

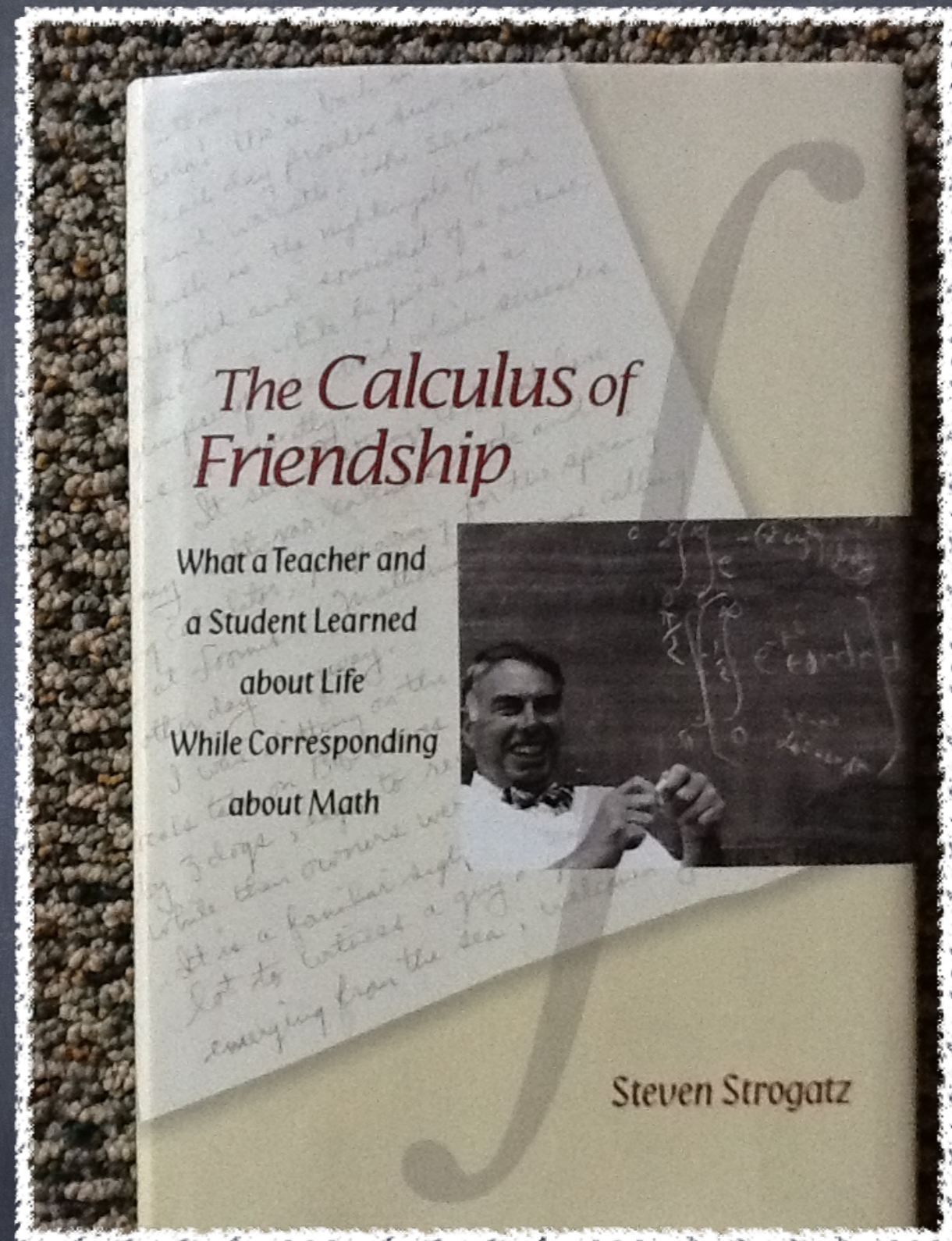
# Problems from The Calculus of Friendship:

Worth Corresponding About  
Carmel Schettino

Inspired by Rick Parris & Ron Lancaster

# A Wonderful Narrative

- Book of relationship
- Read it in '08 gave as gift
- NYT Opinionator Articles
- Spoke at '08 Conf
- Used it in my course this past year



The Calculus of Friendship

What a  
a Stud  
at  
While C  
ab

The Calculus of Friendship

III

6/28/10

To Carmel,

with best wishes to  
a fellow math teacher,

Steven  
Strogatz

Signed by my idol

# Math vis-a-vis Life

• Continuity

• Pursuit

• Relativity

• Irrationality

• Pursuit

• Randomness

• Chaos

• Infinity and  
Limits

# The Monk and the Mountain

A buddhist monk climbs a mountain. The narrow path, no more than a foot wide, spiraled around the mountain to a glittering temple at the summit.

The monk ascended at varying rates, stopping to rest at different times. He reached the top shortly after sunset. A few days later, shortly after sunrise, he began his descent down the mountain along the same path, again walking at variable speeds with pauses in his trip. His average speed was of course greater than his average climbing speed.

Prove that there is a spot along the path that the monk will occupy on both trips at precisely the same time of day.

( p. 71 )



# Solution views

- Graphical perspective
- Calculus - continuous functions of time, IVT guarantees crossing of monotonic curves
- "Dr. Who argument" - two monks at once

# Monty Hall Problem

## Let's Make a Deal:

Monty Knows



Obviously the car is not behind door 2.  
But before I open door 1, the door you selected,  
I'm going to let you switch to door 3 if you like.  
Again, click on the door which you think the car is behind.

- Suppose you are on a game show and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick one door, say #1 and the host opens one of the other 2 doors, say 3, which has a goat. He says to you, "Do you want to switch your choice?" Is it to your benefit to stay with your original choice or switch? (p. 89)





"What most people don't realize is that math itself is a very social activity. We mathematicians talk to each other incessantly. We bounce ideas off one another, cogitate together, and get stuck on the same problems together. When you're working on something as hard as math, it helps to share it with someone who understands." (p. 42)

# A Closed Expression for the $n$ th Fibonacci Number

- Define a shift operator to be an operator that acts on a sequence, all at once, shifting each term by one.

For example, if  $a_n = 1, 3, 5, 7, 9, \dots$  and  $L(a_n) = a_n + 1$ , then the new sequence  $L(\langle a_n \rangle) = 3, 5, 7, 9, 11, \dots$

# Rewriting the Fibonacci Sequence with shift operators

- Similar to the  $D$  operator in a differential equation
- Linear operator
- $Ax + Ay = A(x+y)$  and  $cA(y) = A(cy)$







# The irrationality of the square root of 2

- Traditional proof of irrationality of  $\sqrt{2}$  - by contradiction assuming it can be written as  $m/n$  where  $(m,n)=1$
- Claims this is an "unauthentic" argument that the irrationality of  $\sqrt{2}$  is a "fact about number theory instead of geometry" (p. 27)

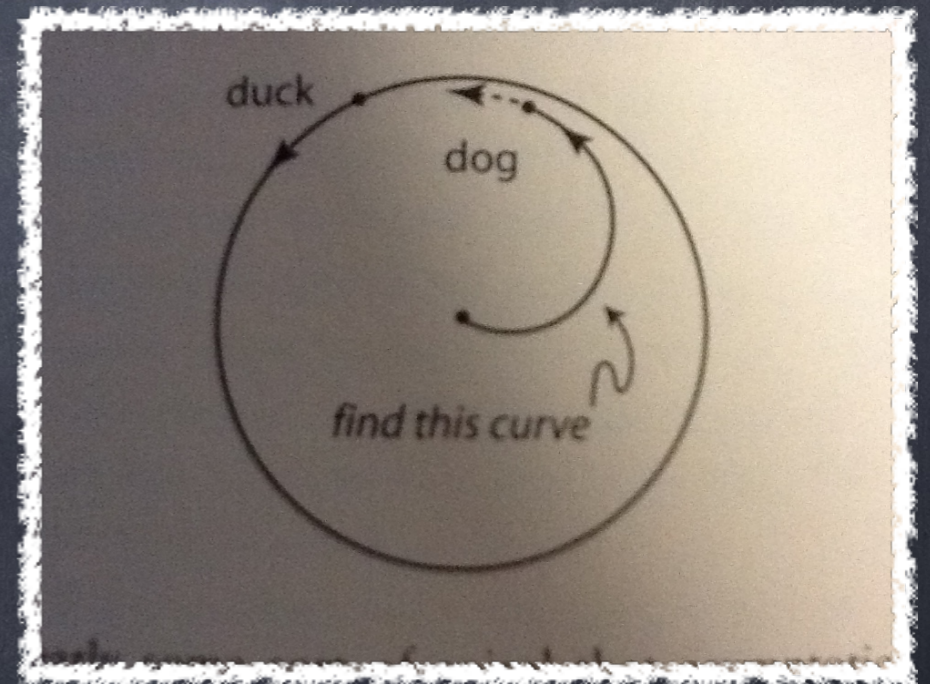
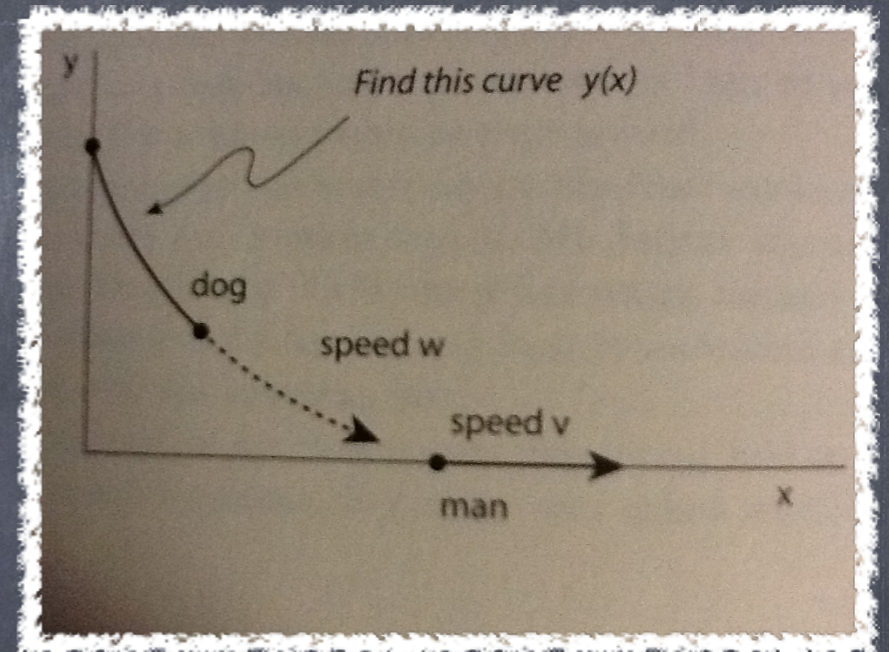






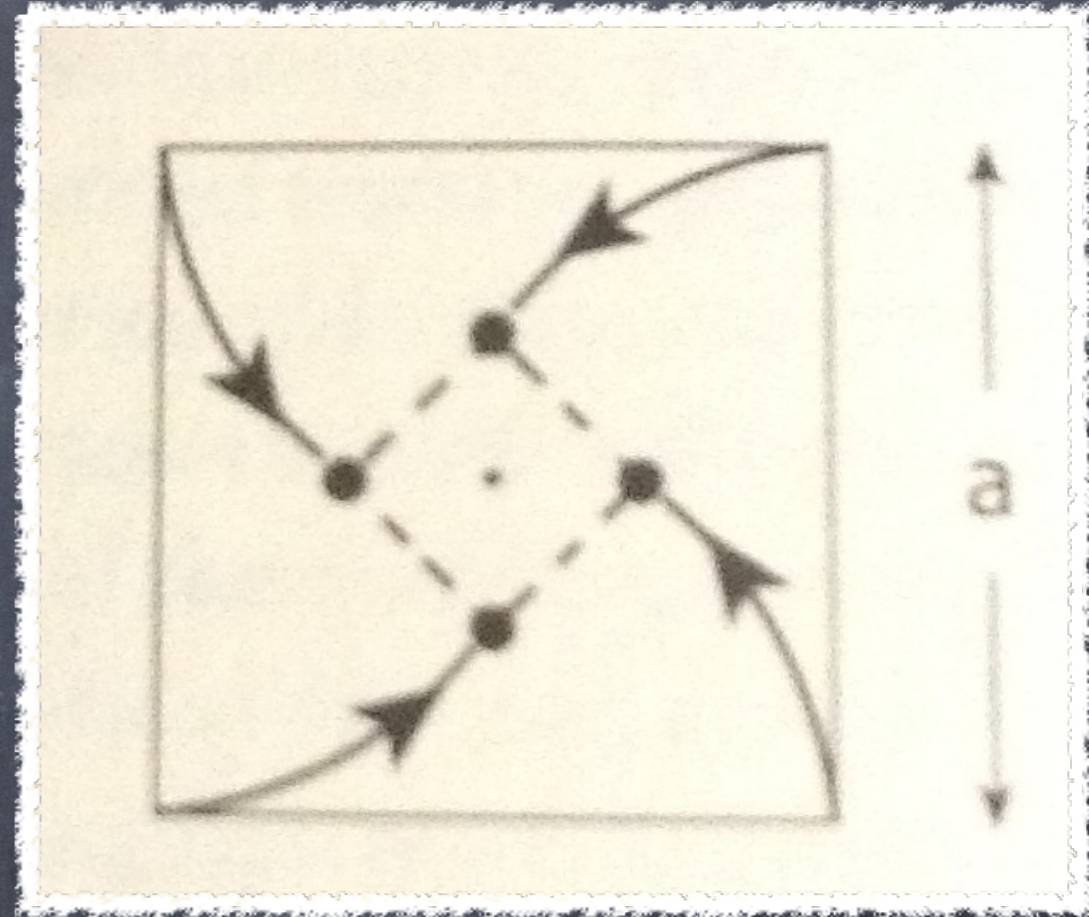
# Pursuit problems

- Initially proposed by Mr. Joffray in his independent study
- He created one of his own



# 4 dogs chase problem

- Starting from the corners of a square of side  $a$ , each dog chases the one counterclockwise from it. If they all start at the same time and run at the same speed, how far does each dog run by the time they collide at the center of the square? (p. 16)





- "The Calculus of Friendship has restored my faith in the intimacy required not only between two people who converse about math but also the intimacy between a person and his intellectual pursuits."
- "To view both the logical and emotional perspective of the people who are using their enthusiasm to solve problems was really refreshing and humanizing."
- "I loved the sense of "happy frustration" because [Mr. Joffray] always seems so happy and enthusiastic about his math even though he has plenty of bad news in his life to be unhappy. I thought that maybe Joff found that being able to converse with a friend about an unvarying topic helped him find shelter from the chaotic aspects of life."