to-tail*. Find the area of the parallelogram.

6. A polygon that is both equilateral and equiangular is called *regular*. Prove that all diagonals of a regular *pentagon* (five sides) have the same length.



7. Solve for x: $\sqrt{x+1}=7$

8. Write an equation that says that the distance from point (*x*, *y*) to (3, 5) is equal to the distance from the point (*x*, *y*) to (7, -1). Do not simplify your formula.

9. Find the area of the parallelogram whose vertices are (2, 5), (7, 6), (10, 10), and (5, 9).

10. Let *E* = (2, 7) and *F* = (10, 1). There are two points on line *EF* that are 3 units from *E*. Use vectors to find coordinates for both of them.

GeoGebra Lab #10

A. Open your sketch titled GeoGebra Perpendicular Bisectors Lab. Hide lines as before.
B. With the segment tool, create the segment connecting the Orthocenter and the Centroid.
C. Select the Slope tool in the Measurements toolbox (5th from the right). Click on the segment.
D. Create another segment, with the segment tool, connecting the Centroid and the Circumcenter. Using the Slope tool, find the slope of this segment, being careful to select the correct segment, if there is an overlap.
E. Make a conjecture about the slope between the Orthocenter and the Circumcenter.
F. Select one vertex of the triangle ABC and move it around on your sketch. What do you conjecture about these three points?

1. Find coordinates for the point equidistant from (-1, 5), (8, 2), and (6, -2).

2. Simplify the equation $\sqrt{(x-3)^2+(y-5)^2}=\sqrt{(x-7)^2+(y+1)^2}$. Interpret your result.

3. Use the diagram at right to help you explain why SSA evidence is not by itself sufficient to justify the congruence of triangles. The tick marks designate segments that have the same length.

4. Let A = (1, 3), B = (7, 5), and C = (5, 9). Answer the item below that is determined by *the first letter of your last name*. Algebraically find coordinates for the requested point.
   (a-e) Show that the three medians of triangle ABC are concurrent at a point G.
   (f-m) Show that the three altitudes of triangle ABC are concurrent at a point H.
   (n-z) Show that the perpendicular bisectors of the sides of triangle ABC are concurrent at a point K. What special property does K have?

5. Find an equation for the line through point (7, 9) that is perpendicular to vector [5, -2].

6. Describe a transformation that carries the triangle with vertices (1, 2), (6, 7), and (10, 5) onto the triangle with vertices (0, 0), (7, -1), and (9, 3). Where does your transformation send (7, 4)?

7. A triangle that has a 13-inch side, a 14-inch side, and a 15-inch side has an area of 84 square inches. Accepting this fact, find the lengths of all three altitudes of this triangle.

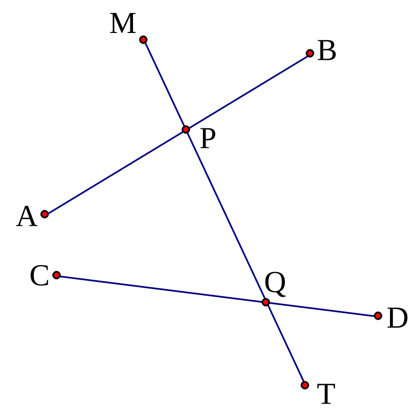
8. Find the lengths of *all* the altitudes of the triangle whose vertices are (0, 0), (3, 0), and (1, 4).

9. Given that (-1, 4) is the reflected image of (5, 2), find an equation for the line of reflection.

10. Point (0, 1) is reflected across the line $2x + 3y = 6$. Find coordinates for its image.

1. The *converse* of a statement of the form "If A then B" is the statement "If B then A." Write the converse of the statement "If it is Tuesday, we have sit down lunch."

2. (Continuation) "If point P is equidistant from the coordinate axes, then point P is on the line $y = x$".
(a) Write the converse of the given statement. Is it true?
(b) Give an example of a true statement whose converse is false.
(c) Give an example of a true statement whose converse is also true.



3. The diagram at right shows lines *APB* and *CQD* intersected by line *MPQT*, which is called a *transversal*. There are two groups of angles: one group of four angles with vertex at P, and another group with vertex at Q. There is special terminology to describe pairs of angles –one from each group. If the angles are on different sides of the transversal, they are called *alternate*, for example, angles *APM* and *PQD*. Angle *BPQ* is an *interior* angle because it is between the lines *AB* and *CD*. Thus, angles *APQ* and *PQD* are called *alternate interior*, while angles *QPB* and *PQD* are called *Same Side Interior*. On the other hand, the pair of angles *MPB* and *PQD* – which are non-alternate angles, one interior, and the other exterior – is called *corresponding*. Refer to the diagram and name
(a) the other pair of alternate interior angles;
(b) the other pair of same side interior angles;
(c) the angles that correspond to *CQT* and to *TQD*.

4. Let $P = (2, 7)$, $B = (6, 11)$, and $M = (5, 2)$. Find a point D that makes $\overrightarrow{PB} = \overrightarrow{DM}$. What can you say about quadrilateral *PBMD*?

5. The diagonals of quadrilateral *ABCD* intersect perpendicularly at O. What can be said about quadrilateral *ABCD*?

6. What do you call **(a)** an *equiangular quadrilateral*? **(b)** an *equilateral quadrilateral*? **(c)** a *regular quadrilateral?*

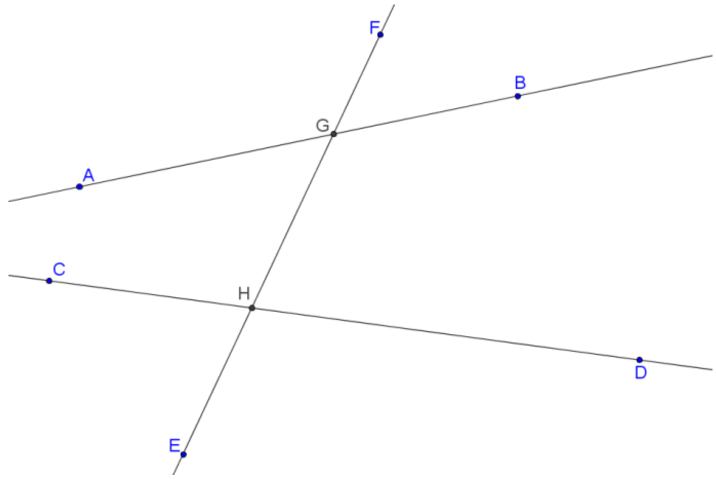
7. In quadrilateral *ABCD*, it is given that vectors $\overrightarrow{AB} = \overrightarrow{DC}$. What kind of a quadrilateral is *ABCD*? What can be said about the vectors $\overrightarrow{AD}$ and $\overrightarrow{BC}$?

8. Find the area of a triangle formed by placing the vectors [3, 6] and [7, 1] tail-to-tail.

9. (Continuation) Describe the same triangle using a different pair of vectors, remaining tail-to-tail.

10. (Continuation) Find the length of the longest altitude of your triangle.

GeoGebra Lab #11



In this lab, we will discover properties of angles formed by two lines (or segments) cut by a transversal.

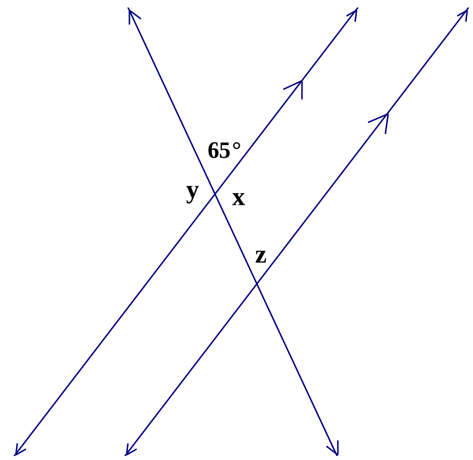
A. Open a GeoGebra file and turn off the axes in the Graphics View.
B. Draw two **lines**, not necessarily parallel, and a transversal, as in the diagram. Construct the intersection points and label as shown in the diagram
C. Measure all eight of the angles using the Angle tool. Recall that in order to measure an angle, you need to select the three points that define that angle in a clockwise order.
D. Move all angle measurement labels so that they are readable.
E. Press escape and select a point on either line (not the intersection). All angle measure labels should remain in view, but the measure of the angles should change.
F. In the Algebra View, make sure the equations of the lines AB and CD are in slope/intercept form and compare the slopes.
G. Drag one of the lines so that they are parallel to each other.
H. Fill in the chart below. Name a pair of each type of angle given and state the angle measurement that you observe in your Graphics View when the lines are parallel. Finally, state the relationship that exists (if any) between the angles in that pair.

Angle Type	**First Pair name and angle measurement**	**Second Pair name and angle measurement**	**Relationship?**
Corresponding			
Alternate Interior			
Same Side Interior			

I. For each type of angles, make a conjecture about the relationship of the lines. What is the requirement for your conjectures to be true?
J. Are the converses of these statements true?

1. You have recently seen that there is no generally reliable SSA criterion for congruence. If the angle part of such a correspondence is a *right* angle, however, the criterion *is* reliable. Justify this so-called *hypotenuse-leg* criterion (which is abbreviated HL).

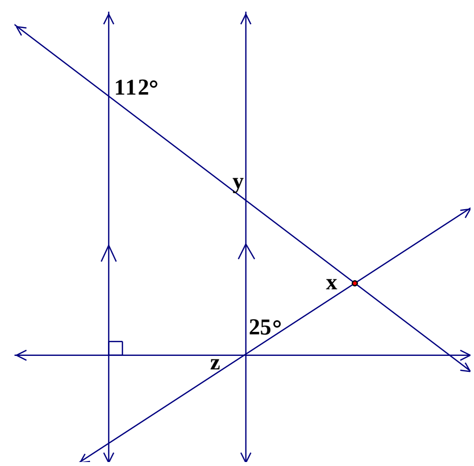
2. For the diagram at the right, find the measure of the angles indicated. Notice the custom of marking arrows on lines to indicate that they are known to be parallel.



3. It is a postulate (assumed without proof) that given two parallel lines cut by a transversal, corresponding angles are congruent. Given two parallel lines cut by a transversal, prove that a pair of alternate interior angles is congruent.

4. Asked to reflect the point $P = (4, 0)$ across the mirror line $y = 2x$, Aubrey reasoned this way: First mark the point $A = (1, 2)$ on the line, then use the vector [-3, 2] from P to A to reach from A to $P' = (-2, 4)$, which is the requested image. What did Aubrey do wrong? Explain and find out where P' should be.

5. For the diagram at the right, find the measure of the angles indicated.



6. Given isosceles triangle *ABC* where $AB = BC = 10$ and the altitude from *B* has length 4. Find the length of the base. Leave your answer in simplest radical form.

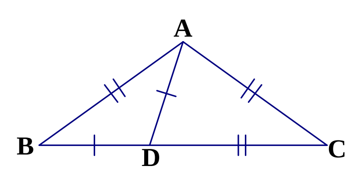
7. You probably know that the sum of the angles of a triangle is a straight angle. One way to confirm this is to draw a line through one of the vertices, parallel to the opposite side. This creates some alternate interior angles. Finish the demonstration.

8. If it is known that one pair of alternate interior angles are congruent, what can be said about **(a)** the other pair of alternate interior angles? **(b)** either pair of alternate exterior angles? **(c)** any pair of corresponding angles? **(d)** either pair of non-alternate interior angles?

9. Suppose that *ABCD* is a square and that *CDP* is an equilateral triangle, with *P outside* the square. What is the size of angle *PAD*?

10. Recall that a quadrilateral that has two pairs of parallel opposite sides is called a *parallelogram*. What can be said about the angles of such a figure?

11. In the figure at right, it is given that *BDC* is straight, $BD = DA$, and $AB = AC = DC$. Find the size of angle *C*.



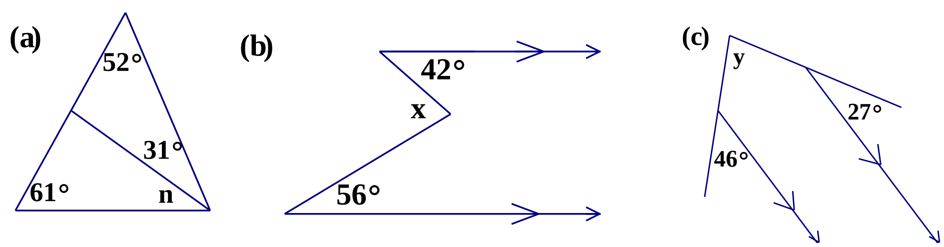
GeoGebra Lab #12

The Exterior Angle Theorem

A. Open GeoGebra and draw an arbitrary triangle ABC. It's helpful to turn off the Axes and Grid so that your drawing space is cleaner.
B. Using the ray tool in the Segment toolbox, extend side AB past B.
C. Place a point, D, on ray AB past B so that B is between A and D.
D. Measure angle DBC, this is called an *exterior angle* of this triangle. Also measure angles BAC, BCA which are called the *remote interior angles* to angle DBC. Why do you think these angles are named this way?
E. You will use the Input bar create a measurement that is the sum of these two angle measures. First, find the first remote interior angle on the Algebra view. Right click it and rename it p (so that you do not have to deal with the Greek letter assigned it). Rename the other remote interior angle as q and the exterior angle as e. Then, in the input bar, type: sum = p + q.
F. Create a text box. In the box, type Exterior Angle = , then notice that under where you are typing are some drop down menus. Select the category Objects, and select "e", the name of your exterior angle. The preview of your text will show up in the bottom of the window. Finally, click OK to close the text box window.
G. Create another text box and type Sum of Remote Interior Angles = and then select sum from the drop down list of Objects.
H. Drag one of the vertices of the triangle. What do you observe?
I. What do you think the relationship between the exterior angle and one of its remote interior angles would be if this was an isosceles triangle? Why?

1. Given an arbitrary triangle, what can you say about the *sum* of the three exterior angles, one for each vertex of the triangle?

2. In the diagrams below, the goal is to find the sizes of the angles marked with letters, using the given numerical information.



3. Prove that the sum of the angles of any quadrilateral is 360 degrees.

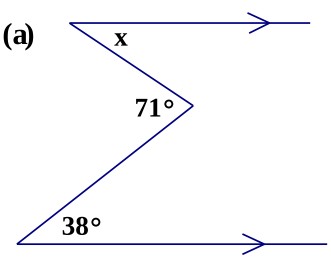
4. Write the Pythagorean Theorem in if…then form. State the converse of the Pythagorean Theorem.

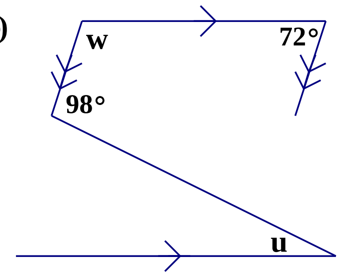
1. Fill in the following table

Number of sides of polygon	3	4	5	6	7	8	…	*n*
Number of non-overlapping triangles	1	2					…	
Total sum of the angles	180°	360°					…	
One Angle on a regular *n*-sided polygon							…	

2. Given parallelogram *PQRS*, let *T* be the intersection of the bisectors of angles *P* and *Q*. Without knowing the sizes of the angles of *PQRS*, calculate the size of angle *PTQ*. Recall that the diagonals of a parallelogram are not necessarily the angle bisectors.

3. In the figures below, find the sizes of the angles indicated by letters:

(a)


(b)


4. Mark the point *P* inside square *ABCD* that makes triangle *CDP* equilateral. Calculate the size of angle *PAD*.

5. If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent. What is the converse of this statement? Is the converse true?

6. Find the measure of an interior angle of a regular decagon.

7. If *ABC* is any triangle, and *TAC* is one of its exterior angles, then what can be said about the size of angle *TAC*, in relation to the other angles of the figure?

8. In regular pentagon *ABCDE*, draw diagonal *AC*. What are the sizes of the angles of triangle *ABC*? Prove that segments *AC* and *DE* are parallel.

9. Given square *ABCD*, let *P* and *Q* be the points outside the square that make triangles *CDP* and *BCQ* equilateral. Prove that triangle *APQ* is also equilateral.

10. The sides of an equilateral triangle are 12 cm long. How long is an altitude of this triangle? What are the angles of a right triangle created by drawing an altitude? How does the short side of this right triangle compare with the other two sides? Please leave your lengths in simplest radical form.

1. If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent. Explain. What is the converse of this statement? Is it true?

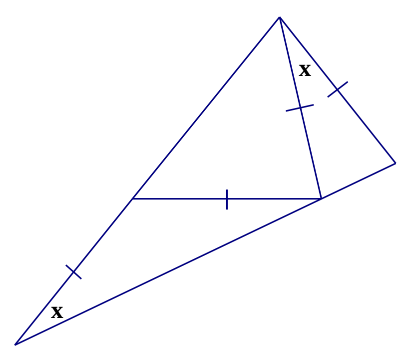
2. In triangle *ABC*, it is given that angle *A* is 59 degrees and angle *B* is 53 degrees. The altitude from *B* to line *AC* is extended until it intersects the line through *A* that is parallel to segment *BC*; they meet at *K*. Calculate the size of angle *AKB*.

GeoGebra Lab #13

A. Using GeoGebra, draw an acute-angled, non-equilateral triangle *ABC* and construct the circumcenter, *K*. Once you have constructed your circumcenter, hide your lines.
B. Measure $\angle A$ and $\angle BKC$, $\angle B$ and $\angle CKA$, and $\angle C$ and $\angle AKB$. What do you conjecture is the relationship between these angles?
C. Select side AB and construct its median. Then, click on side AB and see what its name is. In the input bar, create a new variable, l, that is half the length of AB. Just type "l= " and then the name of side AB divided by two.
D. Create a text box that shows the length of the median and the length of your new variable l. Move the vertices of *ABC* so that the median is equal to l. What is special about this triangle? What can be said about the median to the hypotenuse of a right triangle?
E. Is it true that the midpoint of the hypotenuse of a right triangle is equidistant to all three vertices? If so, why? If not, give a counterexample.

3. Given square *ABCD*, let *P* and *Q* be the points outside the square that make triangles *CDP* and *BCQ* equilateral. Segments *AQ* and *BP* intersect at *T*. Find angle *ATP*.

4. At the zoo, Tate walks along the exterior boundary of a four-sided lion's enclosure, writing down the number of degrees turned at each corner. What is the sum of these four numbers?



5. If the diagonals of a quadrilateral bisect each other then the figure is a parallelogram. Prove that this is so. What about the converse statement?

6. In the figure at right, there are two *x*-degree angles, and four of the segments are congruent as marked. Find *x*.

7. Jackie walks along the boundary of a five-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these five numbers?

8. Marty walks along the boundary of a seventy-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these seventy numbers?

9. The preceding two questions illustrate the *Sentry Theorem*. What does this theorem say, and why has it been given this name?

1. A rectangle with area 540 has one side of length 15. Find the length of the other side and the diagonals.

2. Can two of the angle bisectors of a triangle intersect perpendicularly? Try arguing with what's called a *Proof by Contradiction*. This is when you assume the statement is true and come up with a contradiction to an already known fact.

3. A right triangle has a 24-cm perimeter and its hypotenuse is twice as long as its shorter leg. To the nearest hundredth of a cm, find the lengths of all three sides of this triangle.

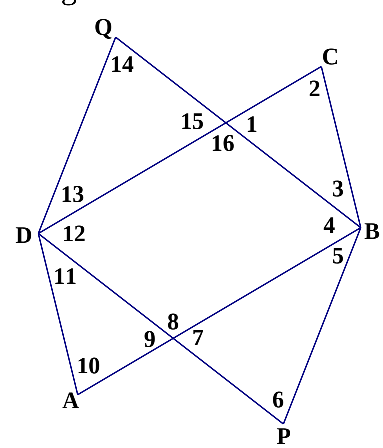
4. Suppose that quadrilateral *ABCD* has the property that *AB* and *CD* are congruent and parallel. Is this enough information to prove that *ABCD* is a parallelogram? Explain.

5. The *midsegment* of a triangle is a segment that connects the midpoints of two sides of the triangle. Given a triangle with coordinates $A(1, 7)$, $B(5, 3)$ and $C(-1, 1)$ find the segment that connects the midpoints of sides *AB* and *AC*, label the midpoints *M* and *N*, respectively.
(a) Find the length of the midsegment *MN* and compare it to the length of *BC*.
(b) What can be said about the lines containing segments *BC* and *MN*?

6. Given rectangle *ABCD*, let *P* be the point outside *ABCD* that makes triangle *CDP* equilateral, and let *Q* be the point outside *ABCD* that makes triangle *BCQ* equilateral. Prove that triangle *APQ* is also equilateral.

7. A regular, *n*-sided polygon has 18-degree exterior angles. Find the integer *n*.

8. Draw a triangle *ABC*, and let *AM* and *BN* be two of its medians, which intersect at *G*. Extend *AM* to the point *P* that makes $GM = MP$. Prove that *PBGC* is a parallelogram.



9. Triangle *EWS* has a perimeter of 26 and $SE = \frac{1}{2} WE$. The midsegment parallel to $WS = 4$. Find the lengths of the three sides of this triangle.

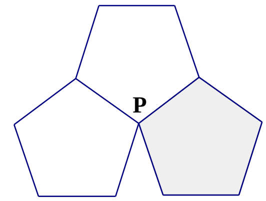
10. In the figure at right, it is given that *ABCD* and *PBQD* are parallelograms. Which of the numbered angles must be the same size as the angle numbered 1? Give a reason for each angle.

11. Triangle *PQR* has a right angle at *P*. Let *M* be the midpoint of *QR* and let *F* be the point where the altitude through *P* meets *QR*. Given that angle *FPM* is 18 degrees, find the sizes of angles *Q* and *R*.

12. Given that *ABCDEFG*. . . is a regular *n*-sided polygon, with angle $CAB = 12$ degrees, find *n*.

13. *Midsegment (Midline) Theorem:* State the properties of the segment that connects the midpoints of two sides of a triangle.
A triangle is created by placing the vectors [7, 4] and [1, 3] tail-to-tail. State a vector that represents a midsegment of this triangle.

1. Draw triangle *ABC* so that angles *A* and *B* are both 42 degrees. Why should *AB* be longer than *BC*? Extend *CB* to *E*, so that *CB* = *BE*. Mark *D* between *A* and *B* so that *DB* = *BC*, then draw the line *ED*, which intersects *AC* at *F*. Find the size of angle *CFD*.



2. The diagram at right shows three congruent regular pentagons that share a common vertex *P*. The three polygons do not quite surround *P*. Find the size of the uncovered acute angle at *P*.

3. (Continuation) If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the value of *n* that makes the three polygons fit perfectly.

4. How can one tell whether a given quadrilateral is a parallelogram? In other words, how much evidence is needed to be sure of such a conclusion? Hint: There are six ways to definitively show that a quadrilateral is a parallelogram.

GeoGebra Lab #14

The Centroid Theorem

A. Using GeoGebra, draw an acute, scalene triangle *ABC*, and two of its medians (as segments), AM and BN. Let G be the point where AM intersects BN.
B. Create a vector from G to M, then press escape.
C. Using the Translate Object By a Vector tool, translate M to a point P so that GM = MP. When selecting the vector, you need to click on it in the Algebra View. Draw a segment MP and change its color in the Object Properties box.
D. Create a vector from G to N and use it to translate N to a point Q so that GN = NQ. Draw a segment NQ and change its color.
E. Draw in segments PC and QC. Why must corresponding sides PC and BG be parallel? Hint: Look at the diagonals of the quadrilateral PCGB. Similarly, why must AG and QC be parallel?
F. What type of quadrilateral is *PCQG*? Justify your answer.
G. Find two segments in your sketch that must have the same length as *BG*. Measure them on your sketch to justify. What property of parallelograms also justifies this?
H. What must be the relationship of *BG* and *GN* considering the fact that *GN* = *NQ*?

5. You are given a square *ABCD* and midpoints *M* and *N* are marked on *BC* and *CD*, respectively. Draw *AM* and *BN*, which meet at *Q*. Find the size of angle *AQB*.

6. Mark *Y* inside regular pentagon *PQRST*, so that *PQY* is equilateral. Is *RYT* straight? Explain.

7. Suppose that triangle *ABC* has a right angle at *B*, that *BF* is the altitude drawn from *B* to *AC*, and that *BN* is the median drawn from *B* to *AC*. Find angles *ANB* and *NBF*, given that angle *C* is 42 degrees.

8. The midpoints of the sides of a triangle are M(3, -1), N(4, 3), and P(0, 5). Find coordinates for the vertices of the triangle.

1. We have discussed medians, perpendicular bisectors, altitudes, midsegments and angle bisectors of triangles.
   (a) Which of these *must* go through the vertices of the triangle?
   (b) Is it possible for a median to also be an altitude? Explain.
   (c) Is it possible for an altitude to also be an angle bisector? Explain.
   (d) Is it possible for a midsegment to be a median? Explain.
   (e) Is it possible for a perpendicular bisector to be an altitude?

2. The diagonals of a rhombus have lengths 18 and 24. How long are the sides of the rhombus?

3. A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is *isosceles*. Make a diagram of an isosceles trapezoid whose sides have lengths 7 in, 10 in, 19 in, and 10 in. Find the *altitude* of this trapezoid (the distance that separates the parallel sides), then find the enclosed area.

4. If a quadrilateral is a rectangle, then its diagonals have the same length. What is the converse of this true statement? Is the converse true? Explain.

5. The diagonals of a parallelogram always bisect each other. Is it possible for the diagonals of a trapezoid to bisect each other? You may use *Proof by Contradiction* here.

6. A trapezoid has a 60-degree angle and a 45-degree angle. What are the other angles?

7. The parallel bases of a trapezoid have lengths 12 and 18 cm. Find the lengths of the two segments into which the midline of the trapezoid is divided by a diagonal.

8. A trapezoid has a 60-degree angle and a 120-degree angle. What are the other angles?

9. An n-sided polygon has the property that the sum of the measures of its exterior angles is equal to the sum of the measures of its interior angles. Find n.

10. Trapezoid $ABCD$ has parallel sides AB and CD, a right angle at D, and the lengths $AB = 15$, $BC = 10$, and $CD = 7$. Find the length DA.

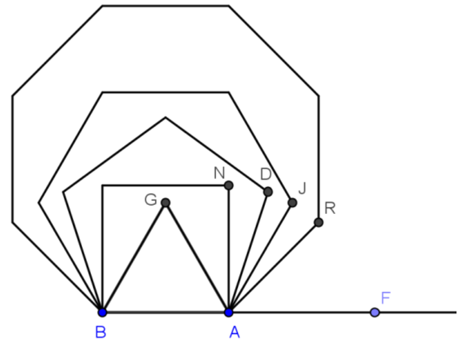
11. The sides of a triangle have lengths 9, 12, and 15. (This is a special triangle!)
    (a) Find the lengths of the medians of the triangle.
    (b) The medians intersect at the centroid of the triangle. How far is the centroid from each of the vertices of the triangle?

12. (Continuation) Apply the same questions to an equilateral triangle of side 6.

13. The *midline* of a trapezoid is the segment that joins the midpoints of the non-parallel sides. Prove that the midline of a trapezoid splits the trapezoid into two new trapezoids.

1. The diagonals of a non-isosceles trapezoid divide the midline into three segments whose lengths are 8 cm, 3 cm, and 8 cm. How long are the parallel sides? From this information, is it possible to infer anything about the distance that separates the parallel sides? Explain

2. In the diagram at right, AGB is an equilateral triangle, $\overline{AN}$ is the side of a square. $\overline{AD}$ is the side of a regular pentagon, $\overline{AJ}$ is the side of a regular hexagon, and $\overline{AR}$ is the side of a regular octagon. $\overline{AB}$ is a side shared by all of the regular polygons. Find
(a) $\angle GAF$ **(b)** $\angle NAR$ **(c)** $\angle JAF$ **(d)** $\angle GAJ$



3. A line of positive slope is drawn so that it makes a 60-degree angle where it intersects the *x*-axis. What is the slope of this line?

4. What can be said about quadrilateral *ABCD* if it has supplementary consecutive angles?

5. Consider the following "trapezoids" and midlines (keeping in mind the definition of a trapezoid):
   a. A "trapezoid" with top base of length 0 and bottom base of length 10. What is the length of its midline? (In actually what is this "trapezoid" and how would find the length of its "midline"?)
   b. A "trapezoid" with top base of length 10 and bottom base of length 10. What is the length of its "midline"?
   c. A trapezoid with top base of length 6 and bottom base of length 10. From the following two examples, how might you conjecture to find the length of the midline? Try to justify your answer.

6. If *MNPQRSTUV* is a regular polygon, then how large is each of its interior angles? If *MN* and *QP* are extended to meet at *A* then how large is angle *PAN*?

7. Is it possible for the sides of a triangle to be 23, 19, and 44? Explain.

8. Suppose that *ABCD* is a square with *AB* = 6. Let *N* be the midpoint of *CD* and *F* be the intersection of *AN* and *BD*. What is the length of *AF*? Hint: Look at triangle ADC.

9. Draw the lines $y = 0$, $y = \frac{1}{2}x$, and $y = 3x$. Use your protractor, GeoGebra to measure the angle that the line $y=\frac{1}{2}x$ makes with the x-axis. Using your intuition, make a guess what the angle is that the line y=3x makes with the x-axis. Now measure it. Explain your conclusions.

10. The parallel sides of trapezoid *ABCD* are *AD* and *BC*. Given that sides *AB*, *BC*, and *CD* are each half as long as side *AD*, find the size of angle *D*.

1. Dana buys a piece of carpet that measures 20 square yards. Will Dana be able to completely cover a rectangular floor that measures 12 ft. 4 in. by 16 ft. 8 in.?

2. The altitudes of an equilateral triangle all have length 12 cm. How long are its sides?

3. Given a triangle, you have found the following result: *The point where two medians intersect (the centroid) is twice as far from one end of a median as it is from the other end of the same median*. Improve the statement of the preceding theorem so that the reader knows which end of the median is which.

4. Let *ABCD* be a parallelogram, with *M* the midpoint of *DA*, and diagonal *AC* of length 36. Let *G* be the intersection of *MB* and *AC* and draw in diagonal *DB*. What is the length of *AG*?

5. The diagonals of a square have length 10. How long are the sides of the square?

6. Triangle *PQR* is isosceles, with $PQ = 13 = PR$ and $QR = 10$. Find the distance from *P* to the centroid of *PQR*. Find the distance from *Q* to the centroid of *PQR*.

7. In triangle *ABC*, let *M* be the midpoint of *AB* and *N* be the midpoint of *AC*. Suppose that you measure *MN* and find it to be 7.3 cm long. How long would *BC* be, if you measured it? If you were to measure angles *AMN* and *ABC*, what would you find?

8. In triangle *SUN*, let *P* be the midpoint of segment *SU* and let *Q* be the midpoint of segment *SN*. Draw the line through *P* parallel to segment *SN* and the line through *Q* parallel to segment *SU*; these lines intersect at *J*. What can you say about the location of point *J*?

9. A bell rope, passing through the ceiling above, just barely reaches the belfry floor. When one pulls the rope to the wall, keeping the rope taut, it reaches a point that is three inches above the floor. It is four feet from the wall to the rope when the rope is hanging freely. How high is the ceiling? It is advisable to make a clear diagram for this problem.

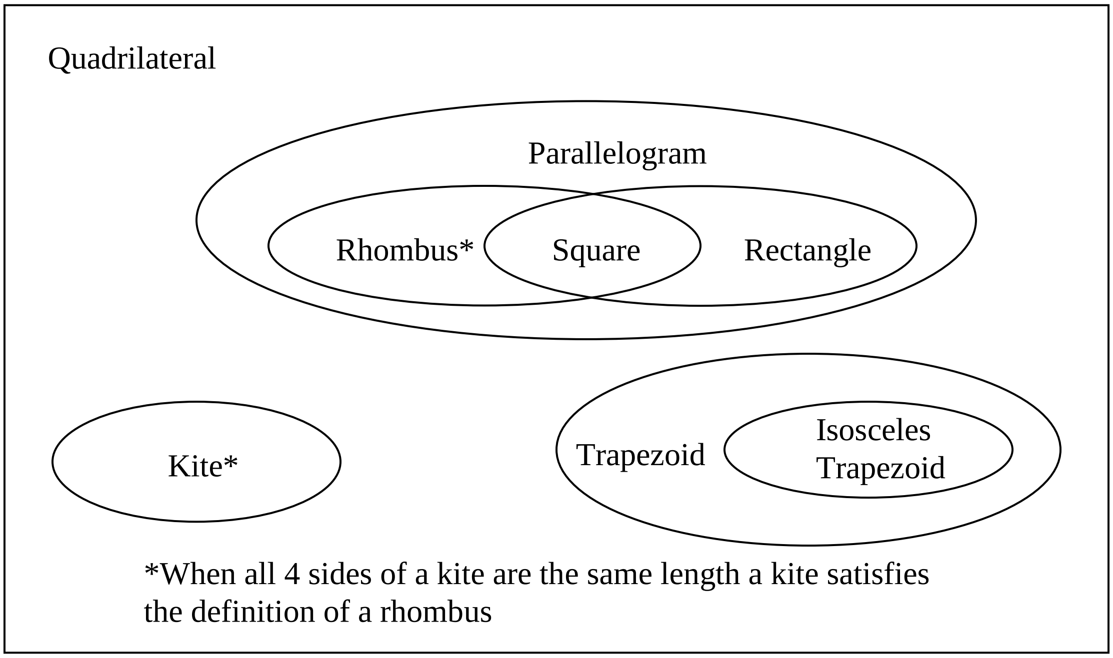
10. A triangle with sides of 5, 12, and 13 must be a right triangle. Keeping the legs constant, how would the triangle change if the hypotenuse was lengthened to 15? 17? 19?

11. (Continuation) What can be said about a triangle if the sum of the squares of the two shorter sides is smaller than the square of the longest side?

12. What if the hypotenuse of a 5-12-13 triangle was shortened to 12? 7? 5?

13. (Continuation) What can be said about a triangle if the sum of the squares of the two shorter sides is larger than the square of the longest side?

Justify the following Venn diagram and check the properties that each type of quadrilateral holds.



Property	Parallel-ogram	Rectangle	Rhombus	Square	Kite	Trapezoid	Isosceles Trapezoid
Opposite sides are parallel							
Opposite sides are congruent							
Exactly one pair of opposite sides is parallel							
Opposite angles are congruent							
Exactly one pair of angles is congruent							
Consecutive angles are supplementary							
Base angles are congruent							
Diagonals bisect each other							
Diagonals are congruent							
Diagonals are perpendicular							
Diagonals bisect opposite angles							
Exactly one diagonal is the perpendicular bisector of the other							

GeoGebra Lab #15

A. Using GeoGebra, draw a horizontal **line** AB and two points, *C* and *E* below AB. Make sure that C is above E (closer to AB).
B. Create lines through C and E that are parallel to AB using the Parallel Lines tool.
C. Draw a transversal (make sure that it's a line, not a segment) by clicking above the parallel lines and finish by clicking below the parallel lines so that the transversal cuts through the three parallel lines. Construct the intersection points, using the Intersect Two Objects tool, of the transversal and the three parallel lines, from bottom to top so that the intersection on the line through E, is G, the point on the line through C, is H, and the intersection of AB and the transversal is I.
D. With the Distance tool, in the Measurement toolbox, measure the distance between points G and H, and H and I. Move the transversal by dragging a point on it that is not an intersection. What happens to these distances when you drag the transversal horizontally thereby changing its slope?
E. In the input bar, type ratio = distance[I,H]/distance[H,G], to create a number that is the ratio of these two segments. Then, in a text box, type ratio = and select ratio from the Objects drop-down list. What happens to the ratio of these distances when you drag the transversal horizontally in the sketch?
F. Draw another transversal, with a different slope than the first, and construct the intersection points with AB, *CD* and EF, called L, M, and N respectively.
G. With the Distance Tool, find the length of segments LM and MN. Then, in the input bar, type in ratio2 = distance[L,M]/distance[M,N]. What conclusions can you draw from this information?

1. Mark $A = (0, 0)$ and $B = (10, 0)$ on your graph paper, and use your protractor to draw the line of positive slope through A that makes a 25-degree angle with AB. Calculate (approximately) the slope of this line by making suitable measurements.

2. (Continuation) Turn on your calculator, press the MODE button, and select the *Degree* option for angles. Return to the home screen, and press the TAN button to enter the expression TAN(25), then press ENTER. You should see that the display agrees with your answer to the preceding item.

3. A line drawn parallel to the side BC of triangle ABC intersects side AB at P and side AC at Q. The measurements $AP = 3.8$ in, $PB = 7.6$ in, and $AQ = 5.6$ in are made. If segment QC were now measured, how long would it be?

4. Given regular hexagon *BAGELS*, show that *SEA* is an equilateral triangle.

5. Given $A = (0, 6)$, $B = (-8, 0)$, and $C = (8, 0)$, find coordinates for the circumcenter of triangle ABC.

6. Rearrange the letters of *doctrine* to spell a familiar mathematical word.

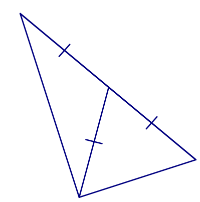
In the following list of true statements, find **(a)** the four pairs of statements whose converses are also in the list; **(b)** the statement that is a definition; **(c)** the statement whose converse is false; **(d)** the Sentry Theorem;**(e)** the Midsegment Theorem;**(f)** The Three Parallels Theorem; **(g)** The Centroid Theorem. Note: Not all statements are used.

1. If a quadrilateral has two pairs of parallel sides, then its diagonals bisect each other.

2. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral must be a parallelogram.

3. If a quadrilateral is equilateral, then it is a rhombus.

4. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



5. If a quadrilateral has two pairs of equal adjacent sides, then its diagonals are perpendicular.

6. If one of the medians of a triangle is half the length of the side to which it is drawn, then the triangle is a right triangle.

7. If a segment joins two of the midpoints of the sides of a triangle, then is parallel to the third side, and is half the length of the third side.

8. Both pairs of opposite sides of a parallelogram are congruent.

9. The sum of the exterior angles of any polygon – one at each vertex – is 360 degrees.

10. The median drawn to the hypotenuse of a right triangle is half the length of the hypotenuse.

11. If two lines are intersected by a transversal so that alternate interior angles are equal, then the lines must be parallel.

12. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is in fact a parallelogram.

13. If two opposite sides of a quadrilateral are both parallel and equal in length, then the quadrilateral is a parallelogram.

14. If three parallel lines intercept equal segments on one transversal, then they intercept equal segments on every transversal.

15. Both pairs of opposite angles of a parallelogram are congruent.

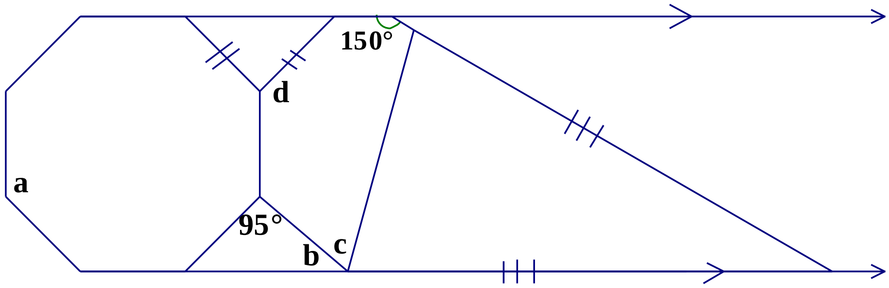
16. The medians of any triangle are concurrent at a point that is two thirds of the way from any vertex to the midpoint of the opposite side.

17. An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

1. How does the value of the tangent of an angle change as an angle increases from 0 to 90 degrees? Is there a direct relationship between the slope and the angle measure?

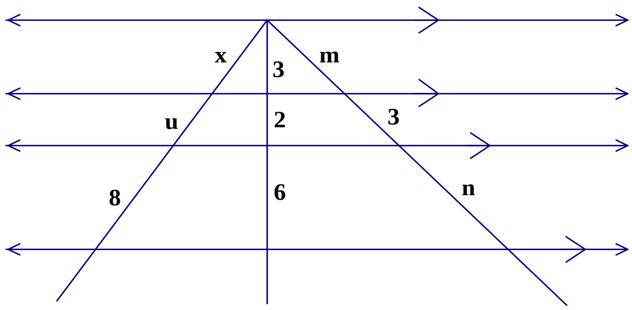
2. Standing 50 meters from the base of a fir tree, Rory measured an *angle of elevation* of 33° to the top of the tree with a protractor. The angle of elevation is the angle formed by the horizontal and the line of sight ray. How tall was the tree?

3. In the diagram below the octagon is regular. Find the measures of the angles labeled a, b, c, and d.



4. When the Sun has risen 32 degrees above the horizon, Sandy casts a shadow that is 9 feet 2 inches long. How tall is Sandy, to the nearest inch?

5. The *Three Parallels* Theorem: If a transversal cuts three parallel lines in a given ratio, then any transversal cuts off segments of the same ratio. Use this to solve for x, u, m, and n in the following diagram.



6. You are building a tent for a sleepover. You have a 36 ft tarp that you are going to use to hang it over a clothesline symmetrically. What are the possible base lengths of the cross-section of your tent?

7. Standing on a cliff 380 meters above the sea, Pocahontas sees an approaching ship and measures its *angle of depression*, obtaining 9 degrees. How far from shore is the ship?

8. (Continuation) Now Pocahontas sights a second ship beyond the first. The angle of depression of the second ship is 5 degrees. How far apart are the ships?

1. What is the radius of the smallest circle that encloses an equilateral triangle with 12-inch sides? What is the radius of the largest circle that will fit inside the same triangle?

2. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Measure angle *CAB* with your protractor. What is the slope of *AC*? Use your calculator to compare the tangent of the angle you measured with the slope. By trial-and-error, find an angle that is a better approximation of the measure of angle *CAB*.

3. (Continuation) On your calculator, ENTER the expression $\text{TAN}^{-1}(0.75)$. Compare this answer with the approximation you obtained for the measure of angle *CAB*. What does the TAN^{-1} button do? (TAN^{-1} is said as "inverse tangent.")

4. A five-foot Deerfield student casts an eight-foot shadow. How high is the Sun in the sky? This is another way of asking for the angle of elevation of the Sun.

5. An isosceles trapezoid has sides of lengths 9, 10, 21, and 10. Find the distance that separates the parallel sides then find the length of the diagonals. Finally, find the angles of the trapezoid, to the nearest tenth of a degree.

6. One day at the beach, Kelly flies a kite, whose string makes a 37-degree elevation angle with the ground. Kelly is 130 feet from the point directly below the kite. How high above the ground is the kite, to the nearest foot?

7. Hexagon *ABCDEF* is regular. Prove that segments *AE* and *ED* are perpendicular.

8. What angle does the line $y = \frac{2}{5}x$ make with the x-axis?

9. Suppose that *PQRS* is a rhombus, with $PQ = 12$ and a 60-degree angle at *Q*. How long are the diagonals *PR* and *QS*?

10. Given a rectangular card that is 5 inches long and 3 inches wide, what does it mean for another rectangular card to have the *same shape*? Describe a couple of examples.

11. *The Varignon quadrilateral*. A quadrilateral has diagonals of lengths 8 and 10. The midpoints of the sides of this figure are joined to form a new quadrilateral. What is the perimeter of the new quadrilateral? What is special about it?

12. The diagonals of rhombus *ABCD* meet at *M*. Angle *DAB* measures 60 degrees. Let *P* be the midpoint of *AD* and let *G* be the intersection of *PC* and *MD*. Given that $AP = 8$, find *MD*, *MC*, *MG*, *CG*, and *GP*.

13. The hypotenuse of a right triangle is twice as long as one of the legs. How long is the other leg? What is the size of the smallest angle?

14. What are the angle sizes in a trapezoid whose sides have lengths 6, 20, 6, and 26?

1. Atiba wants to measure the width of the Hudson River. Standing under a tree *T* on the river bank, Atiba sights a rock at the nearest point *R* on the opposite bank. Then Atiba walks to a point *P* on the river bank that is 50.0 meters from *T*, and makes *RTP* a right angle. Atiba then measures *RPT* and obtains 76.8 degrees. How wide is the river?

2. The legs of an isosceles right triangle have a length of *s*. What is the length of the hypotenuse with respect to *s*?

3. A regular *n*-sided polygon has exterior angles of *m* degrees each. Express *m* in terms of *n*.

4. Sketch an arbitrary quadrilateral *ABCD* with perpendicular diagonals *AC* and *BD* intersecting at point *E*. Let $BE = h_1$ and $DE = h_2$. By finding expressions for the area of triangles *ADC* and *ABC*, find a formula for the area of a quadrilateral that has perpendicular diagonals.

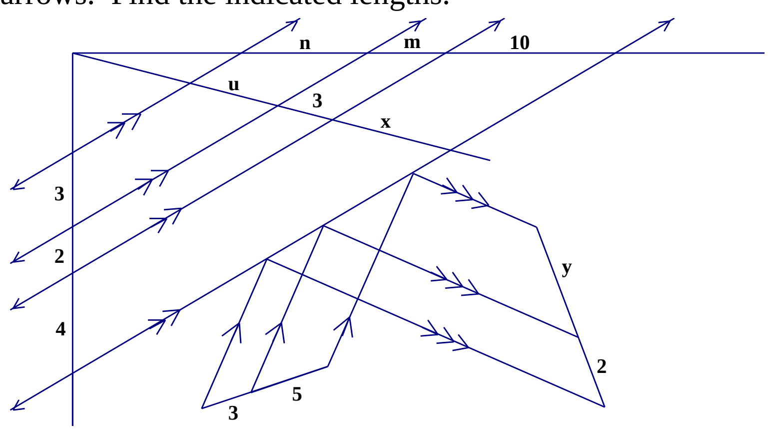
5. How tall is an isosceles triangle, given that its base is 30 cm long and that both of its base angles are 72 degrees?

6. A triangle has sides in the ratio $1:2:\sqrt{3}$. Draw a triangle with this scale. What can you say about this triangle?

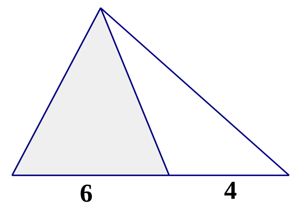
7. In a Deerfield Freshman class there are 105 students, and the day**:** boarder ratio is approximately 1:5.
   a. How many students in that class are boarders?
   b. How many day students would you expect to find in a freshman English class of fifteen students? Explain.

8. Find the equation of a line passing through the origin that makes an angle of 52 degrees with the *x*-axis.

9. In the figure at right, find the lengths of the segments indicated by letters. Parallel lines are indicated by arrows. Find the indicated lengths.



1. *Special Right Triangles*. There are special right triangles with integer valued sides. These are called *Pythagorean Triples*. There are two other special right triangles commonly used in mathematics that do not have integer valued sides. One of these is a 45-45-90 triangle. What is the other one?



2. In the figure at right, the shaded triangle has area 15. Find the area of the unshaded triangle.

3. To the nearest tenth of a degree, how large are the congruent angles of an isosceles triangle that is exactly as tall as it is wide? (There is more than one interpretation).

4. Rectangle *ABCD* has *AB* = 16 and *BC* = 6. Let *M* be the midpoint of side *AD* and *N* be the midpoint of side *CD*. Segments *CM* and *AN* intersect at *G*. Find the length *AG*.

5. An estate of $362880 is to be divided among three heirs, Alden, Blair, and Cary. According to the will, Alden is to get two parts, Blair three parts, and Cary four parts. What does this mean in terms of dollars and cents?

6. What is the relationship between the length of the hypotenuse and the length of the legs in a 45-45-90 triangle?

7. The area of a parallelogram can be found by multiplying the distance between two parallel sides by the length of either of those sides. Explain why this formula works.

8. Using GeoGebra or Geometry Pad, plot the points *A* = (0, 0), *B* = (4, -3), *C* = (6, 3), *P* = (-2, 7), *Q* = (9, 5), and *R* = (7, 19). Measure the angles of triangles *ABC* and *PQR*. Create ratios of the lengths of the corresponding sides. Find justification for any conclusions you make. (If you choose not to use technology, leave all answers in simplest radical form.)

9. The perimeter of a square is 36, what is the length of a diagonal of the square? The area of a square is 36, what is the length of a diagonal of the square?

10. Given that *P* is three fifths of the way from *A* to *B*, and that *Q* is one third of the way from *P* to *B*, describe the location of *Q* in relation to *A* and *B*.

11. Apply the transformation *T* (*x*, *y*) = (3*x*, 3*y*) to the triangle *PQR* whose vertices are *P* = (3, -1), *Q* = (1, 2), and *R* = (4, 3). Compare the sides and angles of the image triangle *P'Q'R'* with the corresponding parts of *PQR*. This transformation is an example of a *dilation*.

12. Show that the altitude drawn to the hypotenuse of any right triangle divides the triangle into two triangles that have the same angles as the original.

1. One figure is *similar* to another figure if the points of the first figure can be matched with the points of the second figure in such a way that corresponding distances are proportional. In other words, there is a *ratio of similarity*, *k*, such that every distance on the second figure is *k* times the corresponding distances on the first figure.

a. Open GeoGebra and plot the points K = (1, -3), L = (4, 1), M = (2, 3), P = (6, 5), Q = (2, 5) and R = (7, -2).

b. Is triangle KLM similar to triangle RPQ? Justify with measurements from GeoGebra.

c. Would it be correct to say that triangle MKL is similar to triangle RQP?

2. You have learned that the formula for the area of a triangle is $\frac{1}{2}bh$. Find a formula for the area of an equilateral triangle in terms of the side, *s*. Hint: Remember that an equilateral triangle is made up of two 30-60-90 triangles.

3. Draw a right triangle that has a 15-cm hypotenuse and a 27-degree angle. To the nearest tenth of a cm, measure the side opposite the 27-degree angle, and then express your answer as a percentage of the length of the hypotenuse. Compare your answer with the value obtained from your calculator when you enter SIN 27 in degree mode.

4. (Continuation) Repeat the process on a right triangle that has a 10-cm hypotenuse and a 65-degree angle. Try an example of your choosing. Write a summary of your findings.

5. In triangle *ABC*, points *M* and *N* are marked on sides *AB* and *AC*, respectively, so that $AM : AB = 1:3 = AN : AC$. Why are segments *MN* and *BC* parallel?

GeoGebra Lab #16

Using GeoGebra, you will consider the transformation called a dilation.

A. Open a new GeoGebra sketch and graph the points *C* = (1, 4), *P* = (5, 2) and *P’* = (13, -2).

B. Find the lengths of segments *CP* and *CP’* by using the Distance tool, then calculate the ratio of *CP’/CP*. What is the ratio? This is called the *scale factor* of the dilation.

C. Now, plot poinnt *Q* = (3, 5). Find the Dilate Object From Point by Factor tool in the Transformations toolbox (4th from the right). Use this tool to dilate Q with C as a center so that the dilation has the same fixed ratio (scale factor) as P→P’.

D. Verify, using distances as above in part B, that the ratio of *CQ’/CQ* is the same as the fixed ratio you entered.

E. What can be said about the relationship between the vectors $\overrightarrow{CP}$ and $\overrightarrow{CP'}$ and $\overrightarrow{CQ}$ and $\overrightarrow{CQ'}$?

F. If *R’* = (-2, -2), what were the coordinates of *R*, its *pre-image*, using the same center of dilation? Hint: dilations can also make images closer to the center of dilation.

1. One way to find the area of a trapezoid is by multiplying its altitude (the distance between the parallel sides) by the average of the bases. Can you come up with the two other ways?

2. What is the length of an altitude of an equilateral triangle with perimeter 36?

3. To actually draw a right triangle that has a 1-degree angle and measure its sides accurately is difficult. To get the sine ratio for a 1-degree angle, however, there is an easy way – just use your calculator. Is the ratio a small or large number? How large can a sine ratio be?

4. If two sides of a triangle are 5 and 10, what is the range of values for the third side?

5. Let $C = (1, 4)$, $P = (5, 2)$, and $P' = (13, -2)$. There is a dilation that leaves C where it is and transforms P into P'. The point C is called the *dilation center*. Explain why the *magnitude* of this dilation is 3. Keeping C as the center of dilation, calculate Q' given that $Q = (3, 5)$. Calculate R, given that $R' = (-6, 7)$, keeping C as the center of dilation.

6. What is the size of the acute angle formed by the x-axis and the line $3x + 2y = 12$?

7. If triangle ABC has a right angle at C, the ratio $AC : AB$ is called the *sine ratio of angle B*, or simply the *sine of B*, and is usually written *sin B*. What should the ratio $BC : AB$ be called? Without using your calculator, can you predict what the value of the sine ratio for a 30-degree angle is? How about the sine ratio for a 60-degree angle?

8. Compare the quadrilateral whose vertices are $A = (0, 0)$, $B = (6, 2)$, $C = (5, 5)$, $D = (-1, 3)$ with the quadrilateral whose vertices are $P = (9, 0)$, $Q = (9, 2)$, $R = (8, 2)$, and $S = (8, 0)$. Show that these two figures are similar. Is there a dilation that takes ABCD to PQRS?

9. Write an equation using the distance formula that says that $P = (x, y)$ is 5 units from (0, 0). Plot several such points. What is the configuration of all such points called? How many are lattice points?

10. (Continuation) Explain how you could use the Pythagorean Theorem to obtain the same result.

11. Does a dilation transform any figure into a similar figure? If you know that two triangles are similar does that mean that they are dilations of one another?

12. What is the length of a side of an equilateral triangle whose altitude is 16?

13. When you take the sine of 30 degrees using your calculator you get 0.5. What do you think $\text{SIN}^{-1}(0.5)$ is? Use your calculator to test your conjecture. Find $\text{SIN}^{-1}(0.3)$ and $\text{SIN}^{-1}\left(\frac{3}{5}\right)$.

    What do these values represent?

1. When triangle *ABC* is similar to triangle *PQR*, with *A*, *B*, and *C* corresponding to *P*, *Q*, and *R*, respectively, it is customary to write $ABC \sim PQR$. Suppose *that* $AB = 4$, $BC = 5$, $CA = 6$, and $RP = 9$. Find *PQ* and *QR*.

2. To the nearest tenth of a degree, find the sizes of the acute angles in the right triangle whose hypotenuse is 2.5 times as long as its short leg.

3. A triangle has a 60-degree angle and a 45-degree angle and the side opposite the 45-degree angle is 240 mm long. To the nearest mm, how long is the side opposite the 60-degree angle?

4. One triangle has sides that are 5 cm, 7 cm, and 8 cm long; the longest side of a similar triangle is 6 cm long. How long are the other two sides?

5. Judy is driving along a highway that is climbing a steady 9-degree slope. After driving for two miles along this road, how much altitude has Judy gained?

6. (Continuation) How far must Judy travel in order to gain a mile of altitude?

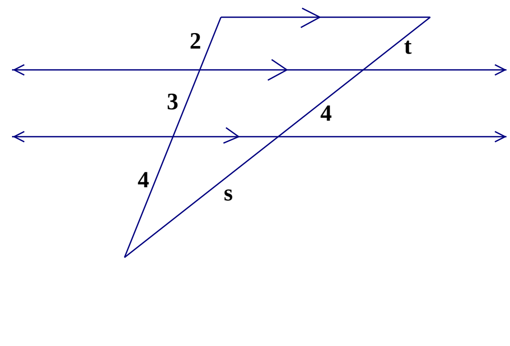
7. The floor plan of a house is drawn with a ratio of 1/8 inch = 1 foot. On the plan, the kitchen measures 2 in. by 2 1/4 in. What is the size of the kitchen?

8. If an altitude is also the side of a triangle, what do you know about the triangle?

9. If two polygons are similar, explain why the corresponding angles are the same size. What is the converse of this statement? Is it true?

10. To the nearest tenth of a degree, find the sizes of the acute angles in a 5-12-13 triangle and in a 9-12-15 triangle. This enables you to calculate the sizes of the angles in a 13-14-15 triangle. Show how to do it then invent another example of this sort.

11. *AA Similarity Postulate:* If two corresponding angles of a triangle are equal in size to the angles of another triangle, then the triangles are similar. Justify this statement. State the converse of this statement. Is it true?



12. In the diagram at the right, find *t* and *s*.

13. You are given a dilation of two triangles. Describe how you would algebraically find the center of dilation. Create an example of GeoGebra to be sure your method works.

14. Is it possible to draw a triangle with the given sides? If it is possible, state whether it is acute, right, or obtuse. If it is not possible, say no and sketch why.

(a) 9, 6, 5 **(b)** $3\sqrt{3}$, 9, $6\sqrt{3}$ **(c)** 8.6, 2.4, 6.2

GeoGebra Lab #17

Discovery of π *:* The Greek scholar Archimedes discovered a constant relationship between the circumference of a circle and its *diameter*. He called this constant π.
Describe the circumference with respect to its diameter with respect to its radius. Create a sketch to validate this relationship. If you have time, confirm the relationship of the circle's area to its radius with respect to π as well.

1. The area of an equilateral triangle with m-inch sides is 8 square inches. What is the area of a regular hexagon that has m-inch sides? (no need to solve for m)

2. A parallelogram has 10-inch and 18-inch sides and an area of 144 square inches.
   (a) How far apart are the 18-inch sides? **(b)** How far apart are the 10-inch sides?
   (c) What are the angles of the parallelogram? **(d)** How long are the diagonals?

3. Write an equation that describes all the points on the circle whose *center* is at the origin and whose *radius* is **(a)** 13; **(b)** 6; **(c)** r.

4. If the lengths of the midsegments of a triangle are 3, 4, and 5, what is the perimeter of the triangle?

5. Graph the circle whose equation is $x^2 + y^2 = 64$. What is its radius? What do the equations $x^2 + y^2 = 1$, $x^2 + y^2 = 40$, and $x^2 + y^2 = k$ all have in common? How do they differ?

GeoGebra Lab #18

Dilation Exploration

A. Open a new GeoGebra Sketch and using the Polygon tool construct any type of triangle XYZ
B. Construct a point somewhere else on the sketch and rename it C.
C. Using the Dilate Object by a Factor tool in the Transformations toolbox, dilate triangle XYZ using C as the center of dilation by a factor of 2.
D. Select the center of dilation, *C*, and move it around the sketch. What happens to the image as you move the center throughout the sketch? In particular, where is it with respect to triangle XYZ? Explain.
E. What happens when the center is moved to one of the vertices of *XYZ*? Explain.
F. What happens when the center is moved to the interior of *XYZ*? Explain.
G. Dilate XYZ again, with C as the center, and choose a factor that is positive, but less than one. What happens to XYZ? How is this different from your first dilation?
H. What happens when you dilate XYZ by multiples of 2?
I. Each time you dilate, how far from each other are the corresponding vertices?
J. What do you notice about the linearity of the corresponding vertices? (X,X', X'' and Y, Y", Y'', etc.)

1. When moving a lot of plates, the dining hall packs our round plates in square boxes with a perimeter of 36 inches. If the plates fit snugly in one stack in the box, one plate per layer, what is the circumference of each plate?

2. (Continuation) Each dining hall saucer has a circumference of 12.57 in. Can four saucers fit on a single layer in the same square box? Justify your answer.

3. Let A = (0, 5), B = (-2, 1), C = (6, -1), and P = (12, 9). Let A', B', and C' be the midpoints of segments PA, PB, and PC, respectively. After you make a diagram, identify the center and the magnitude of the dilation that transforms triangle ABC onto $A'B'C'$.

4. Taylor lets out 120 meters of kite string then wonders how high the kite has risen. Taylor is able to calculate the answer after using a protractor to measure the 63-degree angle of elevation that the string makes with the ground. How high is the kite, to the nearest meter? What (unrealistic) assumptions did you make in answering this question?

5. Find the sine of a 45-degree angle. Use your calculator *only to check your answer*.

6. Using GeoGebra or Geometry Pad: Let A = (1, 5), B = (3, 1), C = (5, 4), A' = (5, 9), B' = (11, -3), and C' = (17, 6). Show that there is a dilation that transforms triangle ABC onto triangle $A'B'C'$. In other words, find the dilation center and the scale factor.

7. (Continuation) Calculate the areas of triangles ABC and $A'B'C'$. Are your answers related in a predictable way?

8. If the central angle of a slice of pizza is 36 degrees, how many pieces are in the pizza?

9. (Continuation) A 12 inch pizza is evenly divided into 8 pieces. What is the length of the crust of one piece?

10. The vertices of triangle ABC are A = (-5, -12), B = (5, -12), and C = (5, 12). Confirm that the circumcenter of ABC lies at the origin. What is the equation for the *circumscribed circle*?

11. If the sides of a triangle are 13, 14, and 15 cm long, then the altitude drawn to the 14-cm side is 12 cm long. How long are the other two altitudes? Which side has the longest altitude?

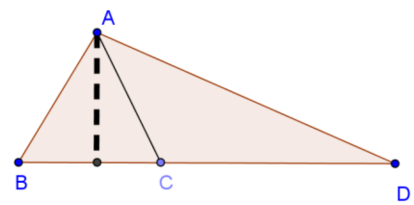
12. (Continuation) How long are the altitudes of the triangle if you double the lengths of its sides?

1. Let $A = (6, 0)$, $B = (0, 8)$, $C = (0, 0)$. In triangle ABC, let F be the point of intersection of the altitude drawn from C to side AB.
a. Explain why the angles of triangles ABC, CBF, and ACF are the same.
b. Find coordinates for F and use them to calculate the exact lengths FA, FB, and FC.
c. Compare the sides of triangle ABC with the sides of triangle ACF. What do you notice?

2. What happens to the area of a triangle if its dimensions are doubled?

3. A rectangular sheet of paper is 20.5 cm wide. When it is folded in half, with the crease running parallel to the 20.5-cm sides, the resulting rectangle is the same shape as the unfolded sheet. Find the length of the sheet, to the nearest tenth of a cm. (In Europe, the shape of notebook paper is determined by this similarity property).

4. Write an equation that describes all the points $P(x,y)$ that are 5 units away from the point $C(1,-4)$. What set of points does this describe?



5. What is the ratio of the areas of ΔABC and ΔADC in the following diagram if BC=5 and CD=8?

6. A regular polygon is inscribed in a circle. What happens to the regular polygon as the number of sides increases?

7. Sketch the circle whose equation is $x^2 + y^2 = 100$. Using the same system of coordinate axes, graph the line $x + 3y = 10$, which should intersect the circle twice – at $A = (10, 0)$ and at another point B in the second quadrant. Estimate the coordinates of B. Now use algebra to find them exactly. Segment AB is called a *chord* of the circle.

8. (Continuation) Find coordinates for a point C on the circle that makes chords AB and AC have equal length.

9. What is the radius of the smallest circle that surrounds a 5-by-12 rectangle?

10. Baking Powder is made up of Baking Soda, Cream of Tartar and sometimes Corn Starch in a ratio of 1:2:1. If you need 2 Tablespoons of Baking Powder, how much of each ingredient do you need?

11. Without doing any calculation, what can you say about the tangent of a k-degree angle, when k is a number between 90 and 180? Explain your response, then check with your calculator.

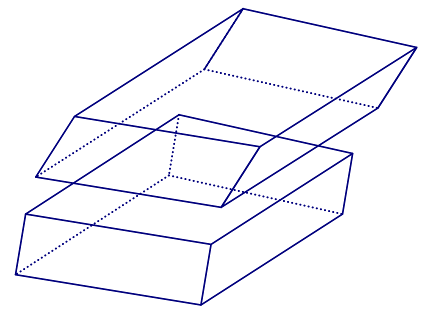
12. Ask your calculator for the sine of a 56-degree angle, then for the cosine of a 34-degree angle. Ask your calculator for the sine of a 23-degree angle, then for the cosine of a 67-degree angle. The word *cosine* is an abbreviation of *sine of the complement*. Explain the terminology.

1. (Continuation) With the knowledge that cosine is the *sine of the complement of an angle,* how can you represent the cosine of an angle in terms of a ratio?

2. A right triangle has a 123-foot hypotenuse and a 38-foot leg. To the nearest tenth of a degree, what are the sizes of its acute angles?

3. The line $y = x + 2$ intersects the circle $x^2 + y^2 = 10$ in two points. Call the third quadrant point R and the first-quadrant point E, and find their coordinates. Let D be the point where the line through R and the center of the circle intersects the circle again. The chord DR is an example of a *diameter*. Show that triangle RED is a right triangle.

4. To the nearest tenth of a degree, find the angles of the triangle with vertices (0, 0), (6, 3), and (1, 8). Use your protractor to *check* your calculations, and explain your method.



5. Playing cards measure 2.25 inches by 3.5 inches. A full deck of fifty-two cards is 0.75 inches high. What is the volume of a deck of cards? If the cards were uniformly shifted (turning the bottom illustration into the top illustration), would this volume be affected?

6. In a right triangle, the 58-cm hypotenuse makes a 51-degree angle with one of the legs. To the nearest tenth of a cm, how long is that leg? Once you have the answer, find some other ways to calculate the length of the other leg. They should all give the same answer, of course.

7. An equilateral triangle ABC is inscribed in a circle centered at O. The portion of the circle that lies above chord AB is called an *arc*. If $AB = BC = AC$, what is the measure of AB? AB is called a *minor arc* and ACB is called a *major arc*. Why do you think they are called this? How are AB and ACB related?

8. (Continuation) *Some Terminology:* A *central angle* is an angle whose vertex is at the center of a circle and whose sides are radii. What is the measure of angle AOB? What is the relationship between a central angle and the arc it intercepts?

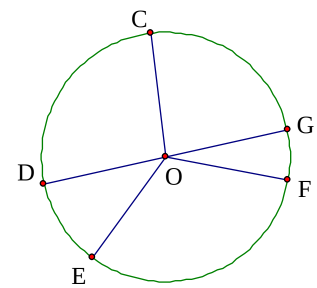
9. What is the angular size of an arc that a diameter intercepts? This arc is called a *semicircle.*

10. If the ratio of the areas of two triangles is 18: 8, what is the ratio of similarity?

11. The vertices of a square with sides parallel to the coordinate axes lie on the circle of radius 5 whose center is at the origin. Find coordinates for the four vertices of this square.

12. Draw a circle and label one of its diameters AB. Choose any other point on the circle and call it C. What can you say about the size of angle ACB? Does it depend on which C you chose? Justify your response.

1. A square *pyramid* is a pyramid with a square base and four triangular lateral faces. The slant height is the distance from the vertex of the pyramid along a *lateral face* to the midpoint of a base edge. If the slant height is 10 and an edge of the square is 12, what is the altitude of this pyramid?



2. Circle O has diameter DG and central angles $COG = 86^\circ$, $DOE = 25^\circ$, and $FOG = 15^\circ$. Find the angular size of minor arcs CG, CF, EF and major arc DGF.

3. A regular pentagon can be dissected into 5 isosceles triangles whose vertex angle is at the center of the pentagon. The height of the triangles is 10 cm. Find the area of this pentagon.

4. Write the equation of a circle centered at C = (-2,-5) with radius of 7. Come up with a general description of how you would write the equation of a circle given the center and radius.

5. *The area of a circle:* A regular hundred-sided polygon with side length of very close to b=2 is inscribed in a circle with radius of 32 (do not attempt to draw this polygon!)
   a. Show that the perimeter of the polygon is approximately equal to the circumference of the circle.
   b. Show that the height of a central triangle of the polygon is approximately equal to the radius of the circle.
   c. You can write the area of the polygon as $A_{polygon} = 100 \cdot$ area of a central triangle. Justify rewriting this expression as $A_{polygon} = 100 \cdot b \cdot \frac{1}{2} \cdot h$.
   d. Now substitute $2\pi r$ for $100 \cdot b$. Why can you do this? Please simplify your expression.
   e. Then substitute r for h. Why can you do this?
   f. Simplify your expression for the area of the circle.

6. If two chords in the same circle have the same length, then their minor arcs have the same length, too. True or false? Explain. What about the converse statement? Is it true? Why?

7. The circle $x^2 + y^2 = 25$ goes through A = (5, 0) and B = (3, 4). To the nearest tenth of a degree, find the angular size of the minor arc AB.

8. (Continuation) Let point O be (0, 0), now find the measure of angle OBA.

9. The sides of a triangle are found to be 10 cm, 14 cm, and 16 cm long, while the sides of another triangle are found to be 15 in, 21 in, and 24 in long. On the basis of this information, what can you say about the angles of these triangles?

10. In mathematical discussion, a *right prism* is defined to be a solid figure that has two parallel, congruent polygonal bases, and rectangular *lateral faces*. How would you find the volume of such a figure? Explain your method.

1. In triangle *ABC*, it is given that angle *BCA* is right. Let $a = BC$, $b = CA$, and $c = AB$. Using *a*, *b*, and *c*, express the sine, cosine, and tangent ratios of acute angles *A* and *B*.

2. The sine of a 38-degree angle is some number *r*. Without using your calculator, you should be able to identify the angle size whose cosine is the same number *r*.

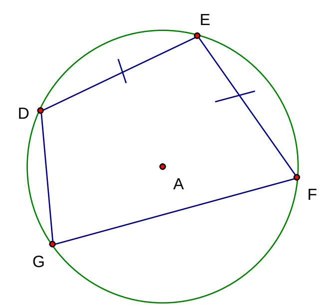
3. On a circle whose center is *O*, using your protractor, GeoGebra or Geometry Pad, mark points *P* and *A* so that minor arc *PA* is a 46-degree arc. What does this tell you about angle *POA*? Extend *PO* to meet the circle again at *T*. What is the size of angle *PTA*? This angle is *inscribed* in the circle, because its vertex is on the circle. The arc *PA* is *intercepted* by the angle *PTA*. Make a conjecture about arcs intercepted by inscribed angles.

4. (Continuation) Confirm your conjecture about inscribed angles and the arcs they intercept using GeoGebra or Geometry Pad. To measure the arc, select one endpoint, then the center of the circle, then the final endpoint, making sure to go clockwise with respect to the vertices.

5. Given *SSS* information about an isosceles triangle, describe the process you would use to calculate the sizes of its angles.

6. If *P* and *Q* are points on a circle, then the center of the circle must be on the perpendicular bisector of chord *PQ*. Explain. Which point on the chord is closest to the center? Why?

7. Suppose that *MP* is a diameter of a circle centered at *O*, and *Q* is any other point on the circle. Draw the line through *O* that is parallel to *MQ*, and let *R* be the point where it meets minor arc *PQ*. Prove that *R* is the midpoint of minor arc *PQ*.



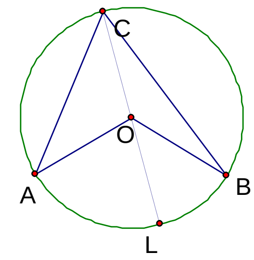
8. Quadrilateral DEFG in inscribed in circle A. $ED \cong EF$, $\angle E = 100°$ and $\angle F = 70°$. Find the measures of the four minor arcs.

9. Given that triangle *ABC* is similar to triangle *PQR*, write the three-term proportion that describes how the six sides of these figures are related.

10. A circle of radius 5 is circumscribed about a regular hexagon. Find the area of the hexagon in simplest radical form.

11. A regular hexagon has an inscribed circle of radius 4. Find the area of the hexagon.

1. *The Star Trek Theorem*: You have found that an inscribed angle is half the measure of the arc that it intercepts.



(a) Given a circle centered at O, let A, B and C be points on the circle such that arc AC is not equal to arc BC and CL is a diameter. Why must triangles AOC and BOC be isosceles?
(b) State the pairs of angles that must be congruent in these isosceles triangles.
(c) Using the Exterior Angle Theorem, find expressions for the measures of $\angle AOL$ and $\angle BOL$.
(d) Based on your statement in part c, explain the statements $\angle ACO = \frac{1}{2}(\angle AOL)$ and $\angle OCB = \frac{1}{2}(\angle BOL)$.
(e) Now find an expression for $\angle ACB$ and simplify to prove that it equals $\frac{1}{2}\angle AOB$.

2. Draw a circle with a 2-inch radius, mark four points randomly (not evenly spaced) on it, and label them consecutively *G*, *E*, *O*, and *M*. Measure angles *GEO* and *GMO*. Could you have predicted the result? Name another pair of angles that would have produced the same result.

3. A circular park 80 meters in diameter has a straight path cutting across it. It is 24 meters from the center of the park to the closest point on this path. How long is the path?

4. Triangle *ABC* is inscribed in a circle. Given that *AB* is a 40-degree arc and $\angle ABC$ is a 50-degree angle, find the sizes of the other arcs and angles in the figure.

5. Algebraically, find the intersections of the line $y = 2x - 5$ with the circle $x^2 + y^2 = 25$. Then use your calculator to find the intersections of the line $-2x + 11y = 25$ with the same circle. Show that these lines create chords of equal length when they intersect the circle. With your protractor, measure the inscribed angle formed by these chords.

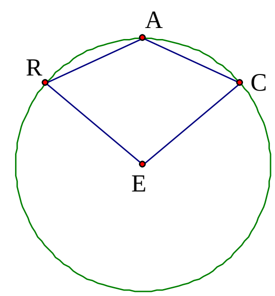
6. (Continuation) Calculate the angle between the chords to the nearest 0.1 degree. What is the angular size of the arc that is intercepted by this inscribed angle?

7. A triangle has a 3-inch side, a 4-inch side, and a 5-inch side. The altitude drawn to the 5-inch side cuts this side into segments of what lengths?

8. A chord 6 cm long is 2 cm from the center of a circle. How long is a chord that is 1 cm from the center of the same circle?

9. By using the triangle whose sides have lengths 1, $\sqrt{3}$ and 2, you should be able to write non-calculator expressions for the sine, cosine, and tangent of a 30-degree angle. Do so. You can use your calculator to check your answers, of course.

10. The figure at right shows points *C*, *A*, and *R* marked on a circle centered at *E*, so that chords *CA* and *AR* have the same length, and so that major arc *CR* is a 260-degree arc. Find the angles of quadrilateral *CARE*. What is special about the sizes of angles *CAR* and *ACE*?



1. Find all the angles in a 5-12-13 triangle.

2. A trapezoid has two 65-degree angles and 8-inch and 12-inch parallel sides. How long are the non-parallel sides? What is the area enclosed by this figure?

3. Two circles of radius 10 cm are drawn so that their centers are 12 cm apart. The two points of intersection determine a *common chord*. Find the length of this chord.

4. What is the sine of the angle whose tangent is 2? First find an answer *without* using your calculator (draw a picture) then use your calculator to check.

5. Find the area of a regular 36-sided polygon inscribed in a circle of radius 20 cm.

6. You are at the scenic overlook at Mt. Sugarloaf in South Deerfield looking through the panoramic viewer that is looking straight ahead. By what degree measure must you rotate the viewer directly downward to see the Mullins Center at UMass, which you know to be 8 miles away if the scenic overlook is 600 feet high?

7. Can the diagonals of a kite bisect each other?

8. Draw trapezoid ABCD so that AB is parallel to CD and the diagonals of the trapezoid. Label the intersection as E. Show that triangles ABE and CDE are similar.

9. The area of an equilateral triangle is $100\sqrt{3}$ square inches. How long are its sides?

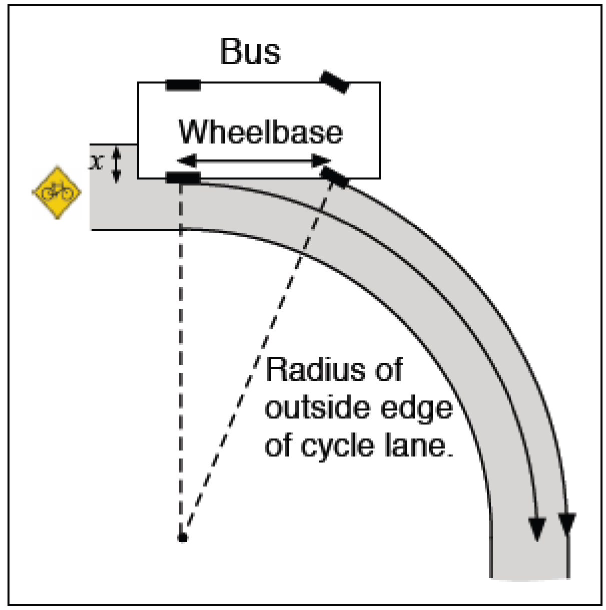
10. Points *E*, W, and *S* are marked on a circle whose center is *N*. In quadrilateral *NEWS*, angles *S* and *W* are found to be 54° and 113°, respectively. What are the other two angles?

11. The points $A = (0, 13)$ and $B = (12, 5)$ lie on a circle whose center is at the origin. Show that the perpendicular bisector of *AB* goes through the origin.

12. The areas of two similar triangles are 24 square cm and 54 square cm. The smaller triangle has a 6-cm side. How long is the corresponding side of the larger triangle?

13. Find the perimeter of a regular 36-sided polygon inscribed in a circle of radius 20 cm.

1. When a bus turns a corner, it must swing out so that its rear wheels don't go into the bicycle lane of the road (see diagram). In this picture, as the bus goes around the corner, the front wheel is on the edge of the bicycle lane, but the rear wheel cuts into the bicycle lane. (the grey area is the bicycle lane of the road).



http://map.mathshell.org/materials

   a. Come up with an equation that relates x, the distance the rear wheels cut into the bicycle lane, r, the radius of the outside edge of the bicycle lane, and w, the wheelbase of the bus.
   b. If a bus has a 10 foot wheelbase and the edge of the bicycle lane if the radius of the outside edge of the bicycle lane is 17 feet, how far does the bus cut into the bicycle lane?
   c. How far does the front wheel have to be from the outside edge of the bicycle lane in order for the rear wheel NOT to cut into the bicycle lane at all?

2. When two circles have a common chord, their centers and the endpoints of the chord form a quadrilateral. What kind of quadrilateral? What special property do its diagonals have?

3. Given that θ (*Greek* "theta") stands for the degree size of an acute angle, fill in the blank space between the parentheses to create a true statement: $\sin \theta = \cos(\ )$.

4. If the ratio of similarity between two triangles is 3: 5, what is the ratio of the areas of these triangles?

5. Let $P = (-25, 0)$, $Q = (25, 0)$, and $R = (-24, 7)$.
   a. Find an equation for the circle that goes through P, Q, and R.
   b. Find at least two ways of showing that angle PRQ is right.
   c. Find coordinates for another point R that would have made angle PRQ right.

6. How much evidence is needed to be sure that two triangles are similar?

7. Trapezoid $ABCD$ has parallel sides AB and CD, of lengths 8 and 24, respectively. Diagonals AC and BD intersect at E, and the length of AC is 15. Find the lengths of AE and EC.

8. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Mark point D so that ACD is a right angle and DAC is a 45-degree angle. Find coordinates for D. Find the tangent of angle DAB.

9. A regular octagon has a perimeter of 64. Find its area.

1. Two circles have a 24-cm common chord, their centers are 14 cm apart, and the radius of one of the circles is 13 cm. Make an accurate drawing, and find the radius for the second circle in your diagram. Are there other possible answers?

2. Triangle *ABC* has *P* on *AC*, *Q* on *AB*, and angle *APQ* equal to angle *B*. The lengths *AP* = 3, *AQ* = 4, and *PC* = 5 are given. Find the length of *AB*.

3. A *cyclic* quadrilateral is a quadrilateral whose vertices are points on a circle. Draw a cyclic quadrilateral *SPAM* in which the size of angle *SPA* is 110 degrees. What is the size of angle *AMS*? Would your answer change if *M* were replaced by a different point on major arc *SA*?

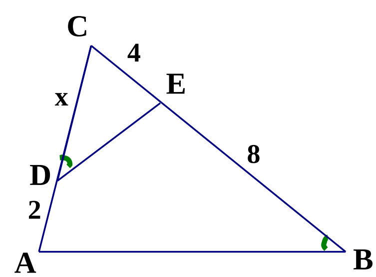
4. If A = (3, 1), B = (3, 4), and C = (8, 1) find the measure of angle B.

5. A 20-inch chord is drawn in a circle with a 12-inch radius. What is the *angular size* of the minor arc of the chord? What is the *length* of the arc, to the nearest tenth of an inch?

6. In an old historic building, there are two large regular octagonal pillars. The edges are 6.5 in and they are 9 feet tall. How much granite was needed to build these pillars?

7. Quadrilateral *WISH* is *cyclic*. Diagonals *WS* and *HI* intersect at *K*. Given that arc *WI* is 100 degrees and arc *SH* is 80 degrees, find the sizes of as many angles in the figure as you can. Note: *K* is not the center of the circle.

8. Refer to the figure, in which angles *ABE* and *CDE* are equal in size and various segments have been marked with their lengths. Find *x*.

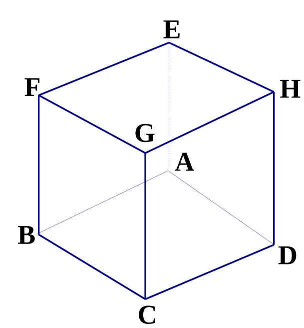


9. Quadrilateral *BAKE* is cyclic. Extend *BA* to a point *T* outside the circle, thus producing the exterior angle *KAT*. Why do angles *KAT* and *KEB* have the same size?

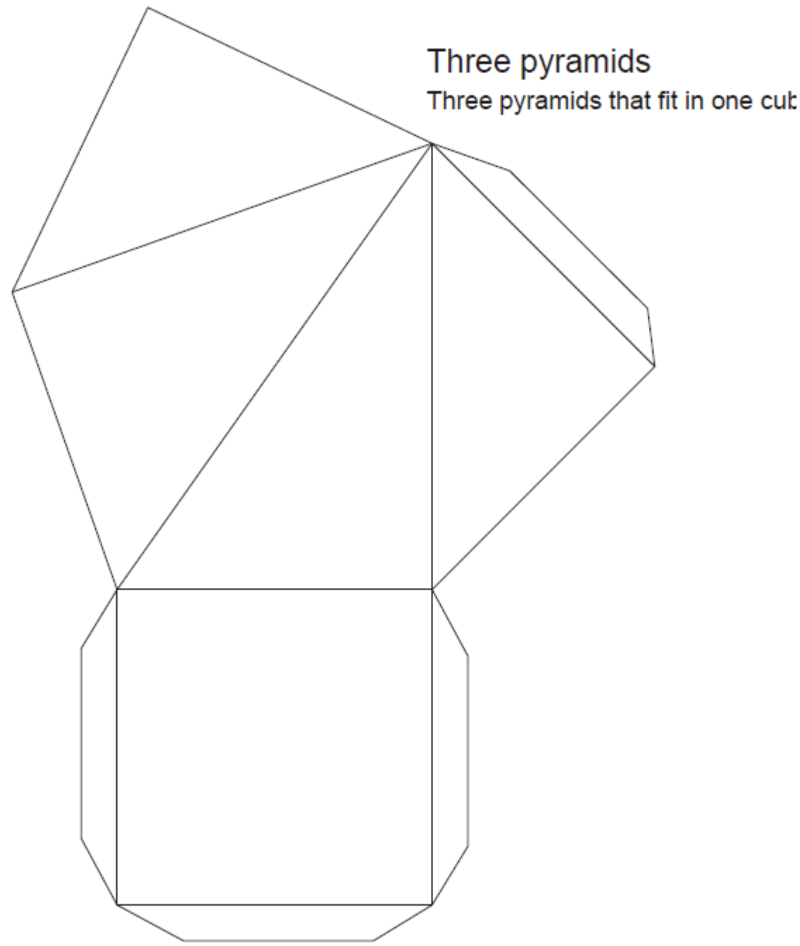
10. Draw the line $y = 2x - 5$ and the circle $x^2 + y^2 = 5$. Use algebra to show that these graphs touch at only one point. It is customary to say that a line and a circle are *tangent* if they have exactly one point in common.

11. (Continuation) Find the slope of the segment that joins the point of tangency to the center of the circle and compare your answer with the slope of the line $y = 2x - 5$. What do you notice?

12. Given that *ABCDEFGH* is a cube (shown at right), what is the relationship between the cube and the three square pyramids *ADHEG*, *ABCDG*, and *ABFEG*?

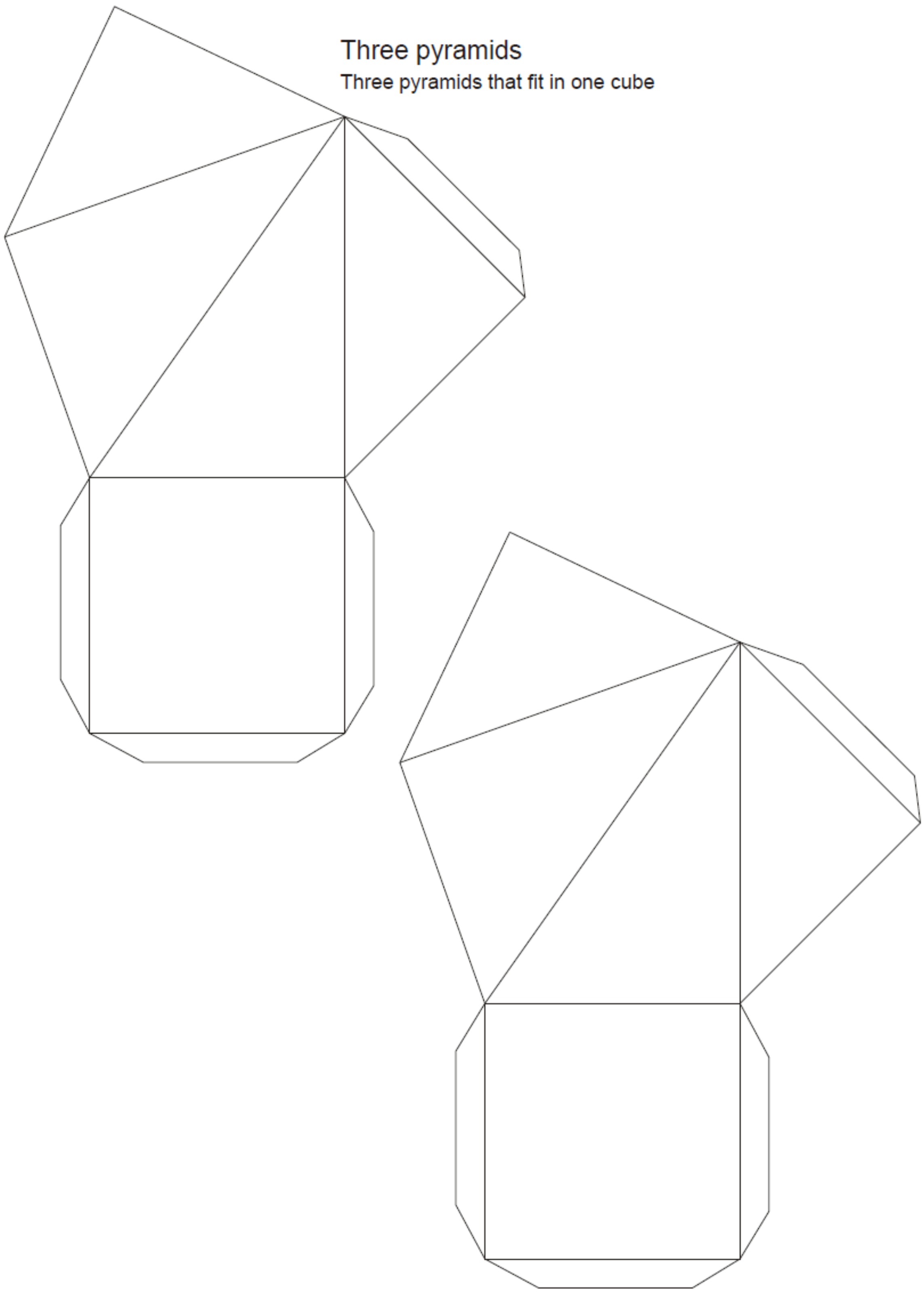


To the right and on the next page there are three copies of two-dimensional networks that you should print cut out and attempt to fold up and tape into three oblique pyramids that will help you to visualize pyramids *ADHEG*, *ABCDG* and *ABFEG*. Attempt to put them together and form the cube *ABCDEFGH* once again and justify the volume formula of a pyramid. Why do you conjecture is volume of a pyramid that has the same base area and height as a cube? Bring your cubes to class.



Three pyramids

Three pyramids that fit in one cube

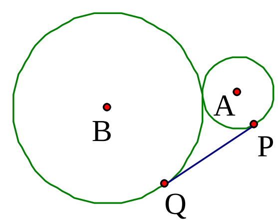


1. The parallel sides of a trapezoid have lengths 9 cm and 12 cm. Draw *one* diagonal, dividing the trapezoid into two triangles. What is the ratio of their areas? If the other diagonal had been drawn instead, would this have affected your answer?

2. Drawn in a circle whose radius is 12 cm, chord *AB* is 16 cm long. Calculate the angular size of minor arc *AB*.

3. Show that the line $y = 10 - 3x$ is tangent to the circle $x^2 + y^2 = 10$. Find an equation for the line perpendicular to the tangent line at the point of tangency. Show that this line goes through the center of the circle.

4. Let $A = (4, 6)$, $B = (6, 0)$, and $C = (9, 9)$. Find the size of angle *BAC*.

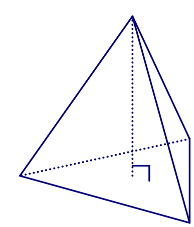


5. A circle with a 4-inch radius is centered at *A* and a circle with a 9-inch radius is centered at *B*, where *A* and *B* are 13 inches apart. There is a segment that is tangent to the small circle at *P* and to the large circle at *Q*. It is a common external tangent of the two circles. What kind of quadrilateral is *PABQ*? What are the lengths of its sides?

6. Segment *AB*, which is 25 inches long, is the diameter of a circle. Chord *PQ* meets *AB* perpendicularly at *C*, where *AC* = 16 in. Find the length of *PQ*.

7. Prove that the arcs between any two parallel chords in a circle must be the same size.

8. *Two Tangents Theorem*. From any point *P* outside a given circle, there are two lines through *P* that are tangent to the circle. Explain why the distance from *P* to one of the points of tangency is the same as the distance from *P* to the other point of tangency. What special quadrilateral is formed by the center of the circle, the points of tangency, and *P*?



9. The altitude of a regular triangular pyramid is the segment connecting a vertex to the centroid of the opposite face. A regular triangular pyramid has edges of length 6 in. How tall is such a pyramid, to the nearest hundredth of an inch?

10. A 72-degree arc *AB* is drawn in a circle of radius 8 cm. How long is chord *AB*?

11. Find the perimeter of a regular 360-sided polygon that is inscribed in a circle of radius 5 inches. If someone did not remember the formula for the circumference of a circle, how could that person use a calculator's trigonometric functions to find the circumference of a circle with a 5-inch radius?

12. The segments *GA* and *GB* are tangent to a circle with center *O* at *A* and *B*, and *AGB* is a 60-degree angle. Given that $GA = 12\sqrt{3}$ cm, find the distance *GO*. Find the distance from *G* to the nearest point on the circle.

1. A circle T has two tangents that intersect at 54° at point *M*. The points of tangency are *A* and *H*. What is the angular size of AH ?

2. A trapezoid has 11-inch and 25-inch parallel sides, and an area of 216 square inches.
(a) How far apart are the parallel sides?
(b) If one of the non-parallel sides is 13 inches long, how long is the other one? (Note: there are two answers to this question. It is best to make a separate diagram for each).

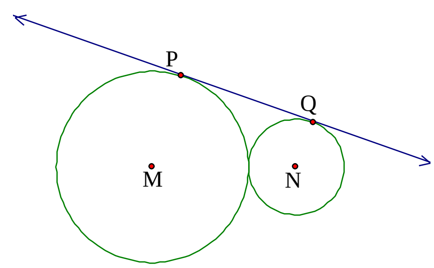
3. The Great Pyramid at Giza was originally 483 feet tall, and it had a square base that was 756 feet on a side. It was built from rectangular stone blocks measuring 7 feet by 7 feet by 15 feet. Such a block weighs seventy tons. Approximately how many tons of stone were used to build the Great Pyramid? The volume of a pyramid is one third the base area times the height.

4. In a group of 12 students, only 4 of them like olives on their pizza. If they are sharing a 16-in pizza what is the area of the part the pizza covered with olives?

5. A triangle that has a 50-degree angle and a 60-degree angle is inscribed in a circle of radius 25 inches. The circle is divided into three arcs by the vertices of the triangle. To the nearest tenth of an inch, find the lengths of these three arcs.

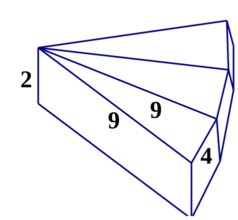
6. Stacy wants to decorate the side of a cylindrical can by using a rectangular piece of paper and wrapping it around the can. The paper is 21.3 cm by 27.5 cm. Find the two possible diameters of the cans that Stacy could use. (Assume the paper fits exactly).

7. The area of a trapezoidal cornfield *IOWA* is 18000 sq m. The 100-meter side *IO* is parallel to the 150-meter side *WA*. This field is divided into four sections by diagonal roads *IW* and *OA*. Find the areas of the triangular sections.



8. PQ is tangent to circles $\odot M$ and $\odot N$. $\odot M$ and $\odot N$ are externally tangent.
   a. If $\odot M$ and $\odot N$ are not congruent; what kind of quadrilateral is *MNQP*?
   b. If $\odot M$ and $\odot N$ are congruent, what kind of quadrilateral is *MNQP*.

9. A wedge of cheese is 2 inches tall. The triangular base of this right prism has two 9-inch edges and a 4-inch edge. Several congruent wedges are arranged around a common 2-inch segment, as shown. How many wedges does it take to complete this wheel? What is the volume of the wheel, to the nearest cubic inch?



1. The segments GA and GB are tangent to a circle at A and B, and AGB is a 48-degree angle. Given that $GA = 12$ cm, find the distance from G to the nearest point on the circle.

2. A 16.0-inch chord is drawn in a circle whose radius is 10.0 inches. What is the angular size of the minor arc of this chord? What is the length of the arc, to the nearest tenth of an inch?

3. The line $x + 2y = 5$ divides the circle $x^2 + y^2 = 25$ into two arcs. Find the lengths of the arcs.

4. (Continuation) A *sector* is a region formed by two radii and an arc of a circle. Find the area of the smaller sector.

5. The equation of a circle is $x^2 + y^2 = 50$, find the area of the circle.

6. If the area of a circle centered at the origin is 40π, write the equation for this circle.

7. Write the equation of the circle that passes through the vertices of the triangle defined by (-1, -7), (5, 5), (7, 1).

8. Alex's dog, Fluffy, is tied with a 20 ft rope to the center of the bottom of the back wall of the shed, which has dimensions of 14 ft by 18 ft. If the back wall is the longer wall, over what area can Fluffy play, to the nearest square foot? Would your answer change if the back wall was 14 ft instead?

9. All triangles have circumscribed circles. Why? What property must a given quadrilateral hold in order to have a circumscribed circle? Explain.

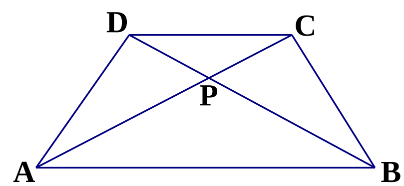
10. Pyramid $TABCD$ has a square base $ABCD$ with 20-cm base edges. The lateral edges that meet at T are 27 cm long. Make a diagram of $TABCD$, showing F, the point of $ABCD$ closest to T. To the nearest 0.1 cm, find the height TF. Find the volume of $TABCD$, to the nearest cm^3.

11. (Continuation) Find the slant height of pyramid TABCD. The slant height is the height of the *lateral face*.

12. (Continuation) Let K, L, M, and N be the points on TA, TB, TC, and TD, respectively, which are 18 cm from T. What can be said about polygon $KLMN$? Explain.

13. Two of the tangents to a circle meet at Q, which is 25 cm from the center. The circle has a 7-cm radius. To the nearest tenth of a degree, find the angle formed at Q by the tangents.

14. Suppose that $ABCD$ is a trapezoid with AB parallel to CD and diagonals AC and BD intersecting at P. Explain why
    a. triangles CDA and CDB have the same area;
    b. triangles BCP and DAP have the same area;
    c. triangles ABP and CDP are similar;
    d. triangles BCP and DAP need not be similar.



1. Which is the better (tighter) fit: A round peg in a square hole or a square peg in a round hole?

2. From the top of Mt Washington, which is 6288 feet above sea level, how far is it to the horizon? Assume that the Earth has a 3962-mile radius, and give your answer to the nearest mile.

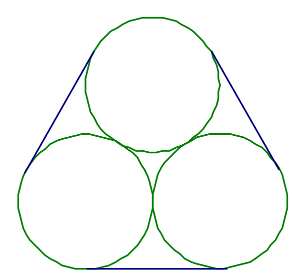
3. What is the minimum amount of wrapping paper needed to wrap a box with dimensions 20 cm by 10 cm by 30 cm?

4. Which polygons can have circumscribed circles? Explain.

5. A paper towel tube has a diameter of 1.7 inches and a height of 11 inches. If the tube were cut and unfolded to form a rectangle, what would be the area of the rectangle?

6. Find the area of a kite whose longer diagonal is divided into two parts that are 4 and 12 and whose shorter side is 5.

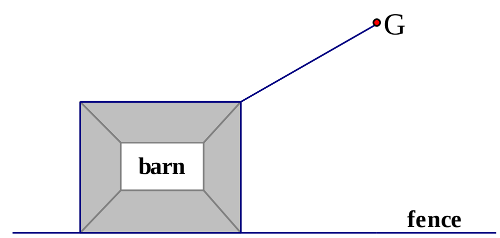
7. The figure shows three circular pipes, all with 12-inch diameters, that are strapped together by a metal band. How long is the band?



8. (Continuation) Suppose that four pipes are strapped together with a snugly-fitting metal band. How long is the band?

9. The lateral edges of a regular hexagonal pyramid are all 20 cm long, and the base edges are all 16 cm long. To the nearest cm^3, what is the volume of this pyramid? To the nearest square cm, what is the combined area of the base and six lateral faces?

10. Find the total grazing area of the goat *G* represented in the figure (a top view) shown at right. The animal is tied to a corner of a 40′ × 40′ barn, by an 80′ rope. One of the sides of the barn is extended by a fence. Assume that there is grass everywhere except inside the barn.



11. *Surface area of a Sphere*: The surface area of a sphere is found using the formula $4\pi r^2$. Find the surface area of the Earth, given that its diameter is 7924 miles.

12. For any pyramid, the volume is $\frac{1}{3} \bullet \text{base area} \bullet \text{height}$. A cone is a pyramid with a circular base. Find the volume of a cone with a slant height of 13 and a diameter of 10.

13. A sector of a circle is enclosed by two 12.0-inch radii and a 9.0-inch arc. Its perimeter is therefore 33.0 inches. What is the area of this sector, to the nearest tenth of a square inch? What is the central angle of the sector, to the nearest tenth of a degree?

1. (Continuation) There is another circular sector – part of a circle of a different size – that has the same 33-inch perimeter and that encloses the same area. Find its central angle, radius, and arc length, rounding the lengths to the nearest tenth of an inch.

2. The radius of the Sun is 109 times the radius of the Earth. Find the surface area of the Sun.

3. The radius of a circular sector is r. The central angle of the sector is θ. Write formulas for the arc length and the perimeter of the sector.

4. Suppose that the lateral faces *VAB*, *VBC*, and *VCA* of triangular pyramid *VABC* all have the same height drawn from *V*. Let *F* be the point in base *ABC* that is closest to *V*, so that *VF* is the altitude of the pyramid. Show that *F* is one of the special points of triangle *ABC*.

5. Schuyler has made some glass prisms to be sold as window decorations. Each prism is four inches tall, and has a regular hexagonal base with half-inch sides. They are to be shipped in cylindrical tubes that are 4 inches tall. What radius should Schuyler use for the tubes? Once a prism is inserted into its tube, what volume remains for packing material?

6. Suppose that chords *AB* and *BC* have the same lengths as chords *PQ* and *QR*, respectively, with all six points belonging to the same circle (they are *concyclic*). Is this enough information to conclude that chords *AC* and *PR* have the same length? Explain.

7. A conical cup has a 10-cm diameter and is 12 cm deep. How much can this cup hold?

8. (Continuation) Water in the cup is 6 cm deep. What percentage of the cup is filled?

9. Imagine the interior of a sphere can be approximated by numerous cones, each with a base area, B, height of r, and vertex at the center of the sphere. What formula do you already know that describes the sum of the base areas?

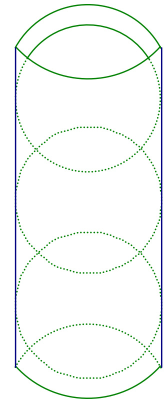
10. (Continuation) Given this approximation of a sphere, develop a formula for its volume.

11. A sphere of ice cream is placed on an ice cream cone. Both have a diameter of 8 cm. The height of the cone is 12 cm. Will all the ice cream, if pushed down into the cone, fit?

12. Dana takes a paper cone with a 10-cm diameter and is 12 cm deep, cuts it along a straight line from the rim to the vertex, then flattens the paper out on a table. Find the radius, the arc length, and the central angle of the resulting circular sector.

13. The base radius of a cone is 6 inches, and the cone is 8 inches tall. To the nearest square inch, what is the area of the lateral surface of the cone? What is the total surface area of the cone?

1. Three tennis balls fit snugly inside a cylindrical can. What percent of the available space inside the can is occupied by the balls?



2. The areas of two circles are in the ratio of 50: 32. If the radius of the larger circle is 10, what is the radius of the smaller circle?

3. Dana takes a sheet of paper, cuts a 120-degree circular sector from it, then rolls it up and tapes the straight edges together to form a cone. Given that the sector radius is 12 cm, find the height and volume of this paper cone.

4. In an effort to make their product seem like a better bargain, the Chock-a-Lot candy company increased the size of their chocolate balls, from a 2-cm diameter to a 3-cm diameter, without increasing the price. In fact, the new balls still contain the same amount of chocolate, because they are *hollow spherical shells*, while the 2-cm balls are solid chocolate. How thick are the spherical chocolate shells that Chock-a-Lot is now selling?

5. The area of a sector of a circle with radius 12 is 16π cm^2. What is the central angle of this sector?

6. Find the perimeter of the semicircle with radius 10.

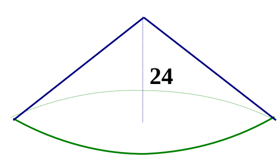
7. You are familiar with the square root notation $\sqrt{81} = 9$. This is because $9x9 = 9^2 = 81$. When looking for the number that appears as a factor three times, the term used is the "cube root" and the notation is $\sqrt[3]{8} = 2$ because $2x2x2 = 2^3 = 8$ (note that the small three denotes the cube root).

8. **(a)** Find $\sqrt[3]{27}$ **b)** Find $\sqrt[3]{125}$

9. A squash ball fits snugly inside a cubical box whose edges are 4 cm long. Guess the percentage of the box's volume that the ball occupies, then calculate that percentage. (This is an example of a *sphere inscribed in a cube*.)

10. There is a park, 27 feet wide, that is between two buildings whose heights are 123 ft and 111 ft. Two Deerfield teachers, Mr. Barnes and Ms.Schettino are standing on top of the shorter building looking at a rare bird perched on top of the taller building. If Mr. Barnes is 74 inches tall and Ms. Schettino is 62 inches tall who has the smaller angle of elevation while looking at the bird? Explain your answer.

11. The highway department keeps its sand in a conical storage building that is 24 feet high and 64 feet in diameter. To estimate the cost of painting the building, the lateral surface area of the cone is needed. To the nearest square foot, what is the area?



1. As a spherical gob of ice cream that once had a 2-inch radius melts, it drips into a cone of the same radius. The melted ice cream exactly fills the cone. What is the height of the cone?

2. A conical cup is 64/125 full of liquid. What is the ratio of the depth of the liquid to the depth of the cup? Conical cups appear fuller than cylindrical cups – explain why.

3. Two similar triangles have medians in a ratio of 5: 6, what is the ratio of their areas?

4. Find the volume of material that is needed to form a spherical shell whose outer radius is 6.0 inches and whose thickness is 0.01 inch. Use your answer to estimate the surface area of the 6-inch sphere.

5. A spider is on the rim of an empty conical cup when it spies a fly one third of the way around the rim. The cone is 36 cm in diameter and 24 cm deep. In a hurry for lunch, the spider chooses the shortest path to the fly. How long is this path?

6. A 10 cm tall cylindrical glass 8 cm in diameter is filled to 1 cm from the top with water. If a gold ball 4 cm in diameter is dropped into the glass, will the water overflow?

7. An *annulus* is defined as the region lying between two *concentric* circles. If the diameter of the larger circle is 20 in and the radius of the smaller circle is 8, find the area of the annulus.

8. A Reese's Big Cup has a diameter of two inches and a height of 0.8 inches. A Reese's bar has dimensions 4 inches by 0.8 inches by 0.5 inches. Using approximations, which candy has more peanut butter?

9. (Continuation) Assuming a uniform chocolate thickness, which candy has more chocolate?

10. Ice cream scoops are often 6 cm in diameter. How many scoops should you get from a half-gallon of ice cream? A half-gallon container can be approximated by a cylinder with a diameter of 12 cm and a height of 14 cm.

11. A spherical globe, 12 inches in diameter, is filled with spherical gumballs, each having a 1-inch diameter. Estimate the number of gumballs in the globe, and explain your reasoning.

12. The altitudes of two similar triangles are 6 cm and 9 cm. If the area of the larger triangle is 36 cm^2, what is the area of the smaller triangle?

13. Seventy percent of the Earth's surface is covered in water. Find the approximate surface area of the Earth that is dry land.

1. Given triangle *EWS* defined by *E*(5, 5), *W*(4, -8), and *S*(-6, 6), write the equation of the median from point *E* to *WS*. How far is it from point *E* to the centroid?

2. The ratio of similarity of two triangular prisms is 3: 5. What is the ratio of their surface areas? What is the ratio of their volumes?

3. The sum of the lengths of the two bases of a trapezoid is 22 cm and its area is 946 cm^2. Find the height of this trapezoid.

4. The volumes of two similar hexagonal prisms are in the ratio of 8: 125. What is the ratio of their heights? If the surface area of the larger prism is 100, what is the surface area of the smaller prism?

5. A triangle is defined by placing vectors [5, 7] and [-21, 15] tail to tail. Find its angles.

6. Find the point that is equidistant from the points (0, 4), (2, 3), and (5, 9).

7. The base of a pyramid is the regular polygon *ABCDEFGH*, which has 14-inch sides. All eight of the pyramid's lateral edges, *VA*, *VB*, etc, are 25 inches long. To the nearest tenth of an inch, calculate the height of pyramid *VABCDEFGH*.

8. An equilateral triangle is inscribed in the circle of radius 1 centered at the origin (the *unit circle*). If one of the vertices is (1, 0), what are the coordinates of the other two? The three points divide the circle into three arcs; what are the angular sizes of these arcs?

9. The surface areas of two cubes are in the ratio of 49: 81. If the volume of the smaller cube is 20, what is the volume of the larger cube?

10. Charlie built a treasure box. Lucy built a treasure box with dimensions twice as large as Charlie's. If it takes one-half gallon of paint to cover the surface of Charlie's box, how many gallons of paint would it take to paint Lucy's box? How many times more volume will Lucy's box hold than Charlie's?

11. Find the area of the regular polygon whose exterior angle is 45 and whose sides are 3.5 inches.

12. Suppose that *DRONE* is a regular pentagon and that *DRUM*, *ROCK*, *ONLY*, *NEAP*, and *EDIT* are squares attached to the outside of the pentagon. Show that decagon *ITAPLYCKUM* is equiangular. Is this decagon equilateral?

13. A 20-inch chord is drawn in a circle with a 12-inch radius. What is the *angular size* of the minor arc of the chord?

14. Find the lengths of both altitudes in the parallelogram determined by [2, 3] and [−5, 7].

1. To the nearest tenth of a degree, find the angle formed by placing the vectors [4, 3] and [-7, 1] tail-to-tail.

2. Suppose that square *PQRS* has 15-cm sides and that *G* and *H* are on *QR* and *PQ*, respectively, so that *PH* and *QG* are both 8 cm long. Let *T* be the point where *PG* meets *SH*. Find the size of angle *STG*, with justification.

3. (Continuation) Find the lengths of *PG* and *PT*.

4. It is given that the sides of an isosceles trapezoid have lengths 3 in, 15 in, 21 in, and 15 in. Make a diagram. Show that the diagonals intersect perpendicularly.

5. Triangle *ABC* has *AB* = *AC*. The bisector of angle *B* meets *AC* at *D*. Extend side *BC* to *E* so that *CE* = *CD*. Triangle *BDE* should look isosceles. Is it? Explain.

6. Find coordinates for the centroid of the triangle whose vertices are
a. (-1, 5), (-2, 8) and (3, 3); **b)** (2, 7), (8, 1) and (14, 11); **(c)** (*a*, *p*), (*b*, *q*) and (*c*, *r*).

7. Segments *AC* and *BD* intersect at *E*, so as to make *AE* twice *EC* and *BE* twice *ED*. Prove that segment *AB* is twice as long as segment *CD*, and parallel to it.

8. Can a circle always be drawn through three given points? If so, describe a procedure for finding the center of the circle. If not, explain why not.

9. Find a triangle two of whose angles have sizes $\text{TAN}^{-1}(1.5)$ and $\text{TAN}^{-1}(3)$. Answer this question either by giving coordinates for the three vertices, or by giving the lengths of the three sides. To the nearest 0.1 degree, find the size of the third angle in your triangle.

10. Let *RICK* be a parallelogram, with *M* the midpoint of *RI*. Draw the line through *R* that is parallel to *MC*; it meets the extension of *IC* at *P*. Prove that *CP* = *KR*.

11. Triangle ABC with A(3, -2), B(6,-1) and C(1,2) is dilated onto A'B'C' with A'(2,-3), B'(3.5, -2,5) and C'(1,-1). Find the center of dilation and the scale factor.

12. Suppose that *PEANUT* is a regular hexagon, and that *PEGS*, *EACH*, *ANKL*, *NUMB*, *UTRY*, and *TPOD* are squares attached to the outside of the hexagon. Decide whether or not dodecagon *GSODRYMBKLCH* is regular and give your reasons.

13. A kite has an 8-inch side and a 15-inch side, which form a right angle. Find the length of the diagonals of the kite.

14. Point P is marked inside regular pentagon *TRUDY* so that triangle *TRP* is equilateral. Decide whether or not quadrilateral *TRUP* is a parallelogram and give your reasons.

1. Find the equation of the line that contains all of the points equidistant from the points A(-2, 7) and B(3, 6).

2. A triangle with sides 6, 8, and 10 and a circle with radius is r are drawn so that no part of the triangle lies outside the circle. How small can r be?

3. Diagonals AC and BD of regular pentagon $ABCDE$ intersect at H. Decide whether or not $AHDE$ is a rhombus, and give your reasons.

4. Let $A = (3, 1)$, $B = (9, 5)$, and $C = (4, 6)$. Your protractor should tell you that angle CAB is about 45 degrees. Explain why angle CAB is in fact exactly 45 degrees.

5. The sides of a polygon are cyclically extended to form *rays*, creating one exterior angle at each vertex. Viewed from a great distance, what theorem does this figure illustrate?

6. In trapezoid $ABCD$, AB is parallel to CD, and $AB = 10$, $BC = 9$, $CD = 22$, and $DA = 15$. Points P and Q are marked on BC so that $BP = PQ = QC = 3$, and points R and S are marked on DA so that $DR = RS = SA = 5$. Find the lengths PS and QR.

7. Triangle ABC has $AB = 12 = AC$ and angle A is 120 degrees. Let F and D be the midpoints of sides AC and BC, respectively, and G be the intersection of segments AD and BF. Find the lengths FD, AD, AG, BG, and BF.

8. The midpoints of the sides of a quadrilateral are joined to form a new quadrilateral. For the new quadrilateral to be a rectangle, what must be true of the original quadrilateral?

9. The vectors [8, 0] and [3, 4] form a parallelogram. Find the lengths of its altitudes.

10. One leg of a right triangle is twice as long as the other and the perimeter of the triangle is 40. Find the lengths of all three sides.

11. Suppose that *PEANUT* is a regular hexagon, and that *PEGS*, *EACH*, *ANKL*, *NUMB*, *UTRY*, and *TPOD* are squares attached to the outside of the hexagon. Decide whether or not dodecagon *GSODRYMBKLCH* is regular and give your reasons.

12. Square $ABCD$ has 8-inch sides, M is the midpoint of BC, and N is the intersection of AM and diagonal BD. Find the lengths of NB, NM, NA, and ND.

13. Parallelogram $PQRS$ has $PQ = 8$ cm, $QR = 9$ cm, and diagonal $QS = 10$ cm. Mark F on RS, exactly 5 cm from S. Let T be the intersection of PF and QS. Find the lengths TS and TQ.

14. The parallel sides of a trapezoid are 12 inches and 18 inches long. The non-parallel sides meet when one is extended 9 inches and the other is extended 16 inches. How long are the non-parallel sides of this trapezoid?

1. Show that the area of a square is half the product of its diagonals. Then consider the possibility that there might be other quadrilaterals with the same property.

2. The dimensions of rectangle $ABCD$ are $AB = 12$ and $BC = 16$. Point P is marked on side BC so that $BP = 5$ and the intersection of AP and BD is called T. Find the lengths of the four segments TA, TP, TB, and TD.

3. The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into two segments, whose lengths are 8 inches and 18 inches. How long is the altitude?

4. A triangle has two 13-cm sides and a 10-cm side. The largest circle that fits inside this triangle meets each side at a point of tangency. These points of tangency divide the sides of the triangle into segments of what lengths?

5. (Continuation) What is the radius of this circle?

6. In the middle of the nineteenth century, octagonal barns and sheds (and even some houses) became popular. How many cubic feet of grain would an octagonal barn hold if it were 12 feet tall and had a regular base with 10-foot edges?

7. The diagonal of a rectangle is 15 cm, and the perimeter is 38 cm. What is the area? It is possible to find the answer without finding the dimensions of the rectangle – try it.

8. A triangle that has a 5-inch and a 6-inch side can be similar to a triangle that has a 4-inch and an 8-inch side. Find all possible examples. Check that your examples really *are* triangles.

9. What is the radius of the circumscribed circle for a triangle whose sides are 15, 15, and 24 cm long?

10. A kite has a 5-inch side and a 7-inch side. One of the diagonals is bisected by the other. The bisecting diagonal has length 8 inches. Find the length of the bisected diagonal.

11. The academy's track is shaped like a rectangle with a semicircle on each of the shorter sides. The distance around the track is one-quarter mile. The straightaway is twice as long as the width of the field. What is the area of the field enclosed by the track to the nearest square foot?

12. Let $A = (1, 1)$, $B = (3, 5)$, and $C = (7, 2)$. Explain how to cover the whole plane with non-overlapping triangles, each of which is congruent to triangle ABC.

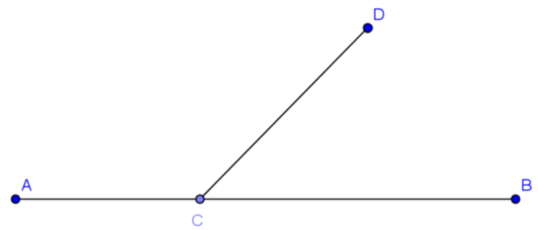
13. (Continuation) In the pattern of lines produced by your *tessellation* you should see triangles of many different sizes. What can you say about their sizes and shapes?

AA similarity: Two *triangles* are sure to be similar if at least two of their angles are equal in size.

adjacent angles: Two angles with a common vertex that share a side but have no common interior points.

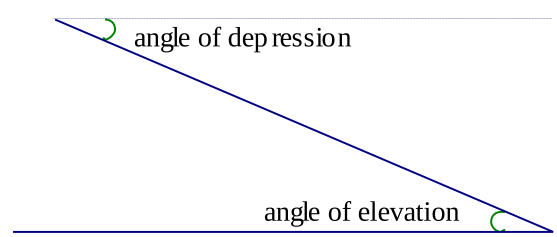
altitude: In a triangle, an altitude is a line through one of the vertices, perpendicular to the opposite side. In obtuse triangles, it may be necessary to extend a side to meet the altitude. The *distance* from the vertex to the point of intersection with the line containing the opposite side is also called an altitude, as is the distance that separates the parallel sides of a trapezoid.

angles can often be identified by a single letter, but sometimes three letters are necessary. The angles shown can be referenced as *C*, *ACD*, or *BCD*.



angle of depression: Angle formed by a horizontal ray and a line-of-sight ray that is below the horizontal. See the diagram at right.

angle of elevation: Angle formed by a horizontal ray and a line-of-sight ray that is above the horizontal. See the diagram at right.



Angle-Angle-Side (corresponding): When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that one pair of corresponding sides has the same length, then the triangles are congruent. This rule of evidence is abbreviated to AAS.

angle bisector: Given an angle, this ray divides the angle into two equal parts.

Angle Bisector Theorem: The bisector of any angle of a triangle cuts the opposite side into segments whose lengths are proportional to the sides that form the angle.

Angle-Side-Angle: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that the (corresponding) shared sides have the same length, then the triangles are congruent. This rule of evidence is abbreviated to ASA.

angular size of an arc: This is the size of the central angle formed by the radii that meet the endpoints of the arc. Also called the measure of the arc.

arc: A section of the perimeter of a circle. Can be a *minor arc* (measure less than 180 degrees) or *major arc* (measure greater than 180 degrees). The portion of a circle that lies to one side of a chord is also called an *arc*.

arc length: The linear distance along the circle from one endpoint of the arc to the other. Given a circle, the length of any arc is proportional to the size of its central angle.

areas of similar figures: If two figures are similar, then the ratio of their areas equals the *square* of the ratio of similarity.

bisect: Divide into two equal parts.

central angle: An angle formed by two radii of a circle.

centroid: The medians of a triangle are concurrent at this point, which is the balance point (also known as the *center of gravity*) of the triangle.

chord: A segment that joins two points on a circle is called a *chord* of the circle.

circle: The set of all points equidistant from a given point, called the *center*. The common distance is the *radius* of the circle. A segment joining the center to a point on the circle is also called a *radius*.

circumcenter: The perpendicular bisectors of the sides of a triangle are concurrent at this point, which is equidistant from the vertices of the triangle.

circumscribed circle: When possible, the circle that goes through all the vertices of a polygon.

collinear: Three (or more) points that all lie on a single line are *collinear*.

common chord: A segment that joins the points where two circles intersect.

complementary: Two angles that fit together to form a right angle are called complementary. Each angle is the *complement* of the other.

components the two parts of a vector that describe how to move from one point to another. Vectors [x,y] describe the motion with the x component (describing the horizontal shift) and the y component (the vertical shift). They are obtained by *subtracting* coordinates, being mindful of the direction of the vector.

concentric: Two figures that have the same center are called *concentric*.

concurrent: Three (or more) lines that go through a common point are *concurrent*.

concyclic: Points that all lie on a single circle are called *concyclic*.

congruent: When the points of one figure can be matched with the points of another figure, so that corresponding parts have the same size, then the figures are called *congruent*, which means that they are considered to be equivalent.

converse: The converse of a statement of the form "if [something] then [something else]" is the statement "if [something else] then [something]."

convex: A polygon is called *convex* if every segment joining a pair of points within it lies entirely within the polygon.

coordinates: Numbers that describe the position of a point in relation to the origin of a coordinate system.

corresponding: They are parts of polygons that are in the same position relative to each other. Corresponding describes parts of figures (such as angles or segments) that could be matched by means of a transformation. In congruent polygons, if the polygons were superimposed, the corresponding parts would be right on top of one another.

cosine ratio: Given a right triangle, the cosine of one of the acute angles is the ratio of the length of the side *adjacent* to the angle to the length of the hypotenuse. The word cosine is a combination of *complement* and *sine*, so named because the cosine of an angle is the same as the sine of the complementary angle.

CPCTC: *Corresponding Parts of Congruent Triangles are Congruent*.

cyclic: A polygon, all of whose vertices lie on the same circle, is called *cyclic*. Also called an *inscribed polygon*.

decagon: A polygon that has ten sides.

diagonal: A segment that connects two nonadjacent vertices of a polygon.

diameter: A chord that goes through the center of a circle is called a *diameter*.

dilation: A similarity transformation, with the special property that all lines obtained by joining points to their images are concurrent at the same *central* point.

distance formula: The distance from (x_1, y_1) to (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This formula is a consequence of the *Pythagorean Theorem*.

dodecagon: A polygon that has twelve sides.

dynamic: A figure is dynamic if it is not fixed in the plane.

equiangular: A polygon all of whose angles are the same size.

equidistant: A shortened form of *equally distant*.
equilateral: A polygon all of whose sides have the same length.

Euclidean geometry (also known as plane geometry) is characterized by its parallel postulate, which states that, *given a line, exactly one line can be drawn parallel to it through a point not on the given line*. A more familiar version of this assumption states that *the sum of the angles of a triangle is a straight angle*.

Euler line: The centroid, the circumcenter, and the orthocenter of any triangle are collinear.

exterior angle: An angle that is formed by a side of a polygon and the extension of an adjacent side. It is supplementary to the adjacent interior angle.

Exterior Angle Theorem: An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

function: A function is a rule that describes how the value of one thing is determined uniquely by the value of another thing.

glide-reflection: An transformation created by a vector translation and reflection. The mirror line contains the transformation vector.

Greek letters appear often in mathematics. Some of the common ones are α (alpha), β (beta), Δ or δ (delta), θ (theta), Λ and λ (lambda), μ (mu), π (pi), and Ω or ω (omega).

head: Vector terminology for the second vertex of a directed segment.

hexagon: a polygon that has six sides.

Hypotenuse-Leg: When the hypotenuses of two right triangles have the same length, and a leg of one triangle has the same length as a leg of the other, then the triangles are congruent. This rule of evidence is abbreviated to HL.

image: The result of applying a transformation to a point *P* is called the *image of P*, often denoted *P'*. One occasionally refers to an *image segment* or an *image triangle*.

incenter: The angle bisectors of a triangle are concurrent at this point, which is equidistant from the sides of the triangle.

included angle: The angle formed by two designated segments.

inscribed angle: An angle formed when two chords meet at a point on the circle. An inscribed angle is *half* the angular size of the arc it intercepts. In particular, an inscribed angle that intercepts a semicircle is a *right* angle.

inscribed polygon: A polygon whose vertices all lie on the same circle; also called a *cyclic polygon*.

integer: Any whole number, whether it be positive, negative, or zero.

intercepted arc: The part of an arc that is found inside a given angle.

isometry: A geometric transformation that preserves distances. The best-known examples of isometries are *translations*, *rotations*, and *reflections*.

isosceles triangle: A triangle that has two sides of the same length. The word is derived from the Greek *iso* + *skelos* (equal + leg)

Isosceles Triangle Theorem: If a triangle has at least two sides of equal length, then the angles opposite the congruent sides are also the same size.

isosceles trapezoid: A trapezoid whose nonparallel sides have the same length.

kite: A quadrilateral that has two pairs of congruent adjacent sides.

labeling convention: Given a polygon that has more than three vertices, place the letters around the figure in the order that they are listed.

lateral face: Any face of a pyramid or prism that is not a base.

lattice point: A point whose coordinates are both integers.

lattice rectangle: A rectangle whose vertices are all lattice points.

leg: The perpendicular sides of a right triangle are called its legs.

length of a vector: This is the length of any segment that represents the vector. Notation: length of $\vec{v}$ is labeled as $|\vec{v}|$. $\vec{v} = [x, y]$, then $|\vec{v}| = \sqrt{x^2 + y^2}$.

linear equation: Any straight line can be described by an equation in the form $Ax + By = C$.

linear pair: Two adjacent angles whose sum is 180 degrees; Two angles that form a straight angle.

magnitude of a dilation: The non-negative number obtained by dividing the length of any dilated segment by its original length. See *ratio of similarity*.

major/minor arc: A non-diameter chord of a circle divides a circle into two parts. Of the two arcs, the smaller one is called *minor* (less than 180 degrees), and the larger one is called *major* (more than 180 degrees). Often, a major arc is described with a label that has 3 letters and a minor arc is described with 2 letters.
median of a triangle: A segment that joins a vertex of a triangle to the midpoint of the opposite side.

midline of a trapezoid: This segment joins the midpoints of the non-parallel sides. Its length is the average of the lengths of the parallel sides, to which it is also parallel. Also known as the *median* in some books.

Midsegment Theorem: A segment that joins the midpoints of two sides of a triangle is parallel to the third side, and is half as long.

midpoint: The point on a segment that is equidistant from the endpoints of the segment. If the endpoints are (a, b) and (c, d), the midpoint is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

Mirror line: In a reflection, the mirror line is the perpendicular bisector of the segments that connect the initial point (pre-image) and its reflected point (image).

Negative (opposite) reciprocal: One number is the negative reciprocal of another if the product of the two numbers is -1.

octagon: a polygon that has eight sides.

opposite: Two numbers or vectors are opposite if they differ in sign. For example, 17.5 is the opposite of -17.5, and $[2, -11]$ is the opposite of $[-2, 11]$.

orthocenter: The altitudes of a triangle are concurrent at this point.

parallel: Coplanar lines that do not intersect. When drawn in a coordinate plane, they are found to have the same slope, or else no slope at all. The shorthand // is often used.

parallelogram: A quadrilateral that has two pairs of parallel sides.

pentagon: a polygon that has five sides.

perpendicular: Coplanar lines that intersect to form a right angle.

perpendicular bisector: Given a line segment, this is the line that is perpendicular to the segment and that goes through its *midpoint*. The points on this line are all *equidistant* from the endpoints of the segment.

point-slope form: A non-vertical straight line can be described by $y - y_0 = m(x - x_0)$

or by $y = m(x - x_0) + y_0$. One of the points on the line is (x_0, y_0) and the slope is *m*.

postulate: A statement that is accepted as true, without proof.

prism: A three-dimensional figure that has two congruent and parallel *bases*, and parallelograms for its remaining *lateral faces*. If the lateral faces are all rectangles, the prism is a *right prism*. If the base is a regular polygon, the prism is also called *regular*.

Proof by Contradiction (Indirect Proof): method of mathematical proof in which the mathematician assumes the opposite of what they are attempting to proof in the hope of coming up with a contradiction of already known fact. This contradiction thereby proves that the assumption that the statement must have been false.

proportion: An equation that expresses the equality of two *ratios*.

pyramid: A three-dimensional figure that is obtained by joining all the points of a polygonal *base* to a *vertex*. Thus all the lateral faces of a pyramid are triangles. If the base polygon is regular, and the lateral edges are all congruent, then the pyramid is called *regular*.

Pythagorean Theorem: The area of the square with a side equal to the hypotenuse of a right triangle equals the sum of the areas of the squares whose sides are the lengths of the legs of the right triangle. If *a* and *b* are the lengths of the legs of a right triangle, and if *c* is the length of the hypotenuse, then these lengths fit the Pythagorean equation
$a^2 + b^2 = c^2$.

quadrant: one of the four regions formed by the coordinate axes. Quadrant I is where both coordinates are positive, and the other quadrants are numbered (using Roman numerals) in a counterclockwise fashion.

quadratic formula: $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ are the two solutions to $ax^2 + bx + c = 0$.

quadrilateral: a four-sided polygon.

ratio of similarity: The ratio of the lengths of any two corresponding segments of similar figures.

ray: A ray is a line bounded at one end and infinite at the other.

rectangle: An equiangular quadrilateral.

reflection: A reflection maps points on one side of the *mirror line* to the other side. If the point is on the mirror line, then it maps onto itself.

regular: A polygon that is both equilateral and equiangular.

rhombus: An equilateral quadrilateral.

right angle: An angle that is its own supplement, in other words, an angle that is 90 degrees.

rotation: A transformation in a plane that moves a figure about a single fixed point. The fixed point is called the *center of rotation*.

SAS similarity: Two triangles are certain to be similar if two sides of one triangle are proportional to two sides of the other, and if the included angles are equal in size.

Same Side interior angles – angles formed by two parallel lines and a transversal which are non-adjacent, interior angles (i.e. they do not share a vertex and are both interior to the parallel lines).

scalar: In the context of vectors, this is just another name for a number that can change the magnitude and/or direction of the vector.

scalene: A triangle that has 3 different side lengths.

segment: That part of a line that lies between two designated points.

Sentry Theorem: The sum of the exterior angles (one per vertex) of any polygon is 360 degrees.

Shared Altitude Theorem: If two triangles share an altitude, then the ratio of their areas is proportional to the ratio of the corresponding bases.

Shared Base Theorem: If two triangles share a base, then the ratio of their areas is proportional to the ratio of the corresponding altitudes.

Side-Angle-Side: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding sides have the same lengths, and so that the (corresponding) angles they form are also the same size, then the triangles are congruent. This rule of evidence is abbreviated to just SAS.

Side-Side-Angle: Insufficient grounds for congruence. See *Hypotenuse-Leg*, however.

Side-Side-Side: When the parts of one triangle can be matched with the parts of another triangle, so that all three pairs of corresponding sides have the same lengths, then the triangles are congruent. This rule of evidence is abbreviated to just SSS.

similar: Two figures are similar if their points can be matched in such a way that all ratios of corresponding lengths are proportional to a fixed *ratio of similarity*. Corresponding angles of similar figures must be equal in size.

sine ratio: Given a right triangle, the sine of one of the acute angles is the ratio of the length of the side *opposite* the angle to the length of the hypotenuse.

skew lines: Non-coplanar lines that do not intersect.

slope: The slope of the segment that joins the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

slope-intercept form: Any non-vertical straight line can be described by an equation that takes the form $y = mx + b$. The slope of the line is m, and the y-intercept is b.

SSS similarity: Two triangles are *similar* if their corresponding sides are proportional.

square: A regular quadrilateral.

supplementary: Two angles whose measures add up to 180 degrees and could be fit together to form a straight line are called *supplementary*. Each angle is the *supplement* of the other.

tail: Vector terminology for the first vertex of a directed segment.

tail-to-tail: Vector terminology for directed segments with a common first vertex.

tangent ratio: Given a right triangle, the tangent of one of the acute angles is the ratio of the side opposite the angle to the side adjacent to the angle.

tangent and slope: When an angle is formed by the x-axis and a ray through the origin, the *tangent* of the angle is the *slope* of the ray. Angles are measured in a counterclockwise sense, so that rays in the second and fourth quadrants determine negative tangent values.

tangent to a circle: A line that has one and only one intersection with a circle. This intersection is called the *point of tangency.* Such a line is perpendicular to the radius drawn to the point of tangency.

tesselate: To fit non-overlapping tiles together to cover a planar region.

Three Parallels Theorem: Given three parallel lines, the segments they intercept on one transversal are proportional to the segments they intercept on any transversal.

transformation: A *function* that maps points to points.

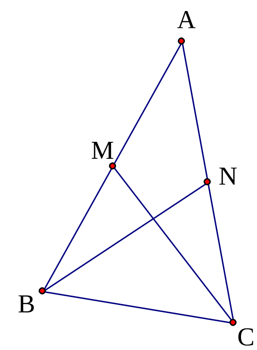
translate: To slide a figure by applying a vector to each of its points.

transversal: A line that intersects two other lines in a diagram.

trapezoid: A quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is called *isosceles*.

triangle inequality: The sum of the lengths of two sides of a triangle is greater than the length of the third side.

two-column proof: A way of outlining a geometric deduction. Steps are in the left column, and supporting reasons are in the right column. For example, here is how one might show that an isosceles triangle ABC has two medians of the same length. It is given that $AB = AC$ and that M and N are the midpoints of sides AB and AC, respectively. The desired conclusion is that medians CM and BN have the same length.



$AB = AC$	given
$AM = AN$	M and N are midpoints
$\angle MAC = \angle NAB$	shared angle
$\Delta MAC \cong \Delta NAB$	SAS
$CM = BN$	CPCTC

Two Tangent Theorem: From a point outside a circle, there are two segments that can be drawn tangent to the circle. These segments have the same length.

unit circle: This circle consists of all points that are 1 unit from the origin, O, of the *xy*-plane. Given a point P on this circle, the coordinates of P are the *cosine* and the *sine* of the counterclockwise angle formed by segment OP and the positive x-axis.

unit square: Its vertices are (0, 0), (1, 0), (0, 1), and (1, 1).

Varignon parallelogram: Given any quadrilateral, this is the figure formed by connecting the midpoints of consecutive sides.

vectors have *magnitude* (size), *direction*, and slope. Visualize them as directed segments (arrows). Vectors are described by *components*, just as points are described by coordinates. The vector from point A to point B is often denoted $\overrightarrow{AB}$ or abbreviated by a boldface letter such as **u**, and its magnitude is often denoted $\left|\overrightarrow{AB}\right|$ or $|\mathbf{u}|$. ***See Components.***

vertex: A labeled point in a figure. The plural is *vertices*, but “vertice” is not a word. The point on a parabola that is closest to the focus is also called the vertex.

vertical angles: Two non-adjacent angles that share a vertex and are formed by the intersection of two lines.

volume of a prism: This is the product of the *base area* and the *height*, which is the distance between the parallel base planes.

volume of a pyramid: This is one third of the product of the *base area* and the *height*, which is the distance from the vertex to the base plane.

volumes of similar figures: If two three-dimensional figures are similar, then the ratio of their volumes equals the *cube* of the ratio of similarity.