Mathematics 325: Semester 2
Emma Willard School
Spring 2011

The problems in this book are adapted from the Phillips Exeter Academy Math 3 Materials and original problems written by Carmel Schettino and members of the Emma Willard Mathematics Department from 2008-2011.

1. *A polynomial function is a function defined as a sum or difference of terms with constants and variables, without any fractional or negative exponents for the variables. Its degree is the largest exponent.* State which of the following is a polynomial functions. If not a polynomial, explain why not. For those that are, state the degree.
   a. $g(x)=5-2x^4$
   b. $h(x)=\sqrt{x}$
   c. $R(x)=\dfrac{x-3}{x^2-9}$
   d. $p(x)=-2x(x+3)^2(2x-5)$
   e. $f(x)=4$

2. Give an example of a fifth degree polynomial; a third degree polynomial; a zero degree polynomial.

3. Give an example of a function that is not a polynomial. Explain why it is not a polynomial function.

4. Given $P(x)=x^3-3x^2+2x-6$, divide $P(x)$ by
   (a) $(x-2)$ **(b)** $(x+1)$ **(c)** $(x-3)$

5. (Continuation) Compare your answers with $P(2)$, $P(-1)$ and $P(3)$. What can you conclude? Test your conjecture with a different polynomial.

6. Simplify and state the domain:
   (a) $\dfrac{3x+2}{x+1}$ **(b)** $\dfrac{x^2-25}{x^2-2x-15}$

7. Determine the degree of the numerator and the degree of the denominator of $\dfrac{x^2-3x+2}{x^3-2x}$.

8. A rational function is any function that can be written as the ratio of two polynomial functions. State the domain, *x*-intercept(s) and *y*-intercept of the following rational functions.
   (a) $R(x)=\dfrac{1}{x}$ **(b)** $g(x)=\dfrac{3}{x-2}$ **(c)** $f(x)=\dfrac{x^2-2}{x^2+4}$ **(d)** $h(x)=\dfrac{2x^2+1}{x^2-5}$

9. (Continuation) The *vertical asymptote* of the function $g(x)=\dfrac{3}{x-2}$ is $x=2$. How does this relate to the domain of *g(x)*?

1. ***The Remainder Theorem.*** If a polynomial $P(x)$ is divided by $(x-c)$, then the reminder is equal to $P(c)$. Find the remainder of $\dfrac{4x^4-6x^2+5}{x+2}$ by using division and by using the Remainder Theorem..

2. Graph $P(x)=x^5-1$. Verify that the points (1, 0) and (-1,-2) are on the graph. How can the Remainder Theorem confirm your findings? What can you conclude when $P(c)=0$?

3. If $P(x)$ is a polynomial of degree 4, what is the degree of the quotient of $\dfrac{P(x)}{x-c}$? Give an example.

4. ***The Factor Theorem.*** A polynomial $P(x)$ has a factor $(x-c)$ if and only if $P(c)=0$. Using division and the factor theorem, determine if $(x+2)$ is a factor of $P(x)=2x^3+3x^2-5x-6$.

5. Is $x+5$ a factor of $x^4+x^3-21x^2-x+20$?

6. Consider the rational function $f(x)=\dfrac{1}{x}$. Complete the following table of values for *f(x).*

X	*f(x)*
-1	
-0.1	
-0.01	
-0.001	
-0.0001	

x	*f(x)*
1	
0.1	
0.01	
0.001	
0.0001	

x	*f(x)*
-1	
-10	
-100	
-1,000	
-10,000	

x	*f(x)*
1	
10	
100	
1,000	
10,000	

(a) What do the values of *f(x)* approach as the value of *x* approaches zero from the negative side?
(b) What do the values of *f(x)* approach as the value of *x* approaches zero from the positive side?
(c) What do the values of *f(x)* approach as the value of *x* approaches negative infinity?
(d) What do the values of *f(x)* approach as the value of *x* approaches positive infinity?
(e) What is the value of *f(0)?*
(f) Using the information from above, sketch a graph of $f(x)=\dfrac{1}{x}$
(g) The line $x=0$ is the vertical asymptote of this function. Explain.
(h) A *horizontal asymptote* occurs if a rational function approaches a *y*-value for *x*-values far from the origin. The line $y=0$ is the *horizontal asymptote* of this function. Explain.

1. Given $(x+5)$ is a factor of $P(x)=x^3+5x^2-49x-245$, factor completely by dividing by the given factor and then factoring the remaining quadratic polynomial. State the zeros of *P(x)*.

2. **<u>Using your calculator</u>**, graph the following functions. Use the graph to determine the equation of all horizontal and/or vertical asymptotes for each function, and add them to your sketch as dashed lines. Find any *x*- and *y*-intercepts.

   (a) $f(x)=\dfrac{2x-2}{x+1}$ **(b)** $g(x)=\dfrac{-2x-5}{4x-1}$

   (c) $R(x)=\dfrac{3x}{3x+2}$ **(d)** $p(x)=\dfrac{x}{x^2-4}$

3. (Continuation) State the domain of each rational function. Examine the equation(s) of the vertical asymptote(s); conjecture how you determine the vertical asymptote(s)? Describe your method.

4. (Continuation) Examine the degree of the numerator, the degree of the denominator and the equation of the horizontal asymptote for each rational equation; conjecture how you would determine the horizontal asymptote? Describe your method.

5. Given $(x+3)$ and $(x-3)$ are factors of $P(x)=x^4-10x^2+9$, factor completely. State the zeros of the polynomial.

6. Given $(-5,0)$ and $(2,0)$ are points on the graph $P(x)=x^4+2x^3-15x^2+4x+20$, factor completely.

7. The ***leading term*** of a polynomial function in *x* is the non-zero term with the largest power of *x*. State the leading term and the degree of each polynomial function.
   (a) $P(x)=x^3+2x^2-11x-12$ **(b)** $K(x)=25-x^4-2x^7$ **(c)** $A(x)=15x^5-1$

8. (Continuation) Using your calculator to assist, create sketches of the graphs of the polynomial functions. What do you notice about the behavior of the graph to the far-left and far-right of the origin?

9. Given $P(x)=2x^3-7x^2+8x-15$ and $P(3)=0$, factor completely.

10. You are given that $(x-a)$ and $(x-b)$ are factors of $P(x)$. What can be said about $P(x)$, if anything.

11. Explain how the Remainder Theorem can be used to determine whether a real number is a zero of a polynomial.

1. Explain the importance of the remainder when dividing a polynomial by $(x-c)$.

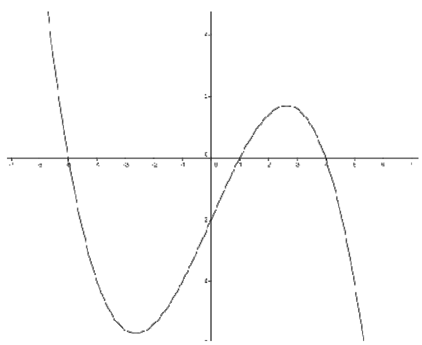
2. A rectangular box has a volume of $V(x)=2x^3+5x^2-22x+15$. The width of the box is $x-1$. Its length is $x+5$. Find the height of the box in terms of x.

3. Graphically and numerically, explain the meaning of the expression: the zero of a polynomial.

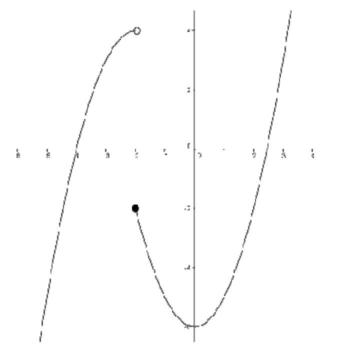
4. What are the advantages and disadvantages of using division rather than substitution to evaluate a polynomial function at $x=c$.

5. A polynomial function must be smooth and continuous, meaning that there are no sharp corners and you can trace it without lifting your pencil. Identify which of the graphs could be a polynomial function. If not a polynomial, explain why not. For those that are, state the zeros.

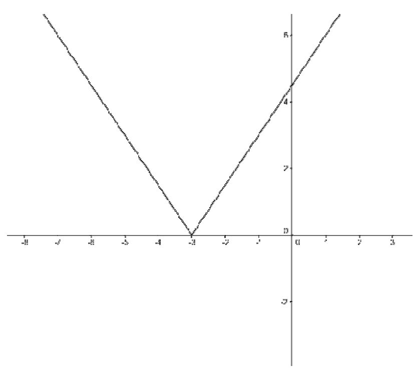
(a)



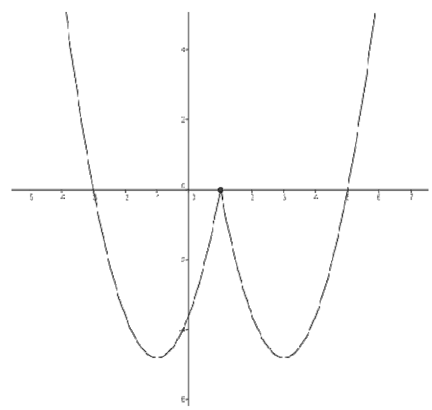
(b)



(c)



(d)



6. Can a rational function ever cross its vertical asymptote? Why or why not?

7. Using your calculator, sketch the graphs $y_1=x^2$, $y_2=x^4$, and $y_3=x^6$ on the same coordinate axes. What do the graphs have in common, and how do they differ?

8. (Continuation) Without your calculator, add the graph $y_4=x^8$. Describe the graph of $y=x^n$, where n is a positive, even integer.

9. (Continuation) Does $y=x^0$ fit this pattern? Why or why not?

1. Use transformations of the graph $y = x^4$ to graph the following. State the transformations applied, in the order they should be applied.
   (a) $t(x) = 2(x-5)^4$ **(b)** $f(x) = (x+2)^4 - 4$ **(c)** $g(x) = 3 - \frac{1}{2}x^4$

2. Can a rational function ever cross its horizontal asymptote? Why or why not?

3. Using what you know, graph each rational function without your calculator. Clearly label all asymptotes with dashed lines and label any intercepts. You may need to use additional points to determine the shape of your graphs.
   (a) $b(x) = \dfrac{2}{x+3}$ **(b)** $f(x) = \dfrac{2x}{x-5}$

4. Using your calculator, sketch the graphs $y_1 = x$, $y_2 = x^3$, and $y_3 = x^5$ on the same coordinate axes. What do the graphs have in common, and how do they differ?

5. (Continuation) Without your calculator, add the graph $y_4 = x^7$. Describe the graph of $y = x^n$, where *n* is a positive, odd integer.

6. Use transformations of the graph $y = x^5$ to graph the following. State, in order, the transformations that you apply.
   (a) $h(x) = -(x+1)^5$ **(b)** $w(x) = (x-2)^5 + 3$ **(c)** $f(x) = 3 - (x+3)^5$

7. Using your calculator to assist you, create a graph of each of the following pairs of polynomial functions on a separate system of coordinate axes:
   (a) $y_1 = x^4 - 2x^2 + 1$ and $y_2 = 1 + x - 3x^2 + x^5$
   (b) $y_1 = x^2 + x + 1$ and $y_1 = x^3 + 2x^2 - 3x - 6$
   What do the pairs of graphs have in common, and how do they differ? What can be said about the far-left and far-right behavior of the graphs?

8. Examine the leading term to determine the far-left and far-right behavior of the graph of each polynomial function.
   (a) $S(x) = 3x^4 - 5x^3 + 20x^2 - 7x + 12$ **(b)** $N(x) = 15 + x^5$ **(c)** $P(x) = \frac{1}{3}x^4 - \frac{1}{4}x^3$

9. What is a rational function? What are some features of a rational function? How do they differ from polynomial functions? How do their graphs differ? Give examples of functions that are rational functions and of functions that are not rational functions.

10. Can the graph of a polynomial function have a vertical asymptote? A horizontal asymptote? If so, find an example. If not, explain why not.

1. Sketch the graph of the rational functions. Label all axis intercepts and asymptotes. Draw asymptotes with dashed lines. Use additional points as necessary. Confirm your graph with your calculator.
   (a) $J(x) = \dfrac{-2}{x+5}$ **(b)** $D(x) = \dfrac{3x+6}{2-3x}$ **(c)** $R(x) = \dfrac{2x}{x-1}$

2. Examine the leading term to determine the far-left and far-right behavior of the graph of each polynomial function.
   (a) $Q(x) = -x^2 - 7x + 12$ **(b)** $K(x) = -x^3 - x + 2$ **(c)** $T(x) = -x^4 + x^5$

3. (Continuation) Examine the leading term to determine the far-left and far-right behavior of the graph of each polynomial function.
   (a) $P(x) = (x-2)(x+3)^2$ **(b)** $Z(x) = (x+2)^2(x-3)^2$

4. (Continuation) Explain how to determine the far-left and far-right behavior of a polynomial. Give an example.

5. A projectile follows the path $h(t) = -16t^2 + 17t + 10$. At what t value is h a maximum? This h value represents an extreme value of the function. Do you think this is a relative, an absolute extreme point or both? Explain.

6. The polynomial $P(x) = 2x^3 - x^2 - 5x - 1$ has two ***extreme points***. Using your calculator, find the coordinates of the extreme points and state whether they are relative or absolute.

7. (Continuation) On the domain values [-2, 2], find the maximum and minimum values. How do the extreme points relate to maximum and minimum values of a graph? Is either value an absolute maximum or minimum value on the graph?

8. What is the domain of $\dfrac{x^2 - 3x}{x-3}$?

9. (Continuation) Graph $f(x) = \dfrac{x^2 - 3x}{x-3}$ on your calculator. Is the graph what you expected? What is the value of $f(3)$? Can you confirm this with your calculator? Explain. It is typical to say there is a "hole" in the graph at x = 3. Explain this terminology.

1. Does the graph of $F(x) = \dfrac{x^2 - 2x - 24}{x^2 - 16}$ have a vertical asymptote at $x = -4$? Explain.

2. Find the extreme points on the polynomial function $P(x) = -x^4 + x^3 + 7x^2 - x - 6$. For each point, state whether it is a relative maximum or relative minimum. Is there an absolute maximum or absolute minimum?

3. What is the maximum number of extreme points a polynomial function of degree 4 can have? Degree *n*?

4. Explain the difference between a relative minimum and an absolute minimum.

5. Given the x-intercepts of a third degree polynomial are $(-1,0)$, $(-3,0)$ and $(2,0)$, write the polynomial as a product of its factors. Is your answer unique?

6. **Using your calculator**, graph the following functions. Use the graph to determine the equation of all horizontal and/or vertical asymptotes for each function. Find any *x*- and *y*-intercepts.

   (a) $p(x) = \dfrac{2x-2}{x+1}$ **(b)** $f(x) = \dfrac{2}{x^2 + 2x - 3}$

   (c) $b(x) = \dfrac{1}{(x+2)^2}$ **(d)** $h(x) = \dfrac{x}{x^2 - 4}$

7. Consider the rational function $R(x) = \dfrac{x^2 + 4x - 5}{2x^2 + 11x + 5}$.

   (a) State the domain of the rational function.
   (b) Factor completely.
   (c) Is there a hole in the graph? If so, where.
   (d) Write the equation(s) of the vertical asymptotes. If none, write "none."
   (e) Write the equation of the horizontal asymptote. If none, write "none."
   (f) Find the *x*- and y-intercepts. If none, write "none."
   (g) Graph the function accurately, using additional points as necessary.

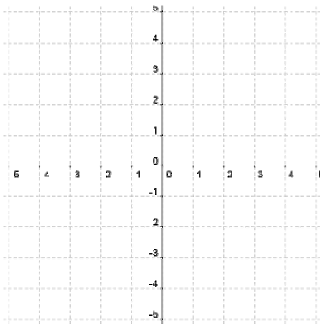
8. Sketch the graph of the function. Label any holes, all intercepts and asymptotes. Please draw your asymptotes with dashed lines. **Confirm your graph with your calculator.**

   (a) $f(x) = \dfrac{x+1}{x^2 + x}$ **(b)** $f(x) = \dfrac{x^3 + 3x^2}{x(x+3)(x-1)}$

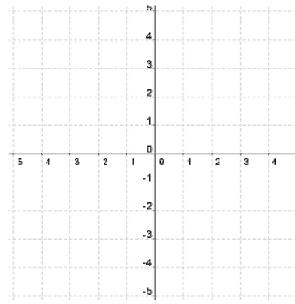
TI-84 Lab: "Pass Through" and "Bounce" Points

Directions: Using your calculator, make a quick sketch of the graph of each of the following polynomial functions. Mark the x- and y-intercepts with their values. It is not important to have the heights drawn to scale. Make certain to draw smooth continuous curves. This is easiest with a window of [xmin, xmax] = [-5, 5].

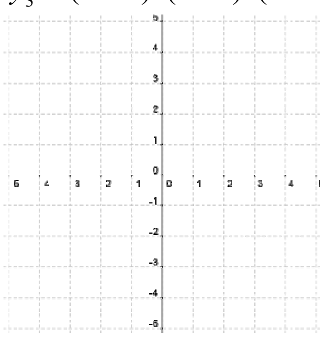
$y_1 = (x-2)(x+1)(x+3)$



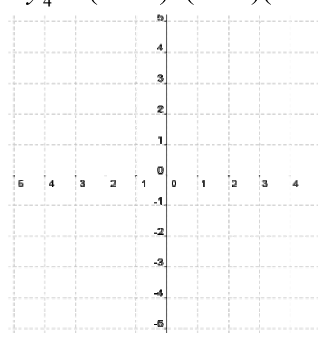
$y_2 = (x-2)^2(x+1)(x+3)$



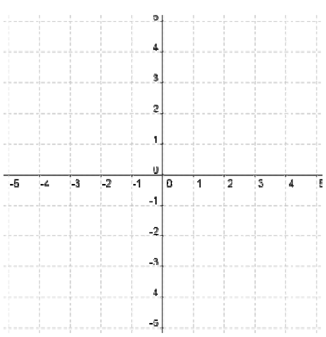
$y_3 = (x-2)^2(x+1)^2(x+3)$



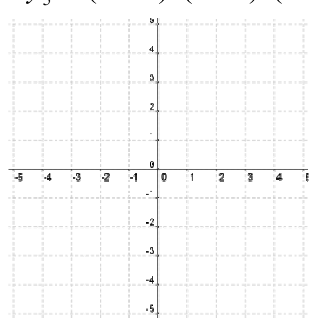
$y_4 = (x-2)^2(x+1)(x+3)^3$



$y_5 = (x-2)^2(x+1)(x+3)^4$



$y_5 = (x-2)^5(x+1)^3(x+3)$



I. When does the graph "pass-through" the x-intercept?

II. When does the graph "bounce" off the x-intercept?

III. Explain the relationship between the exponents on each of the factors in the polynomial functions and the behavior of the graph at $x = 2,\ -1$ and -3.

IV. Is the degree of each polynomial consistent with its far-left and far-right behavior?

1. Give an example of a first degree polynomial equation that intersects the x-axis and one that does not intersect the x-axis. Include a sketch of each of your functions.

2. Give an example of a second degree polynomial equation that intersects the x-axis twice; one that intersects the x-axis exactly once; and one that does not intersect the x-axis. Include a sketch of each of your functions.

3. The ***multiplicity*** of the factor is the degree of a factor. Find the zeros of the polynomial function and state the multiplicity of each zero.
   (a) $B(x)=(x+3)(x-4)^2(x+1)^3$.
   (b) $M(x)=x^2(x-4)^3$
   (c) $T(x)=(2x-5)(x+3)^3$
   (d) $P(x)=(x^2-4)(x+3)^2$

4. (Continuation) Explain how the multiplicity of the factor determines if a graph will "pass-though" or "bounce" at the x-intercept. If you use your calculator, just focus on what happens close to the x-axis – it is not important to see the full function.

5. Give an example of a third degree equation that intersects the x-axis three times; one that intersects the x-axis twice; one that intersects the x-axis exactly once; and one that does not intersects the x-axis. Include a sketch of each of your functions.

6. Give an example of a fourth degree equation that intersects the x-axis four times; one that intersects the x-axis three times; one that intersects the x-axis twice; one that intersects the x-axis exactly once; and one that does not intersect the x-axis. Include a sketch of each of your functions.

7. Find all of the asymptotes of the rational function $F\left(x\right)=\dfrac{x^2-4}{x^2+5x+4}$. Do not use your calculator.

8. (Continuation) Consider the rational function from the previous problem. Change the numerator so that a vertical asymptote becomes a hole.

9. Write a fourth degree polynomial whose zeros are -1, 1 with multiplicity two, and 3 such that $P(0)=-6$.

10. Write a fourth degree polynomial whose zeros are 3 with multiplicity two, and -2 with multiplicity two such that $P(1)=6$.

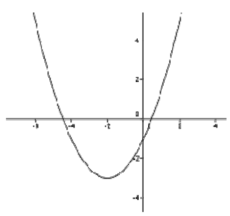
11. What information would be helpful before graphing a polynomial function?

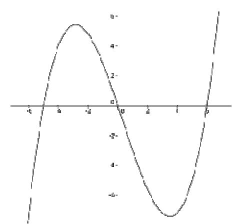
1. Does the graph of every rational function have at least one vertical asymptote? If so explain why. If not, give an example of a rational function without a vertical asymptote.

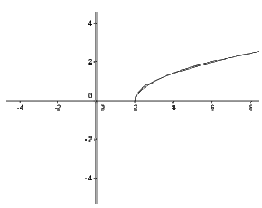
2. Does the graph of every rational function have at least one horizontal asymptote? If so explain why. If not, give an example of a rational function without a horizontal asymptote.

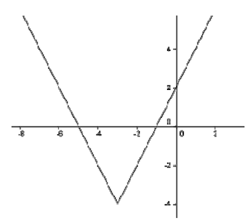
3. Determine the equation of the horizontal asymptote from the graph of
$F(x)=\dfrac{5x^2+2x-1}{3x^2+7}$

4. A *one-to-one* function is a function that has exactly one x-value associated with each y-value. In other words, there is only one x-value that gives each y-value and exactly one y-value that gives each x-value. State which of the following is a one-to-one function. Explain.

(a)


(b)


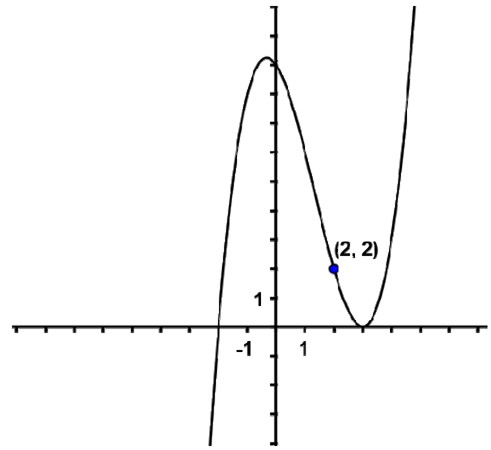
(c)


(d)


5. Without your calculator, sketch a graph the following polynomial functions. It is not important to have the heights drawn to scale.
(a) $U(x)=-x^3-2x^2+3x$ **(b)** $S(x)=x^4-6x^3+8x^2$ **(c)** $P(x)=x^3+4x^2-4x-16$

6. What happens to the graph of $F(x)=\dfrac{x^2+x-6}{2x^2-2x+8}$ at $x=2$? Explain.

7. Without using your calculator, graph the polynomial functions. It is not important to have the heights drawn to scale.
(a) $Q(x)=(x-5)^2(x+1)$
(b) $P(x)=(x^2-4)(x+3)^2$
(c) $h(x)=(3-x)(4+x)^3$



1. Given the polynomial to the right such that the point $(0,9)$ is on the graph, write an equation that represents this curve. Is your answer unique?

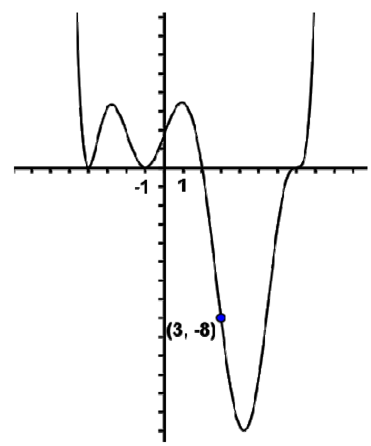
2. Explain the statement "*a one-to-one function passes the vertical and horizontal line tests.*"

3. Give an example of a one-to-one function. Of a function that is not one-to-one.

4. Without using your calculator, graph each equation. Make certain to find the equations of the vertical and horizontal asymptotes (if any), the *x*- and *y*- intercepts (if any), and indicate any holes. You may need to use additional points.

   (a) $G(x)=\dfrac{2x}{x-3}$ **(b)** $F(x)=\dfrac{x-5}{x^2-25}$

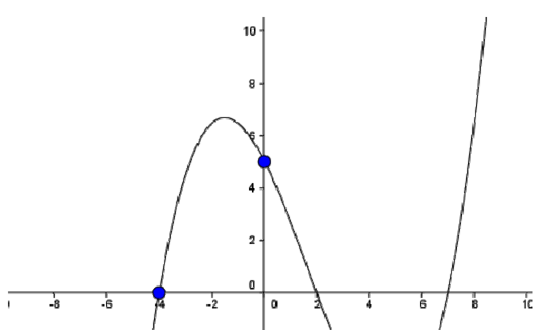
   (c) $L(x)=\dfrac{6x-30}{x-5}$ **(d)** $h(x)=\dfrac{12x+1}{4x+1}$



5. Given the polynomial at right such that $P(3)=-8$ is on the graph, write an equation that represents this curve.

6. A rectangular piece of cardboard measures 10 inches by 14 inches. An open box is formed by cutting squares that measure x inches by x inches from each of the corners of the cardboard box and folding up the sides. Express the volume of the box as a function of *x*. What value of *x* will maximize the volume?

7. The points (-4, 0), (0, 5), and (5, -5) are on a function, pictured below. Reflect this function over the line $y=x$, by switching the coordinates of each point. For instance, (-4, 0) reflected becomes (0, -4). What are the coordinates of the reflected points? Is the reflection of the curve also a function?



1. Is $(x-2)$ a factor of $4x^4-2x^2-4x-5$?

2. Explain how the factors of a polynomial are related to its x-intercepts.

3. Write an equation of a rational function such that it has a vertical asymptote at $x=2$, a horizontal asymptote at $y=3$, and intersects the x-axis at $(4,0)$.

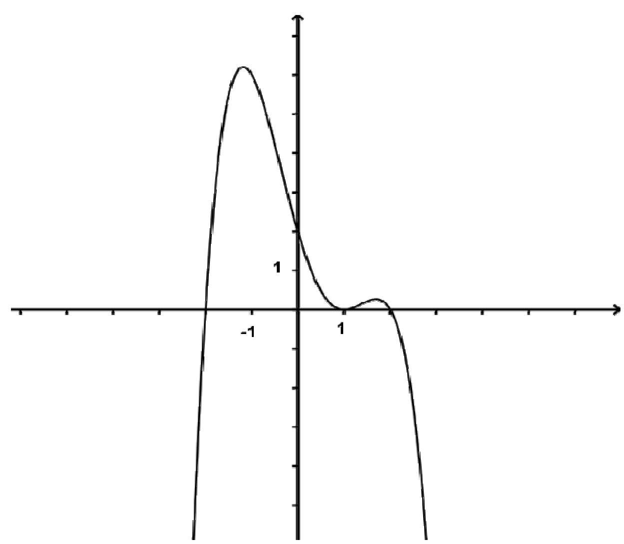
4. If two polynomials has the same zeros, do the graphs of the polynomial functions look the same? Include a sketch supporting your conclusion.

5. Does the graph of every polynomial function have at least one x-intercept? At least one y-intercept?

6. Is $f(x)=x^3$ a one-to-one function? Reflect the graph $f(x)=x^3$ over the line $y=x$. Is the reflection a function?

7. Graph the function $f(x)=2x+1$.
   (a) Evaluate $f(2)$ and $f(-2)$.
   (b) Reflect $f(x)$ over the line $y=x$. What is the equation of the reflected line $g(x)$?
   (c) Evaluate $g(5)$ and $g(-3)$. How do these compare to $f(2)$ and $f(-2)$? Explain.
   (d) Find the intersection of $f(x)$ and $g(x)$. Did you expect this result?

8. Write an equation of the polynomial with the lowest degree that represents the graph at right.



9. Write an equation of a rational function such that it has a vertical asymptote at $x=-4$, a horizontal asymptote at $y=1$, and intersects the y-axis at $(0,2)$.

10. Using your calculator to your advantage, express $y=3x^3-6x^2-15x+18$ in the form $y=a(x-r_1)(x-r_2)(x-r_3)...(x-r_n)$

11. Write a third degree polynomial whose zeros are -1, 1, and 3 such that $P(0)=6$.

1. Determine the x-intercepts of $P(x)=(x+3)^5(5x+2)^4$ and state whether the graph of P crosses the x-axis or intersects but does not crosses the x-axis at each x-intercept.

2. Graph, by hand, $f(x)=x^2$ and $f(x)=\sqrt{x}$ in the domain $x \geq 0$. Add to your graph the line $y=x$. What do you notice?

3. (Continuation) List four points on $f(x)=x^2$, $x \geq 0$ and four points on $f(x)=\sqrt{x}$. How are the x- and y-coordinates of the corresponding points related?

4. One of the properties of inverse functions is that they are reflections of one another across a specific line. What is the equation of this line? Knowing this, sketch the inverses of the following functions, and state whether the reflection is a function.
   (a) $f(x)=2x-3$ **(b)** $f(x)=|x|$ **(c)** $f(x)=4$

5. (Continuation) Which of the functions above are one-to-one?

6. A function, $f(x)$, maps every letter of the alphabet to a number from 0 to 9 in order to enable texting on a cell phone. Using your phone as a guide, what is $f(A)$? What about $f(L)$? Is this function one-to-one? Is the inverse of this function a function?

7. A food scientist needs to combine 100 mL of 15% acetic acid with water to get a solution that is 5% acetic acid. How much water should she add?

8. What is the equation of the horizontal line $y=2$ when reflected over the line $y=x$? Is its reflection a function?

9. Draw the graph of a one-to-one function that contains the points $(-3,-2)$, $(0,0)$ and $(2,6)$. Now draw the graph of its inverse.

10. (Continuation) Compare your results with your peers. What are the similarities? Differences?

11. What can be said about the inverses of one-to-one functions? What can be said about the inverse of a function that is not one-to-one?

12. Solve for a: $2a^2+9a+10>1$. Use the Critical Values Method, and express your answer in interval notation.

1. Identify at least four points on the function $f(x) = x^2 - 1$. Using your knowledge of quadratic equations and reflections, sketch both $f(x)$ and its inverse on the same set of axes. How might you change the domain of $f(x)$ from all real numbers by putting a restriction on it so that its inverse is a function?

2. There is a river near your house that has a swift current of 8 miles per hour. You rent a kayak and paddle upstream for 2 miles and then 2 miles back to your starting spot. The entire trip took you 6 hours. What is your paddling speed? What do you need to assume when answering this question?

3. Explain why it is necessary to include the restriction $x \geq 0$ when stating that $f(x) = x^2$, $x \geq 0$ and $f(x) = \sqrt{x}$ are inverses of one another. Note that, without this restriction on $f(x)$, most textbooks will say that $f(x)$ does not have an inverse.

4. Identify four points on the function $f(x) = \dfrac{3x+1}{x-2}$ and four corresponding points on its inverse. Is $f(x)$ one-to-one? Is its inverse a function? How do the asymptotes relate on the graph and its inverse?

5. (Continuation) Summarize what you have discovered when comparing points on a function and its inverse. If you know that points $(1,3)$ and $(10,0)$ are on $f(x)$, state the two corresponding points on $f^{-1}(x)$.

6. (Continuation) The output (y-coordinate) of a function becomes the input (x-coordinate) of its inverse function. Because of this, we can write an equation for the inverse of a function, by substituting *x* for *y* and *y* for *x* then solving for *y*. Find the equation for the inverse of $f(x) = 4x - 1$ and call your result $f^{-1}(x)$.

7. Write an equation for the circle, in Standard Form, $r^2 = (x-h)^2 + (y-k)^2$, for the circle that has a diameter with endpoints at (-4, -5) and (2, 3).

1. Recall that rules of exponents state that $x^{-1} = \frac{1}{x}$, however, this is easily misleading while working with inverse functions, since $f^{-1}(x) \neq \frac{1}{f(x)}$. If $f(x) = -2x + 4$, what is the value of $f^{-1}(3)$? $[f(3)]^{-1}$?

2. Under the definition that many textbooks have about inverses: Name a function that has an inverse and a function that does not have an inverse.

3. Given $f(-2) = 5$ and $f(5) = 7$, find $f^{-1}(5)$.

4. Solve for a:

   (a) $4a + ax = 5$ (b) $\frac{a+1}{a+3} = 5$

5. Find the equation of the inverse of $f(x) = \frac{3x+1}{x+2}$. What does excluding -2 in the domain of f(x) imply for the range of $f^{-1}(x)$? Sketch both $f(x)$ and $f^{-1}(x)$ on the same set of axes to verify your answer.

6. (Continuation) How are the domain and range of $f(x)$ and $f^{-1}(x)$ related to each other? Considering how you found the equation for $f^{-1}(x)$, is this surprising?

7. Simplify: $\frac{x}{x-2} - \frac{x}{x+2}$

8. Is every odd function one-to-one? Explain.

9. Solve for x: $3 - \sqrt{x+3} = 1$

10. Find the equation of the inverse of $f(x) = \sqrt{x+2}$.

    (a) Is 5 in the domain of $f(x)$? How about -5? Why or why not?
    (b) Is 5 in the range of $f^{-1}(x)$? How about -5? Why or why not?
    (c) State the domain and range of both $f(x)$ and $f^{-1}(x)$. Explain how this supports what you found in parts a and b.

1. Given an equilateral triangle of side length 1, find the length of its altitude.

2. What is the perimeter of an equilateral triangle with an altitude of 10?

3. How are the domain and range of a one-to-one function related to the domain and range of its inverse function?

4. Choose a domain for which $f(x)=x^2-4$ is one-to-one. What is the range of this restricted function? Find the inverse of this restricted function, and state its domain and range.

5. Starting at the same spot on a circular track that is 80 meters in diameter, Hayley and Kendall run in opposite directions, at 300 meters per minute and 240 meters per minute, respectively. They run for 50 minutes. What distance separates Hayley and Kendall when they finish? There is more than one way to interpret the word *distance* in this question.

6. Find the side lengths of a square whose diagonal is length 1.

7. Find the diagonal of a square whose area is 10.

8. $f(x)=x^2,\ x\geq 0$ and $g(x)=\sqrt{x}$. Find $f(g(x))$ and $g(f(x))$. Explain your results in the context of inverse functions.

9. (Continuation) Finish the following statement: If $f(x)$ and $g(x)$ are inverses, then $f(g(x))=g(f(x))=$_________. Why is this statement helpful for checking if two functions are inverses of one another?

10. Are $f(x)=3x+1$ and $g(x)=\frac{1}{3}x-1$ inverses? Answer this question using at least two different methods.

11. Commercial jets usually land using a 3 degree angle of descent. At what ground distance from the destination airport should the pilot begin the descent, if the cruising altitude of the jet is 37000 feet?

12. State the side ratios of the special right triangles, 30-60-90 and 45-45-90.

13. Find the inverse of $f(x)=3x-7$. State any restrictions on the domain or range of $f^{-1}(x)$
.

1. Given the lineup for Basic Time Warner Cable service, each channel on a television maps to a network. Is this mapping function one-to-one? Is the inverse a function? Explain.

BASIC SERVICE

2 / 570	C-SPAN
3 / 585	Time Warner Cable Sports
4	WNYA
5 / 405	TBS
6	WRGB/CBS
8	WXXA/FOX
9 / 509	YNN Your News Now
10	WTEN/ABC
11	WMHT/PBS
12 / 650	OTB
13	WNYT/NBC
14	Movies On Demand (w/ converter)
15	WCWN
16	Education Access
17	Government Access
18	Public Access
19	TV Guide Channel
20	ION Television
21 / 1115	HSN
22 / 1109	QVC

2. Scientists can estimate the depth of craters on the moon by studying the lengths of their shadows in the craters. Shadows' lengths can be estimated by measuring them on photographs. An astronaut was standing at the edge of a crater when the sun had an angle of elevation of 48 degrees. The shadow was estimated to be 400 meters, how deep was the crater?

3. State the exact value of the sine, cosine, and tangent of
(a) 30 degrees **(b)** 60 degrees **(c)** 45 degrees

4. Find the inverse of $f(x)=(x-2)^2$. State any restrictions on the domain or range of both $f(x)$ and $f^{-1}(x)$.

5. An equilateral triangle is inscribed in a circle of radius 1 centered at the origin (the *unit circle)*. If one of the vertices is $(1,0)$, what are the coordinates of the other two? The three points divide the circle into three arcs; what are the angular sizes of these arcs?

6. Dana takes a sheet of paper, cuts a 120-degree circular sector from it, then rolls it up and tapes the straight edges together to form a cone. Given that the sector radius is 12 cm, find the circumference of the base. What is the height and volume of this paper cone?

7. Write the components of a vector that is 8 units long and that makes a 30 degree angle with the x-axis.

8. In general, what condition must a function meet for its inverse to be a function?

9. Find the inverse of $f(x)=\dfrac{x-3}{x+5}$. State any restrictions on the domain or range of $f^{-1}(x)$.

10. Place the tail of the vector $\left[5,5\sqrt{3}\right]$ at the origin. What is the angle formed with the x-axis? Hint: This is a special right triangle.

11. What are the steps in finding an inverse of a function?

12. Without your calculator find:
(a) $\sin 45^\circ \sin 30^\circ$ **(b)** $\cos 30^\circ + \cos 45^\circ$ **(c)** $\tan 45^\circ \left(\cos 60^\circ + \tan 30^\circ\right)$

1. Find the inverses of the following functions. State the domain and range for each function and its inverse.
   (a) $f(x)=\dfrac{4x-1}{3x+5}$ **(b)** $f(x)=\dfrac{3}{4}x-\dfrac{2}{3}$
   (c) $f(x)=2x^2-3$ **(d)** $f(x)=4+\sqrt{x+5}$

2. Are $f(x)=3x^2+1$ and $g(x)=\dfrac{1}{3}\sqrt{x-1}$ inverses? Explain.

3. The angle of elevation from a sailboat to the top of a 243 foot tall lighthouse on the shore measures 14 degrees. How far is the sailboat from the shore?

4. (Continuation) A lighthouse guard spots a sailboat out at sea. If the guard is looking down at an angle of 10 degrees, how far out is the boat?

5. Determine whether $f(x)=5x$ and $g(x)=\dfrac{1}{5x}$ are inverses of one another. Explain your method.

6. Its center at $O=(0,0)$, the unit circle $x^2+y^2=1$ goes through $P=(1,0)$. The line $y=0.6$ intersects the circle at A and B, with A in the first quadrant. The angles *POA* and *POB* are said to be in *standard position*, because their *initial ray OP* points in the positive x-direction. (Their *terminal rays* are *OA* and *OB*.) Find the sizes of these angles. How are they related?

7. (Continuation) If we restrict ourselves to a single revolution, there are actually *two* angles in standard position that could be named *POB*. The one determined by minor arc *PB* is said to be *positive*, because it opens in the counterclockwise direction. Find its degree measure. The one determined by major arc *PB* is said to be *negative*, because it opens in the clockwise direction. Find its degree measure.

8. Use a graph of $f(x)=-x+3$ to explain why f is its own inverse. Explain. Then confirm this conclusion algebraically. Find at least one other function that is its own inverse. What must be true about the graph for this to be true?

9. A function has an inverse function. If the graph of the function lies in the first quadrant, in which quadrant does the graph of its inverse function lie?

10. Place the tail of the vector $[-7,-7]$ at the origin.
    (a) What is the angle formed with the x-axis?
    (b) What is the angle formed with the *positive* x-axis?

1. Your clock is broken and the hour hand is stuck on the 3. If the minute hand is on the 10, what is angle formed by the hands?

2. (Continuation) Assuming that your minute hand still works, how many degrees does it rotate through moving from 3 to 10? In what direction does it travel? How does your answer relate to the previous problem?

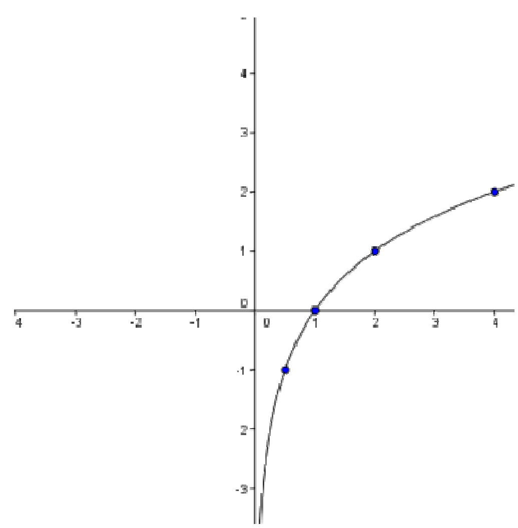
3. The point B is $(6,0)$ and A is $(0,0)$. Find points P in the first and fourth quadrants that make angle PAB 60 degrees.

4. If the graph of a one-to-one function lies in the third quadrant, in which quadrant does the graph of its inverse function lie?

5. Is $g(x)=5x-35$ the inverse function of $f(x)=\frac{1}{5}x+7$? Justify algebraically.

6. Rob notices that the Sun can barely be covered by closing one eye and holding an aspirin tablet, whose diameter is 7 mm, at arm's length, which means 80 cm from Rob's eye. Find the *apparent size* of the Sun, which is the size of the angle *subtended* by the Sun. An angle subtended by an arc is one whose rays pass through the endpoints of an arc.

7. The graph at right is a function f(x). On the same set of axes graph the corresponding part of its inverse.



8. A *reference angle* is the angle formed by the terminal side of the angle and the x-axis. Find the reference angles created by the following coordinates:
(a) $(-3,2)$ **(b)** $(4,-3)$ **(c)** $(-2,-2)$

9. Barbara and Sue are contra dancing. They both start at $(5,0)$ and travel around in a circle, centered at the origin, going opposite directions and end at $(-4,-3)$. Barbara promenades clockwise and Sue promenades counterclockwise. Write an angle that describes each dancer's motion.

10. Gerry, Sandy, and Pat all start out at $(0,0)$ to explore a beach. Gerry goes 20 meters east and 10 meters north, Sandy goes 15 meters west and 45 meters south, and Pat goes 30 meters east and 23 meters south. Describe each of their positions relative to their starting position using a positive angle, a negative angle, and a reference angle.

11. Find the inverse function of $f(x)=ax+b$. Make certain to state any restrictions on the domain or range.

1. A merry-go-round in a playground has a radius of $\sqrt{13}$. You start at $\left(\sqrt{13},0\right)$ and your friend is at $\left(-3,2\right)$. Your friend wants to slap your hand each time as you go around. How many degrees have you rotated at the moment to give your third high-five?

2. Explain the similarities and differences between positive and negative angles.

3. Find the inverse function of $f(x)=\dfrac{4x+3}{x};x\neq 0$.

4. Take vector $\left[4,-4\sqrt{3}\right]$:

   (a) This vector can be the radius of a circle centered at the origin. What is the radius length?
   (b) Scale this vector so that the radius is 1. Recall that a vector of length 1 is called a *unit vector*. What do you think you can call this circle?
   (c) Name three other vectors whose tails are at the origin and whose head is on the unit circle such that their reference angle is 60 degrees.

5. Name the four unit vectors whose reference angles are 45 degrees. What do you notice about their components?

6. Choose a positive number θ (*Greek* "theta") less than 90.0 and ask your calculator for sin θ and cos θ. Square these numbers and add them. Could you have predicted the sum?

7. The point $\left(\dfrac{1}{2},\dfrac{\sqrt{3}}{2}\right)$ lies on the unit circle.

   (a) Name its angle and reference angle.
   (b) Name another angle greater than 360 degrees that would also describe its position.
   (c) Name the coordinates of the point on the unit circle in the second quadrant that have the same reference angle.

8. (Continuation) What is the cosine of 60 degrees? What is the sine of 60 degrees? How do the cosine and sine of 60 degrees relate to the coordinates of the point in the previous problem?

9. The function $f(x)=x^4$ is not one-to-one. Find a suitable restriction on the domain of the function and then find its inverse function.

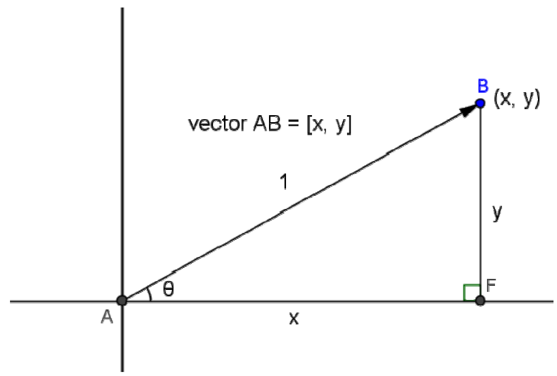
10. Evaluate 4^0, $\left(\dfrac{1}{3}\right)^0$, and $(-5)^0$. What is true about any number to the zero power? Why is this true?

1. What are the components of the vector that makes a 30 degree angle with the positive x-axis? How does this relate to the sine and cosine of 30 degrees?

2. Determine whether the following points are on the circle $x^2 + y^2 = 1$

   (a) $(0,1)$ **(b)** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ **(c)** $\left(\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right)$

3. A circular saw blade with a 5 cm radius is mounted so that one half of it shows above the table and is rotating at one degree per second. One tooth of the saw has been painted red. This tooth starts at $(5,0)$, what is the height of the saw tooth after 30 seconds? After 90 seconds? After 420 seconds?



4. Given the vector $AB = [x, y]$ with length 1 that makes angle θ with the x-axis, how do the coordinates of point B relate to the cosine and sine of angle θ?

5. Although it may seem like a strange request, ask your calculator for sine and cosine values for a 120 degree angle. Try to make sense of the answers.

6. Two observers who are 5 km apart simultaneously sight a small airplane flying between them. One observer measures a 51 degree inclination angle, while the other measures a 40.5 degree inclination angle. At what altitude is the airplane flying? Include a diagram with your solution.

7. What are the similarities and differences between $f(x) = x^2$ and $f(x) = 2^x$.

8. Is $y = (-2)^x$ a function? Explain.

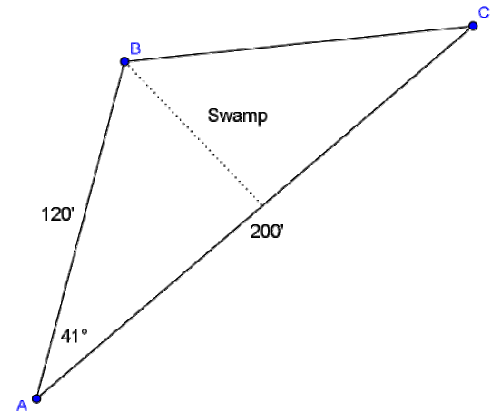
9. We have seen that when a vector of length 1 is in the first quadrant the cosine of θ is the x coordinate and the sine of θ is the y coordinate. What happens to the coordinates as the vector is placed in each of the quadrants?

10. Given that we have already learned that the tangent of θ is the slope of the line that forms the angle of θ with the x-axis, write an expression for tangent of θ in terms of $\cos\theta$ and $\sin\theta$ and in terms of x and y.

11. Consider the unit circle, describe a process to find the points on the unit circle that are half a unit away from the: **(a)** y-axis **(b)** x-axis

1. An exponential function with base b is defined by $f(x) = b^x$. What restrictions should be placed on b? Explain. Are there any restrictions on x?

2. A triangular plot of land has the SAS description indicated in the figure shown below. Although a swamp in the middle of the plot makes it awkward to measure the altitude that is dotted in the diagram, it can at least be calculated. Show how. Then use your answer to find the area of the triangle, to the nearest square foot.



3. A conical cup has a 10-cm diameter and is 12 cm deep.
(a) How much can this cup hold?
(b) Water in the cup is 6 cm deep. What percentage of the cup is filled?

4. (Continuation) Dana takes a paper cone of the given dimensions, cuts it along a straight line from the rim to the vertex, then flattens the paper out on a table. Find the radius, the arc length, and the central angle of the resulting circular sector.

5. Given $f(x) = 5^x$, create a table of values for $f(2), f(1), f(0), f(-1), f(-2)$. Assume this set of points is representative of graphs for which b > 1, describe the graph of $f(x) = b^x,\ b > 1$.

6. (Continuation) Given $f(x) = \left(\frac{1}{2}\right)^x$, evaluate for $x = -4, -2, 0, 2$ and 4. Assume this set of points is representative of graphs for which 0< b <1, describe the graph of $f(x) = b^x$, 0<b<1

7. Graph $f(x) = 2^x$. What is the domain and range of the function? State the x- and y-intercepts. Are there any asymptotes?

8. Graph $f(x) = \left(\frac{1}{5}\right)^x$. What is the domain and range of the function? State the x- and y-intercepts. Are there any asymptotes?

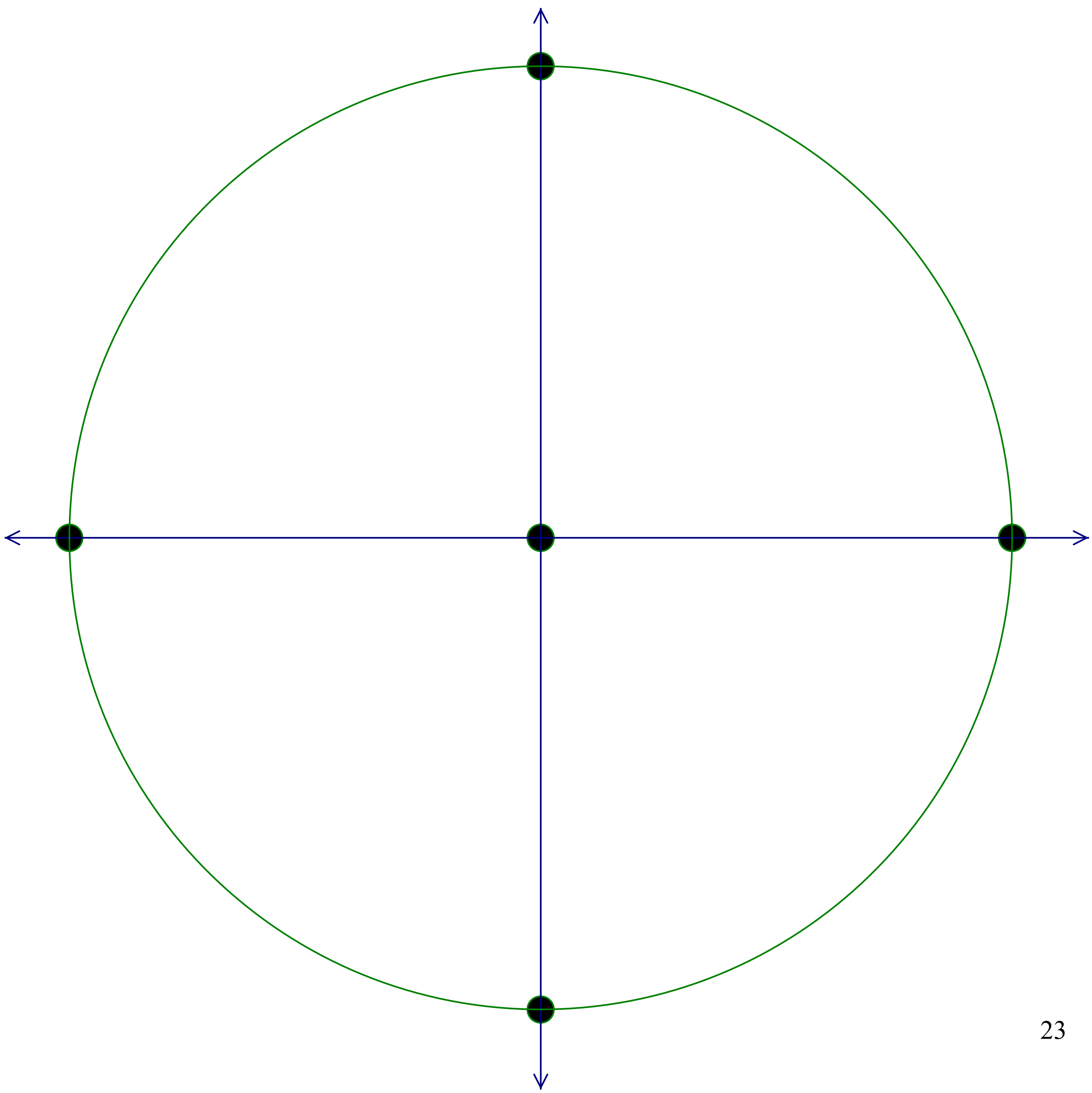
Paper Folding Lab

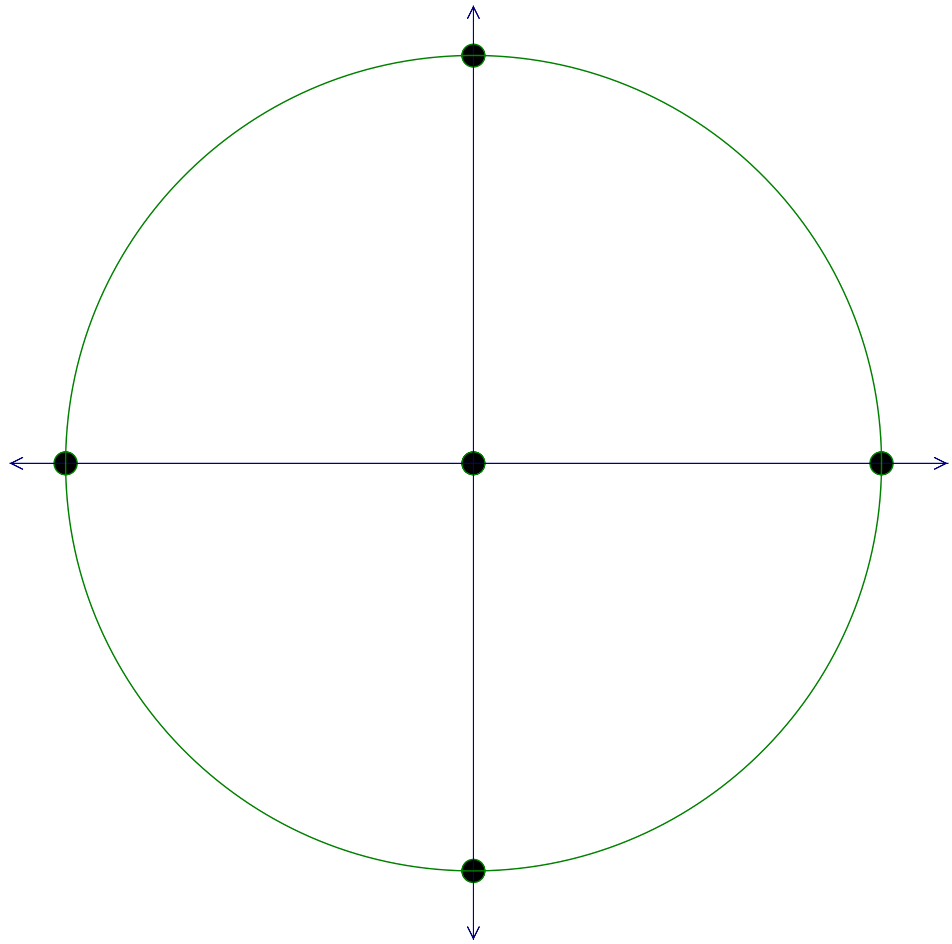
I. Fold your circle to create 45 degree angles – use the large dots to guide your folds. Trace your folds (radii) using a blue pen.

II. Fold your circle again, this time to find the four 30 degree reference angles. You may want to consider the side ratios of a 30-60-90 triangle to help you make the folds. Again, the large dots will be useful. Draw in the four radii this time using a green pen.

III. Fold your circle a third time – to find the 60 degree reference angles. Draw in the four radii using a red pen.

IV. Label the angles and coordinates of each point on the unit circle using the same color pen as the radii, or use a pencil and circle with the corresponding color.





1. For the special right triangles on the unit circle:
   a. All short legs have length _______
   b. All 45-45-90 degree triangles have legs of length _______
   c. All long legs have length _______
   d. The hypotenuse always has length _______

2. The reference angle associated with — Degrees
   a. a long horizontal leg is _______
   b. a short vertical leg is _______
   c. an isosceles right triangle is always _______
   d. a short horizontal leg is _______
   e. a long vertical leg is _______

List the (x, y) and angle coordinate of the point on the Unit Circle that corresponds to:

		(x, y)	Angle
a.	a long horizontal leg and short vertical leg in the first quadrant	_______	_______
b.	a long horizontal leg and short vertical leg in the second quadrant	_______	_______
c.	a long horizontal leg and short vertical leg in the third quadrant	_______	_______
d.	a long horizontal leg and short vertical leg in the fourth quadrant	_______	_______
e.	equal length legs in the first quadrant	_______	_______
f.	equal length legs in the second quadrant	_______	_______
g.	equal length legs in the third quadrant	_______	_______
h.	equal length legs in the fourth quadrant	_______	_______
i.	a short horizontal leg and long vertical leg in the second quadrant	_______	_______
j.	a short horizontal leg and long vertical leg in the third quadrant	_______	_______
k.	a short horizontal leg and long vertical leg in the fourth quadrant	_______	_______

1. Evaluate:
   (a) $\sin 150^\circ$ **(b)** $\tan 300^\circ$

2. Find at least two values for θ that fit the equation $\sin\theta = \frac{\sqrt{3}}{2}$. How many such values are there?

3. The *Quadrantal Angles* on the unit circle are the angles that correspond to the points that are the intersection points of the unit circle and the coordinate axes. State and justify the sine, cosine and tangent of:
   (a) 0 degrees **(b)** 90 degrees, **(c)** 180 degrees, **(d)** 270 degrees, **(e)** 360 degrees

4. A wheel of radius one foot is placed so that its center is at the origin, and a paint spot on the rim is at $(1,0)$. The wheel is spun θ degrees in a counterclockwise direction. Now what are the coordinates of the paint spot, in terms of θ?

5. Graph each of the following pairs of functions on a separate system of coordinate axes, and justify what you see.
   (a) $f(x) = 3^x$ and $g(x) = \left(\frac{1}{3}\right)^{-x}$ **(b)** $f(x) = \left(\frac{1}{3}\right)^{x}$ and $g(x) = 3^{-x}$

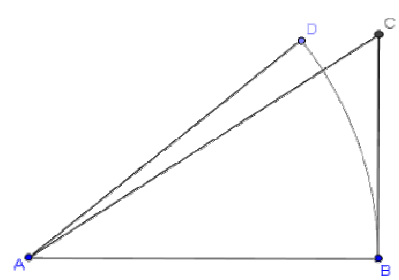
6. What are the properties of the graph $f(x) = b^x$ when $b > 1$? When $0 < b < 1$? Include examples. State the domain and range of $f(x) = b^x$.

7. MacKenzie is on a Ferris wheel that is one decameter in radius. This Ferris wheel loads passengers at its lowest point, 1 m off the ground. What is MacKenzie's height off of the ground after the wheel has gone 120 degrees from where the ride began?

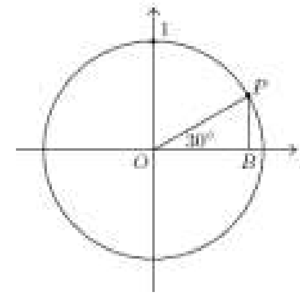
8. (Continuation) The ride continues and the Ferris wheel has rotated a total of five times from the beginning of the ride and MacKenzie is now at the top of the wheel and wonders how many degrees total the wheel has rotated. Explain how this could be calculated.

9. Choose an angle θ and calculate $(\cos\theta)^2 + (\sin\theta)^2$. Repeat with several other values of θ. Explain the coincident results. It is customary to write $\cos^2\theta + \sin^2\theta$ instead of $(\cos\theta)^2 + (\sin\theta)^2$.

10. In the figure at right, arc *BD* is centered at *A*, and it has the same length as tangent segment *BC*. Explain why sector *ABD* has the same area as triangle *ABC*.



1. Consider the unit circle, whose center is the origin O and whose radius is 1. Suppose that P is on the circle in the first quadrant, and that the angle formed by segment OP and the positive x-axis is 30 degrees. Mark B on the x-axis so that angle OBP is right. Find PB and OB, expressed in exact form. What are the x- and y-coordinates of P? Express the x- and y-coordinates of P using trigonometric functions of a 30 degree angle.



2. State the quadrant in which θ lies:
   (a) $\sin\theta > 0$ and $\cos\theta < 0$ **(b)** $\tan\theta > 0$ and $\cos\theta < 0$ **(c)** $\sin\theta < 0$ and $\cos\theta > 0$

3. Is the graph $g(x) = b^x$ one-to-one? What are the restrictions on b? Explain.

4. (Continuation) Find the inverse of $h(x) = 3^x$. Are you able to solve for y in terms of x?

5. (Continuation) Explain the phrase *y=the power of b that produces x*. How does this relate to your expression for the inverse function?

6. Find the exponent that gives you the answer when the base is raised to that power:

Base (b)	Answer Produced (x)	Exponent (y)
3	27	
10	1000	
2	1/8	
25	5	
1/2	1/16	

7. In most mathematics books, the notation $\sin^2 A$ is often used in place of the clearer $(\sin A)^2$, or $\cos^3 B$ in place of $(\cos B)^3$. Why do you think that writers of mathematics fell into this strange habit? It is unfortunate that this notation is *inconsistent* with notation commonly used for inverse functions. Explain.

8. Write without parentheses:
   (a) $(xy)^2$ **(b)** $(x+y)^2$ **(c)** $(a\sin B)^2$ **(d)** $(a+\sin B)^2$

9. Use the unit circle to find $\sin 240^\circ$ and $\cos 240^\circ$, without using a calculator. Then use your calculator to check your answers. Notice that your calculator expects you to put parentheses around the 240°, which is because sin and cos are functions. Except in cases where the parentheses are required for clarity, they are often left out.

1. John Napier (1550-1617) created the notation $\log_b x$ to replace the phrase *the power of b that produces x.* $\log_b x$ is read as "the logarithm (or log) base b of x." How would you write "4 is the power of 2 that produces 16?"

2. Rewrite the equation $\log_3 81 = 4$ as an exponential equation.

3. The radius of a circular sector is r. The central angle of the sector is θ. Write formulas for the arc length and the perimeter of the sector.

4. Given the following information,
   (a) $\sin\theta = -\frac{1}{2}$ and $\cos\theta > 0$. Find $\tan\theta$.
   (b) $\tan\theta = 1$ and $\sin\theta < 0$. Find $\cos\theta$.
   (c) $\cos\theta = -\frac{1}{2}$ and $\tan\theta < 0$. Find $\sin\theta$.
   (d) $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$. Find $\cos\theta$.

5. Write each equation in its exponential form.
   (a) $\log_4(\frac{1}{16}) = -2$ **(b)** $\log_b 16 = 2$ **(c)** $\log_2 8 = x$ **(d)** $\log_5 1 = 0$

6. Quinn is running around the circular track $x^2 + y^2 = 10000$, whose radius is 100 meters. Quinn starts at the point $(100, 0)$ and runs in the counterclockwise direction for 1605 degrees. What are Quinn's coordinates?

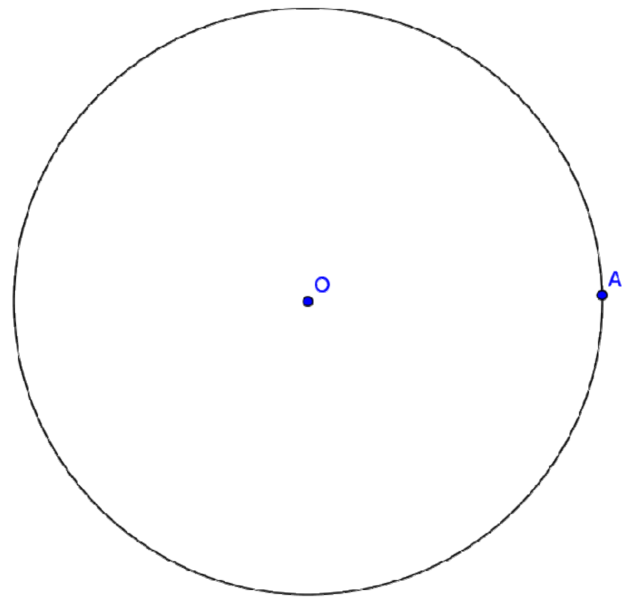
7. If sin A is known to be 0.96, then what can be said about cos A? What if it is also known that A is an obtuse angle?

8. Find the exact value (no decimals) of each expression.
   (a) $\sin 225^\circ$ **(b)** $\tan 330^\circ$ **(c)** $\cos 585^\circ$ **(d)** $\sin 510^\circ$ **(e)** $\cos 765^\circ$ **(f)** $\tan 1485^\circ$

9. Write $\log_3 x = y$ in its exponential form.

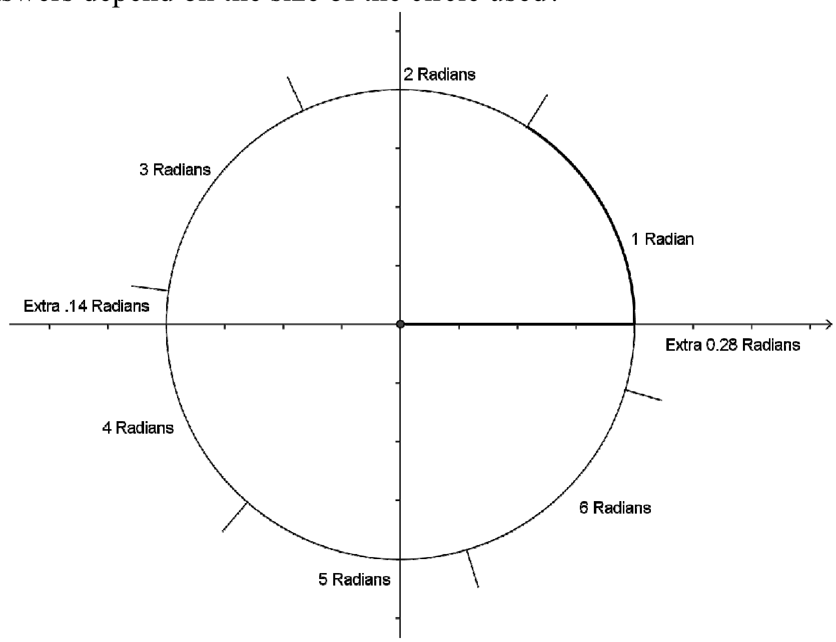
10. (Continuation) Find the inverse of: $3^y = x$; Do you recognize this function? What is the inverse function of $f(x) = \log_b x$?

11. Evaluate the following expressions. Please give an exact answer for each.
    (a) $\sin(-180^\circ)$ **(b)** $\tan(-210^\circ)$ **(c)** $\cos(-570^\circ)$



1. The circle shown at right is centered at *O*. Use a licorice strip (or other appropriate measuring device) to find a point *B* on this circle for which minor arc *AB* has the *same length* as radius *OA*. Draw radius *OB* and use a protractor to measure the size of angle *AOB*. Your answer should be close to 60 degrees. By considering triangle *AOB* and the relation between the arc *AB* and its chord, explain why angle *AOB* must in fact be smaller than 60 degrees.
   Angle *AOB* is an example of a *radian* – a central angle whose arc has the same length as the radius of the circle in which it is drawn.

2. How many 1-radian arcs does it take to fill a complete circle? First make an estimate using the licorice-strip approach, then look for a theoretically exact answer. Do any of your answers depend on the size of the circle used?



3. Consider the formula for the circumference of a circle, how does this prove that there are 2π radians in the unit circle? Is this true for any circle?

4. (Continuation) How many radians are there in half a circle, i.e. 180 degrees = _________ radians. 90 degrees = __________ radians. Justify your answers.

5. (Continuation) 60 degrees is what fraction of half a circle? What is 60 degrees equivalent to in radians? 30 degrees? 45 degrees?

1. The reciprocal of the sine is *cosecant* (abbreviated csc), useful for expressing answers to trigonometry problems without using a division sign. Use this function to express the hypotenuse of a right triangle that has a 12.8 inch side opposite a 25 degree angle.

2. Find equivalent ways to rewrite (without using a calculator) the following expressions:
   (a) $\frac{6a^8}{3a^4}$ **(b)** $\left(3p^3q^4\right)^2$ **(c)** $b^{1/2}b^{1/3}b^{1/6}$ **(d)** $\left(\frac{2x^3}{3y^2}\right)^2$ **(e)** $\left(d^{1/2}\right)^6$

3. Express $3^2 = 9$ in logarithmic form.

4. Change each equation to its logarithmic form.
   (a) $5^{-2} = \frac{1}{25}$ **(b)** $7^0 = 1$ **(c)** $2^4 = 16$ **(d)** $6^1 = 6$

5. Counting by fractions of π, add radian measures for each angle to your unit circle from the Paper Folding Lab.

6. A 6-inch arc is drawn using a 4-inch radius. Describe the angular size of the arc **(a)** using radians; **(b)** using degrees

7. A 2.5-radian arc is drawn using a 6-inch radius. How long is the arc?

Introduction to Radians using GSP

I. Open Radians.gsp

II. Press the *Animate Points* button. Observe what happens. Make a note of how the points *A* and *B* move differently and explain the motion of the path of each point.

III. Press *Reset*, then, press *Animate Points* to start the points moving again. Press the button again to stop the motion when the positions of A and B are the same. If your timing is off – try again.

IV. Because points A and B move at the same rate, the arc traced out by B should equal the length of the radius of the circle. Confirm this by measuring the arc length by selecting the arc and choosing MEASURE → ArcLength.

V. Press *Reset* again. This time, *Animate Points* and let B travel all the way around the circle exactly once. Stop it and notice what you see – how many “petals”?

VI. For each petal formed, how far does B travel in terms of the radius of the circle?

VII. Based on your answers to the last two questions, knowing the circumference of a circle, how many lengths of radius *r* (approximately) are traced out by point B as it moves once around the circumference? Explain how your answer makes sense based on the formula for the circumference of a circle.

1. Find all solutions t between 360 and 720 degrees, inclusive:
   (a) $\cos t = \sin t$ **(b)** $\tan t = -4.3315$ **(c)** $\sin t = -0.9397$

2. Given that $\tan\theta = 2.4$, with $180^\circ < \theta < 270^\circ$, find the values of $\sin\theta$ and $\cos\theta$. Are your answers *rational numbers*?

3. Find the following:
   (a) $\sin\left(\frac{\pi}{3}\right)$ **(b)** $\tan\left(\frac{\pi}{4}\right)$ **(c)** $\cos\left(\frac{\pi}{6}\right)$

4. Graph the function $f(x) = 2^x$ by hand, and label at least four points. Graph the inverse of $f(x)$, and label the corresponding points. How are the coordinates of the function and its inverse related? What is the significance of the line $y = x$?

5. (Continuation) Is the inverse a function? What is the domain and range?

6. Evaluate $\log_2 16$. Explain your procedure.

7. The logarithmic function $y = \log_b x$ is defined as the inverse function of $y = b^x$. What is the domain and range of $y = \log_b x$.

8. If $\csc A = \frac{5}{3}$, then what can be said about $\tan A$? What if I know that A is an obtuse angle?

9. Give an exact answer for each of the following:
   (a) $\cos\frac{\pi}{2}$ **(b)** $\sin\frac{11\pi}{6}$ **(c)** $\sin\frac{4\pi}{3}$

10. Find the size of central angle *AOB*, given that the length of arc *AB* is 16 cm and the length of radius *OA* is 12 cm. Please give your answer in both degrees and radians.

11. (Continuation) Find the size of angle *AOB*, given that the length of arc *AB* is 8 cm and the length of radius *OA* is 6 cm.

12. (Continuation) To find the size of the central angle *AOB*, it is enough to know the value of what ratio?

13. Give an exact answer for each of the following:
    (a) $\tan\left(-\frac{5\pi}{2}\right)$ **(b)** $\cos\left(-\frac{31\pi}{6}\right)$

1. Evaluate:

   (a) $\log_2 1$ **(b)** $\log_6 6$ **(c)** $\log_3\left(\frac{1}{9}\right)$ **(d)** $\log_{\frac{1}{2}} 16$ **(e)** $\log_5\left(5^4\right)$

2. With your calculator in degree mode, using the window $0 \le x \le 360$ and $-2 \le y \le 2$, examine the graph of $y = \sin x$. Explain why its first positive x-intercept has the value it does. Then graph the same equation with the calculator *in radian mode*. The display will show many more oscillations than before (but not as many as exist), making it difficult to see the first positive x-intercept. To make that intercept visible, reduce the window to $0 \le x \le 10$ and $-2 \le y \le 2$, then obtain an accurate reading of this x-value. Once you recognize it, explain how its value could have been predicted.

3. The radius of a circle is 9, and arc PQ has length 22. Find the length of chord PQ.

4. (Continuation) With your calculator in *radian* mode, evaluate $2 \cdot 9 \cdot \sin\left(11/9\right)$. Notice that this expression provides the correct answer to the chord-length question. Why is this so? In particular, what angle does the number 11/9 describe?

5. Give an exact answer for the following:
   (a) $\sin\frac{10\pi}{4}$ (b) $\sin\left(-\frac{3\pi}{2}\right)$

6. Explain the statement “a logarithm is an exponent.”

Construct a Radian

I. Select EDIT→Preferences and change your angle precision to hundredths and your distance precision to thousandths.

II. In GSP construct a circle of the radius of your choosing.

III. Place another point on the circle. Create a minor arc by selecting one endpoint, the circle, then the other endpoint. If you create a major arc, start with the other endpoint instead.

IV. Measure the radius of the circle and the arc length of the minor arc you created. Change your points until the radius and the arc length are equal. You may find that a bigger circle is easier to work with.

V. Measure the central angle of your arc.

VI. Save your sketch for comparison with your classmates’ sketches.

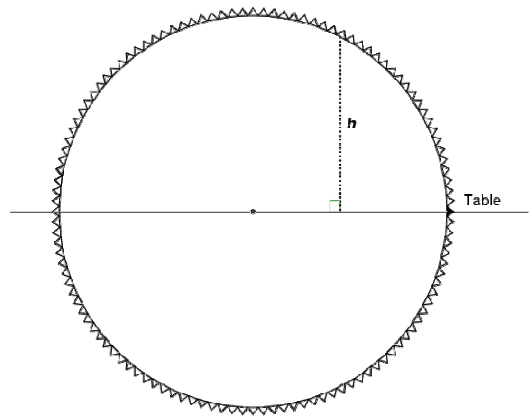
VII. After class discussion - summarize the conclusions – Explain the common result even though the initial circles had different radii.

1. Draw a graph of the exponential function $f(x) = 3^x$ and add to your graph $g(x) = \log_3 x$. Explain your procedure.

2. There is a LOG button on your calculator. Try LOG 3, LOG .001, LOG 100, LOG 10. Explain the meaning of this button. What assumption does your calculator make when evaluating the log?

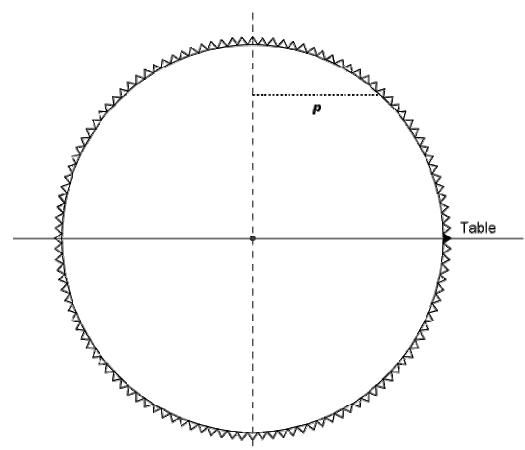
3. Draw the unit circle and a first-quadrant ray from the origin that makes an angle θ with the positive x-axis. Let B be the point on this ray whose x-coordinate is 1, and let $A = (1, 0)$. Segment AB is tangent to the circle. In terms of θ, find its length.

4. If two angles are supplementary, then their sines are equal. Explain why. What about the cosines of supplementary angles? If you are not sure, calculate some examples.



5. How many degrees are there in 1 radian?

6. A large circular saw blade with a 1-foot radius is mounted so that exactly half of it shows above the table. It is spinning slowly, at one degree per second. One tooth of the blade has been painted red. This tooth is initially 0 feet above the table, and rising. What is the height after 37 seconds? After 237 seconds? After t seconds? Draw a graph that shows how the height h of the red tooth is determined by the elapsed time t. It is customary to say that h is a function of t.



7. (Continuation) Now explore the position of the red saw tooth in reference to an imaginary vertical axis of symmetry of the circular blade. The red tooth is initially one foot to the right of the dotted line. How far to the right of this axis is the tooth after 37 seconds? After 237 seconds? After t seconds? Draw a graph that shows how the displacement p of the red tooth with respect to the vertical axis is a function of the elapsed time t.

8. (Continuation) The graphs of the height h and the horizontal displacement p of the red saw tooth are examples of *sine* and *cosine* curves, respectively. Graph the equations $y = \sin x$ and $y = \cos x$ on your calculator, and compare these graphs with the graphs that you drew in the preceding exercises. Use these graphs to answer the following questions: **(a)** For what values of t is the red tooth 0.8 feet above the table? 0.8 feet below the table? **(b)** When is the tooth 6 inches to the right of the vertical axis? When is it farthest left?

1. Find all solutions between 0 and 2π of $\cos t < \frac{\sqrt{3}}{2}$

2. Give an exact answer for the following:
   (a) $\cos\frac{11\pi}{4}$ **(b)** $\tan\frac{7\pi}{2}$ **(c)** $\sin\frac{9\pi}{2}$

3. A javelin lands with six feet of its length sticking out of the ground, making a 52 degree angle with the ground. The sun is directly overhead. The javelin's shadow on the ground is an example of a *projection*. Find its length, to the nearest inch.

4. Sketch the function $f(x) = \log_4 x$, manually. Be certain to include at least three points. State the domain and range.

5. The *secant* is the reciprocal of the cosine: $\sec t = \frac{1}{\cos t}$. Show how to convert the Pythagorean identity $\cos^2 t + \sin^2 t = 1$ into the form $(\sec t)^2 - (\tan t)^2 = 1$.

6. Asked to simplify the expression $\sin(180^\circ - \theta)$, Rory volunteered the following solution: $\sin(180^\circ - \theta) = \sin 180^\circ - \sin\theta$ and, because $\sin 180^\circ$ is zero, it follows that $\sin(180^\circ - \theta)$ is the same as $-\sin\theta$. Is this answer correct? If not, what is a correct way to express $\sin(180^\circ - \theta)$ in simpler form? Answer the same question for $\cos(180^\circ - \theta)$.

7. What is domain and range of $y = \log_b x$. How does this compare with the domain and range of $y = b^x$?

8. In triangle *ABC*, it is given that *BC* = 7, *AB* = 3, and cos *B* = $\frac{11}{14}$. Find the length of the projection of segment *AB* onto line *BC*.

9. Name a specific exponential function of your choosing. Find its inverse function. State the domain and range of this function and its inverse.

10. Find simpler, equivalent expressions for the following. Justify your answers.
    (a) $\sin(180^\circ + \theta)$ **(b)** $\cos(180^\circ + \theta)$ **(c)** $\tan(180^\circ + \theta)$

11. Consider the graph of the sine function and sketch its inverse graph. Is this graph a function? Explain.

Vertical Stretch and Period Change of Sine

Directions:

1. Using a point-by-point plot as suggested below, graph the parent function (i.e. the first function in each set) in pencil and try to memorize its shape as you do so that, in the future, the equation will motivate an immediate mental picture.
2. Also using a point-by-point plot, graph the second function in blue or black ink.
3. Predict what the third graph will look like and sketch it in red.
4. Verify your predication by entering all three functions on your graphing calculator.

<u>VERTICAL</u> <u>STRECTH/SHRINK</u> $y = a \cdot f(x)$

The AMPLITUDE of a trigonometric function is defined to be half the difference between the maximum and minimum values.

Use Degree Mode & Zoom 7.

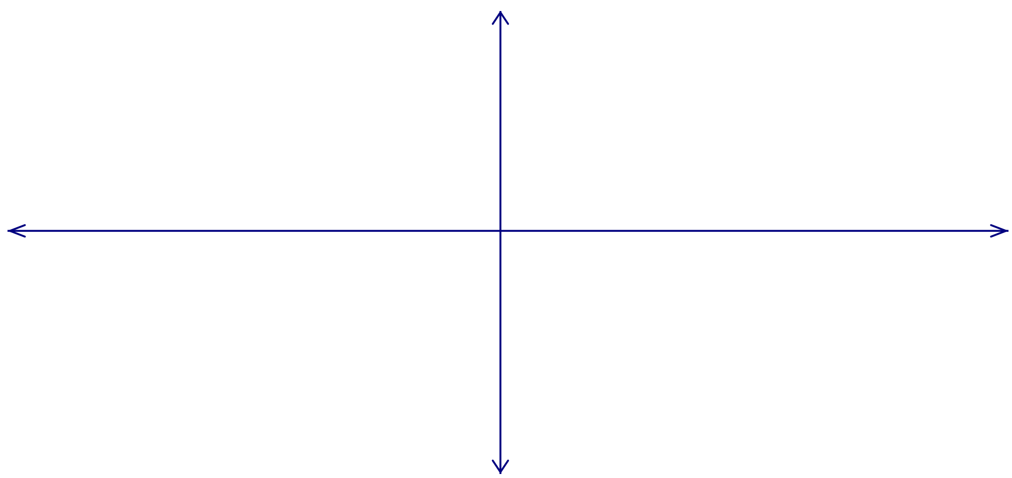
a.) $y = \sin x$

(0,)
(90,) (-90,)
(180,) (-180,)
(270,) (-270,)
(360,) (-360,)

b.) $y = \frac{1}{2}\sin x$

(0,))
(90,) (-90,)
(180,) (-180,)
(270,) (-270,)
(360,) (-360,)

c.) $y = 2\sin x$



Use Degree Mode & Zoom 7.

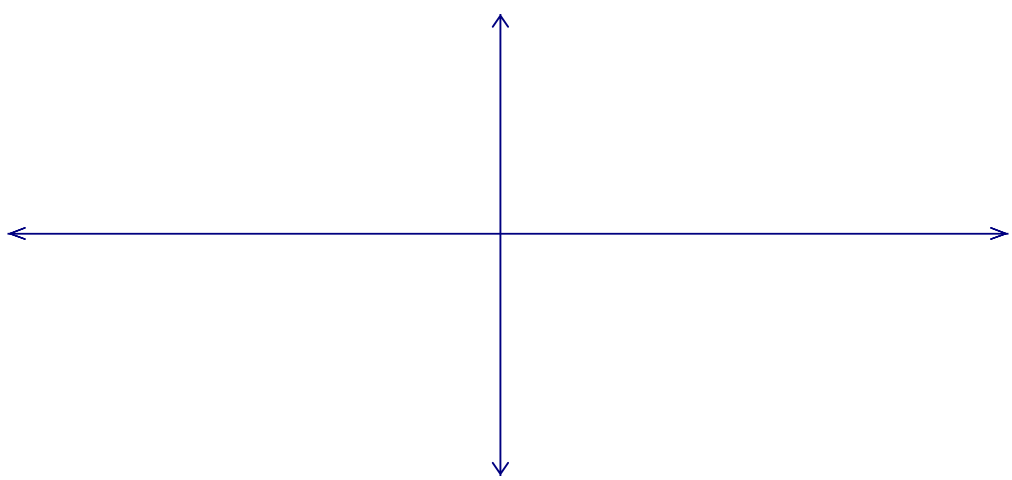
d.) $y = \sin x$

(0,)
(90,) (-90,)
(180,) (-180,)
(270,) (-270,)
(360,) (-360,)

e.) $y = -2\sin x$

(0,)
(90,) (-90,)
(180,) (-180,)
(270,) (-270,)
(360,) (-360,)

f.) $y = -3\sin x$

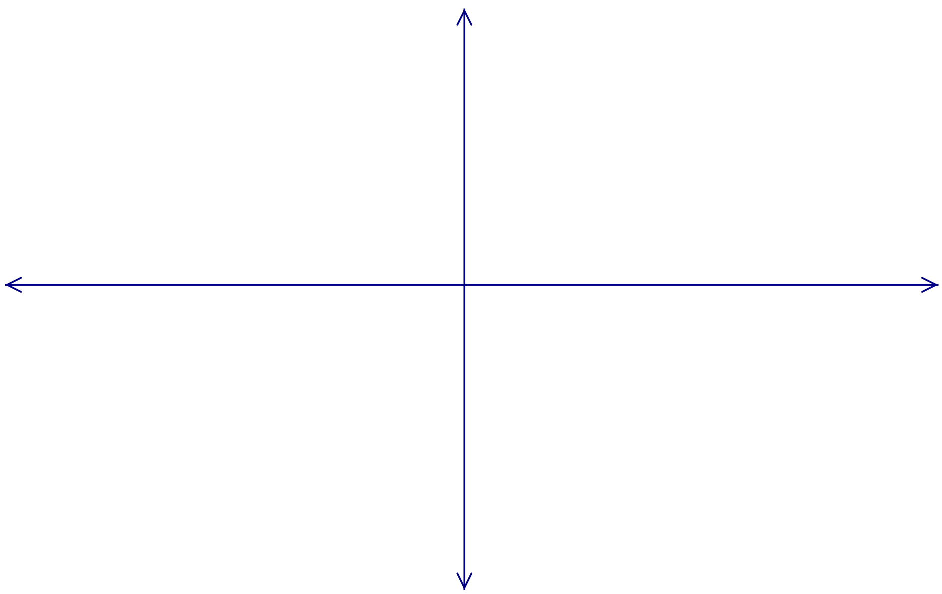


*The amplitude of $y = a \cdot \sin x$ is ____

<u>HORIZONTAL STRECTH/SHRINK</u> $y = f(b \cdot x), b > 0$
The PERIOD of a function is one cycle of the graph.

Use degree mode

a.) $y = \sin x$	b.) $y = \sin 2x$	c.) $y = \sin \frac{1}{2} x$
	(0,)	(0,)
	(45,)	(180,)
	(90,)	(360,)
	(135,)	(540,)
	(180,)	(720,)



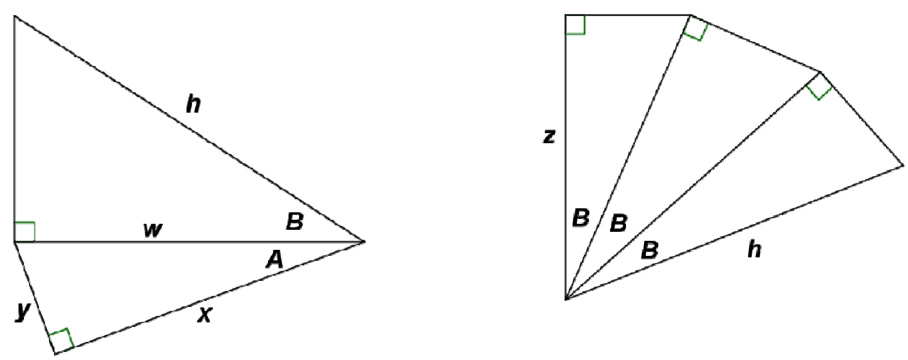
*The period of $y = \sin b \cdot x$ is ____

Complete the table

Function	$y = a \sin bx$	$y = 2 \sin 4x$	$y = -2 \sin \frac{3}{4} x$
Amplitude			
Period			

1. A wheel whose radius is 1 is placed so that its center is at $(3,2)$. A paint spot on the rim is found at $(4,2)$. The wheel is spun θ degrees in the counterclockwise direction. Now what are the coordinates of that paint spot, in terms of θ?

2. Using the figures below, express the lengths *w*, *x*, *y*, and *z* in terms of length *h* and angles *A* and *B*.



3. When working in degree mode, we say that the *period* of the graph $y = \sin x$ is 360. What does this statement mean? What is the period of the graph of $y = \cos x$? What is the period of the graph $y = \tan x$? What is the period of the graph $y = \sin 2x$? What is the period of the graph $y = \sin bx$?

4. When working in radian mode, we say that the *period* of the graph $y = \sin x$ is 2π. What does this statement mean? What is the period of the graph of $y = \cos x$? What is the period of the graph $y = \tan x$? What is the period of the graph $y = \sin 2x$? What is the period of the graph $y = \sin bx$?

5. Given $\tan\theta = -\frac{3}{2}$ and $\sin\theta < 0$, find $\sec\theta$ and $\csc\theta$.

6. Centered 6 meters above the ground, a Ferris wheel of radius 5 meters rotates at 1 degree per second. Assuming that Jamie's ride begins at the lowest point on the wheel, find how far Jamie is above the ground after 29 seconds; after 331 seconds; after *t* seconds.

7. (Continuation) Use your calculator to graph the equation $y = 6 - 5\cos x$. What does this picture tell you about Jamie's ride? Would a graph of $y = 6+5\cos x$ mean anything?

8. Find the third side of a triangle in which a 4 in side and a 6 in side are known to make a 56 degree angle. Round your answer to the nearest thousandth.

9. At constant speed, a wheel rotates once counterclockwise every 10 seconds. The center of the wheel is $(0,0)$ and its radius is 1 foot. A paint spot is initially at $(1,0)$; where is it *t* seconds later?

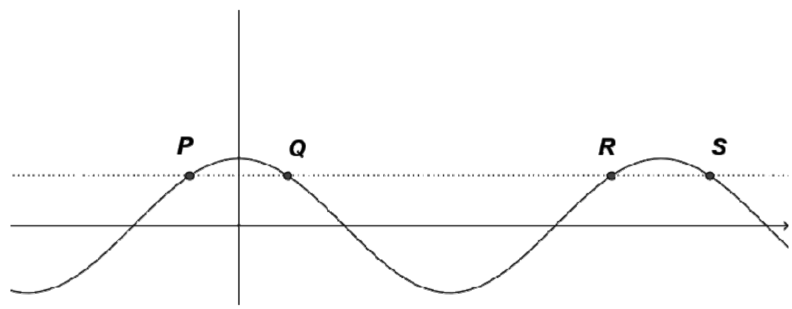
1. Using your calculator, evaluate $\sin^{-1}\left(-\frac{1}{2}\right)$. What are the coordinates on the graph of $y=\sin x, 0\le x\le 2\pi$, that have -0.5 as a y-coordinate. Explain.

2. Jamie rides a Ferris wheel for five minutes. The diameter of the wheel is 10 meters, and its center is 6 meters above the ground. Each revolution of the wheel takes 30 seconds. Being more than 9 meters above the ground causes Jamie to suffer an anxiety attack. For how many seconds does Jamie feel uncomfortable?

3. Find all solutions *w* between 0 and 360 degrees, inclusive:
   (a) $\cos w=\cos\left(-340^\circ\right)$ **(b)** $\cos w=\sin 20^\circ$ **(c)** $\sin w=\cos\left(-10^\circ\right)$
   (d) $\sin w<-\frac{1}{2}$ **(e)** $1<\tan w$

4. Two fire wardens are stationed at locations *P* and *Q*, which are 45 km apart. Each warden sights the forest fire at *F*. Given that angle *FPQ* is 52 degrees and angle *FQP* is 43 degrees, find the distance from *F* to the nearer warden, to the nearest 0.1 km.

5. At right you see the graphs of $y=\cos x$ and $y=0.7431$ (dotted). Given that Q = $\left(42^\circ, 0.7431\right)$, find coordinates for the intersection points *P, R,* and *S* without using a calculator. Use a calculator to check your answers.



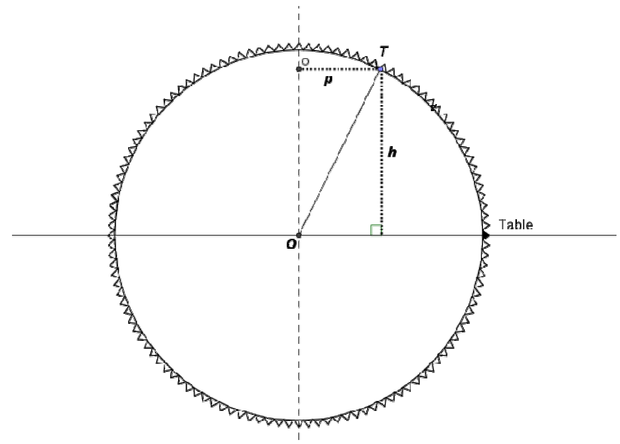
6. Given that $\sin\theta=k$, and that $90^\circ<\theta<180^\circ$, find expressions for $\cos\theta$ and $\tan\theta$ in terms of *k*.

7. Graph each pair of functions on a separate system of coordinate axes:
   (a) $y=\sin x$ and $y=\sin 4x$ **(b)** $y=\sin x$ and $y=\sin 2x$ **(c)** $y=\sin x$ and $y=\sin 0.25x$

8. (Continuation) What do the graphs of $y=\sin bx$ and $y=\sin x$ have in common, and how do they differ?

9. In order to make the sine function *one-to-one*, mathematicians choose a restricted domain for the function. Try it out – state intervals for the domain and range for which the sine graph is one-to-one.

1. Revisit the circular saw blade with the one red tooth. Look at the ratio m of the height h to the horizontal displacement p. (So, $m = h/p$.) The red tooth starts at the rightmost point of the saw and rotates at one degree per second. What is m after 37 seconds? After 137 seconds? After 237 seconds? After t seconds? Draw a graph that shows how m is a function of the elapsed time t. What does the ratio $m = h/p$ tell you about the line OT from the saw center to the tooth?



2. (Continuation) The graph of m versus t is an example of a *tangent* curve. Graph $y = \tan x$ on your calculator and compare it to the graph you drew in the previous exercise. Use this graph to determine the values of t for which m takes on the following values: 0, 0.5, and -2. How large can m be? Is m defined for *all* values of t?

3. Without using a calculator, choose **(a)** the larger of $\cos 40^\circ$ and $\cos 50^\circ$; **(b)** the larger of $\sin 40^\circ$ and $\sin 50^\circ$. Be prepared to explain your reasoning.

4. Find the exact value:
(a) $\sin^{-1}(1)$ **(b)** $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ **(c)** $\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ **(d)** $\sin\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

5. Given that $\sin\theta$ is 12/13, with $90^\circ < \theta < 180^\circ$, find the values of $\cos\theta$ and $\tan\theta$. Try to do this without finding θ first.

6. Given that $\sin\theta$ is a/b, with $0 < \theta < 90^\circ$, find expressions for $\cos\theta$ and $\tan\theta$.

7. Without using a calculator, choose the larger of $\sin 76^\circ$ and $\sin 106^\circ$. Explain.

8. To the nearest degree, find at least three solutions to each of the following:
(a) $\sin A = 0.80902$ **(b)** $\cos B = -0.80902$ **(c)** $\tan C = 1.96261$

9. Sasha tried to graph both $y = \sin x$ and $y = \sin^{-1} x$ on the same coordinate-axis system, using degree mode for the calculations. Sasha found this exercise to be difficult and confusing. Explain how it might have been clearer to put the calculator into *radian* mode first. How large a graphing window is needed to see all the necessary detail?

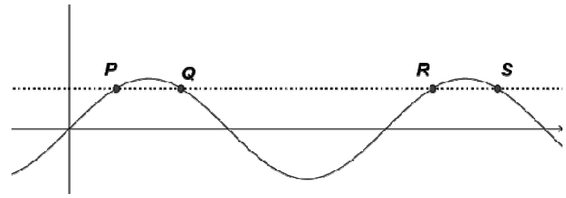
10. Reflect the graph $y = 2\sin x$ across the x-axis. Find an equation to describe the curve that results. Use your calculator to check your answer.

11. Is it possible for $\sin\theta$ to be exactly twice the size of $\cos\theta$? If so, find such an angle θ. If not, explain why not.

1. Given that $\cos\theta$ is 7/25, with $270^\circ < \theta < 360^\circ$, find $\sin\theta$ and $\tan\theta$, without finding θ.

2. Using no calculator, choose the larger of $\cos 310^\circ$ and $\cos 311^\circ$. Explain your reasoning.

3. The graphs of $y = \sin x$ and $y = k$ (dotted) are shown at right. Given that the coordinates of *P* are (θ, k), find the coordinates of *Q, R,* and *S,* in terms of θ and *k.*



4. Simplify the following:
(a) $\cos(360^\circ - \theta)$ **(b)** $\sin(360^\circ - \theta)$ **(c)** $\tan(360^\circ - \theta)$

5. On the graph of $y = \cos x$, many points have 0.39073 as their *y*-coordinate. Among them, find the three that have the smallest positive *x*-coordinates.

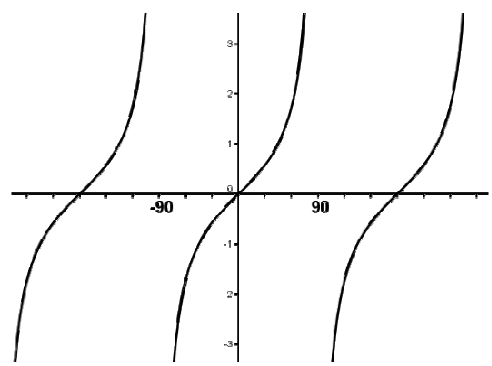
6. A parallelogram has a 5-inch side and an 8-inch side that make a 50 degree angle. Find the area of the parallelogram and the lengths of its diagonals.

7. A ladder leans against a side of a building, making a 63 degree angle with the ground, and reaching over a fence that is 6 feet from the building. The ladder barely touches the top of the fence, which is 8 feet tall. Find the length of the ladder.

8. For what values of θ is it true that $\sin\theta = \cos\theta$? Find two equivalent ways to express the slope of the vector $[\cos\theta, \sin\theta]$.

9. The equation $\tan\theta = 0.9004$ has infinitely many solutions. Find a way of describing all these values θ.

10. A graph of $y = \tan x$ is shown at right, drawn in degree mode. Confirm that one of the points on this graph is $(63.423^\circ, 1.999)$. Recall that the number 1.999 can be interpreted as the *slope* of a certain ray drawn from the origin. What ray? In contrast to a sine graph, which is a connected curve, this graph is in pieces. Explain why.



11. Find the three smallest positive solutions to $2\sin\theta = -1.364$.

12. Explain why equation $\tan\theta = -2$ has solutions, but equation $\sin\theta = -2$ does not.

13. Graph each of the following pairs of functions on a separate system of coordinate axes:
(a) $y = 2\cos x$ and $y = 1 + 2\cos x$ **(b)** $y = -3\cos x$ and $y = 1 - 3\cos x$
What does each pair of graphs have in common? How do the graphs differ?

1. For each of the following, there are two points on the unit circle that fit the given description. Without finding θ, describe how the two points are related to each other.
   (a) $\cos\theta = -0.4540$ **(b)** $\sin\theta = 0.6820$ **(c)** $\tan\theta = -1.280$

2. Graph the equation $y = 1 + 2\sin x$. This curve crosses the x-axis in several places. Identify all the x-intercepts with $0 < x < 2\pi$.

3. For what values of x is $y = \tan 2x$ undefined?

4. State the period of each function.
   (a) $y = 7\tan\dfrac{x}{2}$ **(b)** $y = -4\tan 5x$ **(c)** $y = 5\tan 3\pi x$

5. The point $\left(75^\circ, 0.2588\right)$ is on the graph of $f\left(x\right) = \cos x$. What is the corresponding point on the graph $f\left(x\right) = \cos^{-1}\left(x\right)$? Explain why there is no point on the graph of $f\left(x\right) = \cos^{-1}\left(x\right)$ that corresponds to the point $\left(-75^\circ, 0.2588\right)$.

6. On the graph of $y = \sin x$, there are many points that have 0.39073 as the y-coordinate. Among these points, find the three that have the smallest positive x-coordinates.

7. Sketch one full period of the graph of each function.
   (a) $y = -4\tan\dfrac{x}{4}$ **(b)** $y = 2\tan 2x$ **(c)** $y = \tan\pi x$

8. Establish a restricted domain and range for $f\left(x\right) = \cos x$ so that it is one-to-one. Is this the same restricted domain that applies to $f\left(x\right) = \sin x$? Explain.

9. On the graph $y = \tan x$, many points have 1 as their y-coordinate. Find the three that have the smallest positive x-coordinate.

10. Graph one full period of the function as defined by each equation. Be certain to state the amplitude and period.
    (a) $y = 2\sin x$ **(b)** $y = -5\cos 5x$ **(c)** $y = 10\sin\dfrac{2\pi}{3}x$

11. Simplify the expressions $\cos\left(180^\circ - \theta\right)$ and $\sin\left(180^\circ - \theta\right)$.

12. If the equations $y = \sin x$ and $y = \cos x$ are both graphed on the same xy-axis system, the curves will intersect many times. Find coordinates for at least two intersection points.

1. What is the difference between the solutions to $\sin x = \frac{1}{2}$ and $\sin^{-1}\left(\frac{1}{2}\right) = x$. Explain your answer based on the domains and ranges of these functions.

2. Sketch at least one full period of the graph of each function.
(a) $y = 2\tan x$ **(b)** $y = -4\tan 2x$ **(c)** $y = 3\tan \frac{1}{4}x$

3. The function $f(t) = a\sin bt$ has amplitude of 3 and a period of 4. Write an equation of the function.

4. Centered 7 meters above the ground, a Ferris wheel of radius 6 meters is rotating with angular speed 24 degrees per second. Assuming that Harley's joyride began at time $t = 0$ seconds at the lowest point on the wheel, write a formula for the function that describes the distance $h(t)$ from Harley to the ground (in meters) after t seconds of riding.

5. (Continuation) Draw a graph of $h(t)$ for the restricted *domain* $0 \le t \le 30$, and find coordinates for two points on your graph that both represent the situation when Harley is 10 meters above the ground and climbing. Interpret the domain restriction in context.

6. The *cotangent* is the reciprocal of the tangent: $\cot t = \frac{1}{\tan t}$. Write the Pythagorean Identity in terms of cotangent and cosecant.

7. The graph $y = -3 + 5\cos x$ intersects the x-axis repeatedly. Find all the x-intercepts on the domain of $-2\pi < x < 2\pi$. Can you come up with a way to describe all of the x-intercepts of this graph?

8. With your calculator in degree mode, set the window $-270 \le x \le 270$ and $-5 \le y \le 5$. Graph $y = \tan x$. Does your calculator accurately represent the graph? Explain.

9. The function $f(t) = a\cos bt$ has amplitude of 1.5 and a period of $\frac{\pi}{2}$. Write an equation of the function.

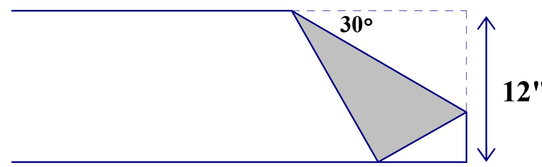
10. Find the radian measure of the angle subtended by an arc of 20 cm on a circle whose radius is 50.

11. Calculate sin 72 and sin(−72). Explain why $\sin(-\theta)$ is always the same as $-\sin\theta$. What can be said about $\cos(-\theta)$? What about tan(-θ)?

12. Find the exact value of $\left(\tan\frac{\pi}{6}\right)\left(\cos\frac{\pi}{3}\right) - \sin\frac{3\pi}{2}$.

1. The function $f(x) = \tan bx$ has period of 8π. Write an equation of the function.

2. The figure at right shows a long rectangular strip of paper, one corner of which has been folded over to meet the opposite edge, thereby creating a 30 degree angle. Given that the width of the strip is 12 inches, find the length of the crease.



3. (Continuation) Suppose that the size of the folding angle is θ degrees. Use trigonometry to express the length of the crease as a function of θ. Check using the case θ = 30 degrees.

4. Find a way of describing all the intersections of the line y = 1.5399 and the graph of $y = \tan x$. For what numbers m is it possible to solve the equation $m = \tan x$?

5. Given $\cot\theta = -1$ and $\frac{\pi}{2} < \theta < \pi$, evaluate $\sin\theta$.

6. You have used the function $\tan^{-1}$ many times to find angles. Now it is time to consider the rule that makes $\tan^{-1}$ a function.
(a) The point $(45.0, 1.0)$ is on the graph of $y = \tan x$ (working in degree mode). There is a corresponding point that is on the graph of $y = \tan^{-1} x$. What is it?
(b) Write four other pairs of points of this sort — two with negative coordinates and two with positive coordinates.
(c) Notice that there is *no* point on the graph of $y = \tan^{-1} x$ that corresponds to the point $(135.0, -1.0)$. Explain why.

7. Graph $y = 2\sin 4x$

8. If θ is in the second quadrant and $\tan\theta = -\frac{4}{7}$, find $\sec\theta$

9. Devon's bike has wheels that are 27 inches in diameter. After the front wheel picks up a tack, Devon rolls another 100 feet and stops. How far above the ground is the tack?

10. (Continuation) How many degrees does the wheel turn for each foot that it rolls?

11. Draw the unit circle and graph of $y = \sin x$ and $y = \sin 2x$. How does the unit circle axis intercepts match up with the axis intercepts of the graphs of these two functions? When you change the period, how does this affect the motion of the angle around the unit circle in general? What would you expect to happen with the unit circle for y = sin 4x?

1. Quinn is running around the circular track $x^2 + y^2 = 10000$, whose radius is 100 meters, at 4 meters per second. Quinn starts at the point $(100, 0)$ and runs in the counterclockwise direction. After 30 minutes of running, what are Quinn's coordinates?

2. Draw the graph of $x = \tan y$, then explain why it is not the same as the graph of $y = \tan^{-1} x$.

3. Avery is riding a Ferris wheel that turns once every 24 seconds, and whose radius is 8 meters. The function $h(t) = 9 - 8\cos(15t)$ describes Avery's distance from the ground (in meters) after *t* seconds of riding. For example, $h(8) = 13$ means that Avery is 13 meters above the ground after 8 seconds of riding. By the way, "*h* of 8" or "*h* at 8" are two common ways to read $h(8)$.
(a) Evaluate $h(0)$, and explain its significance.
(b) Explain why $h(16) = h(8)$.
(c) Find a value for *t* that fits the equation $h(t) = 10$. Interpret this *t*-value in the story.
(d) Explain why $h(t+24) = h(t)$ is true, no matter what value t has.
(e) What is the complete range of values that $h(t)$ can have?

4. The function $f(x) = \tan bx$ has period of 4. Write an equation of the function.

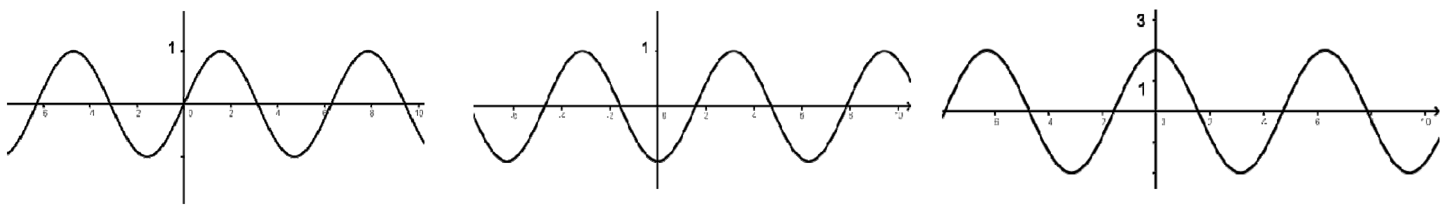
5. Graph each of the following pairs of functions on a separate system of coordinate axes, and account for what you see:
(a) $y = \sin x$ and $y = \sin(-x)$ **(b)** $y = \cos x$ and $y = \cos(-x)$

6. The *domain* of the function $\sin^{-1}$ consists of all numbers between -1 and 1, inclusive. What is the domain of the function $\cos^{-1}$? What is the domain of $\tan^{-1}$? What is the domain of $f(x) = 2 - 7\sin 45x$?

7. When working in degree mode, the *range* of the function $\sin^{-1}$ consists of all numbers between -90 and 90, inclusive. What is the range of the function $\cos^{-1}$? What is the range of $\tan^{-1}$? What is the range of $f(x) = 2 - 7\sin 45x$?

8. Without using a calculator, simplify the following (in degrees):
(a) $\sin(\sin^{-1} 0.32)$ **(b)** $\tan(\tan^{-1} 1.61)$ **(c)** $\cos^{-1}(\cos 223)$ **(d)** $\sin^{-1}(\sin 137)$

1. The sinusoidal graphs shown below appeared on a calculator that was operating in radian mode. Find equations that might have produced the graphs. The period of each curve is 2π.

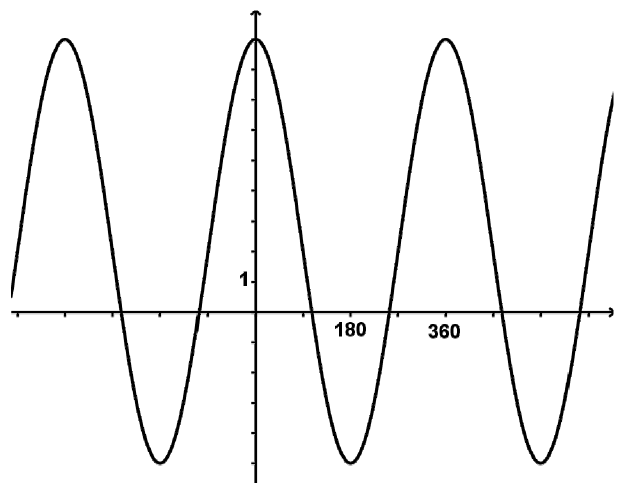


2. Draw the angle described by $\tan^{-1}\left(\frac{12}{5}\right)$.

3. If the cosine of an acute angle is some number k, then what is the *sine* of the same angle? In terms of k, what is the cosecant of this angle? Simplify the expression $\tan\left(\cos^{-1} k\right)$.

4. Write the expression $\dfrac{1}{\sin^2\theta}+\dfrac{1}{\cos^2\theta}$ as a single term.

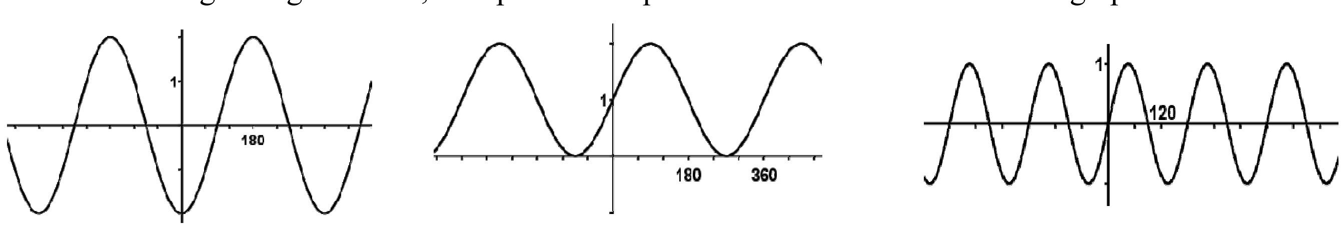
5. The equation whose graph is shown at right has the form $y = k + a\cos x$. Working in degree mode, find believable values for the coefficients a and k, and explain how these numbers affect the appearance of the graph.



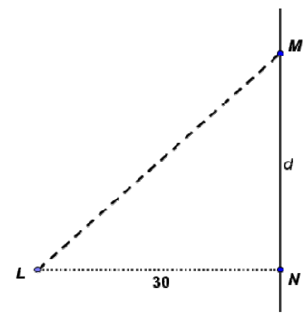
6. In a circle of radius 5 cm, how long is a 1 radian arc? How long is a 2.2 radian arc?

7. An object moves around $x^2 + y^2 = 25$, which represents a circle whose radius is 5 meters, at a constant speed. At time $t = 0$ seconds, the object is at $(5,0)$. When $t = 1$, it is at $(4,3)$. Where is the object when $t = 2$? When $t = 3$? When $t = n$? What is the object's speed? At what time does the object return to $(5,0)$?

8. Working in degree mode, find plausible equation for each of the sinusoidal graphs below:



1. A prison guard tower is 30 feet from the nearest wall of the prison. The diagram shows this arrangement from above, as if the viewer were in a helicopter. The spotlight L on top of the tower rotates counterclockwise, once every six seconds, casting a moving beam of light onto the wall. Let N be the point on the wall that is nearest the spotlight. Let M be the moving spot. Let d be the distance from N to M, and let t be the time, in seconds, since M last passed N. Find d when $t = 0.00$, $t = 0.30$, $t = 0.75$, and $t = 1.49$. Are d and t related linearly? What does the graph of this relationship look like?

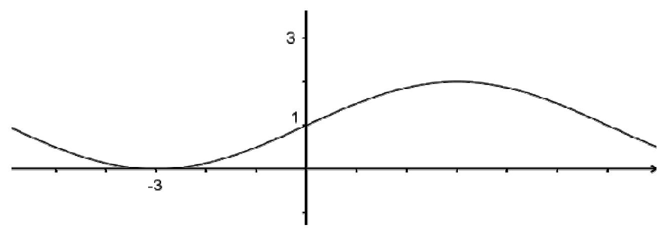


2. How are the graphs of $y = \cos x$ and $y = \cos\left(x - 90^\circ\right)$ related?

3. In radian mode, state the domain and range of:
(a) $f(x) = \sin^{-1}(x)$ **(b)** $f(x) = \cos^{-1}(x)$ **(c)** $f(x) = \tan^{-1}(x)$

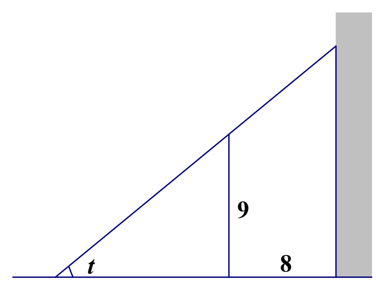
4. What is the relationship between the graph of $y = \sin x$ and the graph of $y = \sin\left(x + 90^\circ\right)$? Work in degree mode.

5. The graph to the right touches the x-axis at -3. Write an equation that could have produced it. Does it affect your answer whether you choose to work in degree mode or radian mode?

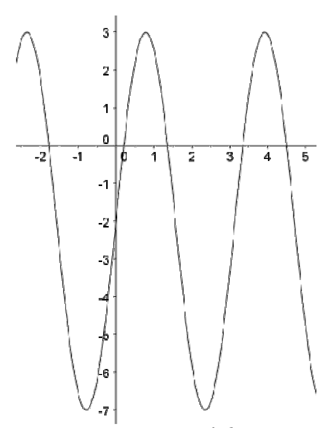


6. Verify that $1 - 2\sin^2 x = 2\cos^2 x - 1$ by manipulating one side of the equation to look like the other side.

7. A fence that is 9 feet tall is situated 8 feet from the side of a tall building. As the figure at right shows, a ladder is leaning against the building, with its base outside the fence. It so happens that the ladder is touching the top of the fence. Find the length of the ladder, given that
(a) it makes a 60 degree angle with the ground
(b) it makes a t degree angle with the ground.
(c) Apply your calculator to the answer for part (b) to find the length of the shortest ladder that reaches the building from outside the fence.



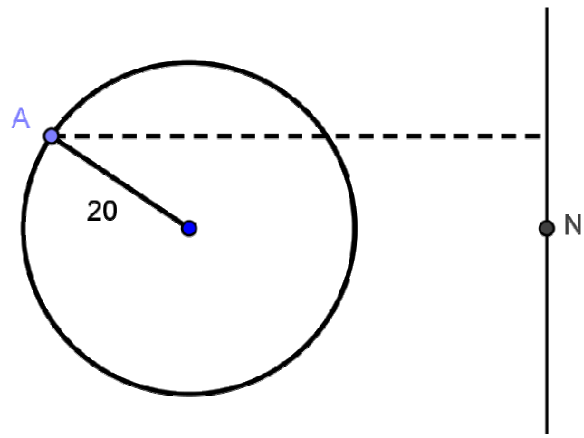
8. The equation whose graph is shown at right has the form $y = k + a\sin 2x$. To three decimal places, one of the x-intercepts is 1.365. Working in radian mode, find values for the other x-intercepts shown in the figure.



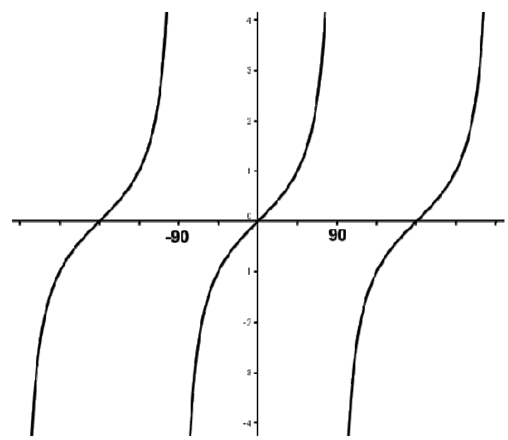
1. Write an equation of the form $y = k + a\sin bx$ for a curve that has a maximum point at $(30, 6)$ and that has 2 as its y-intercept. Use a calculator to check that the graph of your equation fits the given description. Because you have the freedom to work in either degree mode or radian mode, and because there are many curves that fit the given description, there are many correct answers to this question. Can you find another one?

2. Are there real numbers for which $\cos\left(\cos^{-1}(x)\right) \neq x$? Explain.

3. Illuminated by the rays of the setting Sun, Andy rides alone on a merry-go-round, casting a moving shadow on a wall. The merry-go-round is turning 40 degrees per second. As the top view shows, Andy is 20 feet from its center, and the Sun's rays are perpendicular to the wall. Let N be the point on the wall that is closest to the merry-go-round. Write an equation that represents the shadow's distance from N as a function of time.



4. The graph at right appeared on a calculator that was operating in degree mode. Find an equation that might have produced this graph.



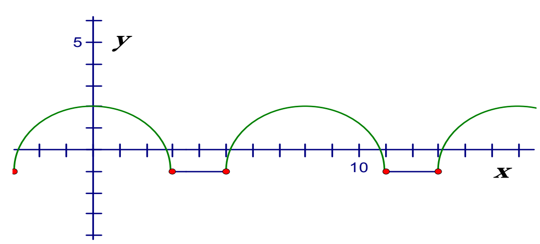
5. Write an equation of the form $y = k + a\cos bx$ for a curve that has a maximum point at $(20, 4)$ and that has -2 as its y-intercept. Use a calculator to check that the graph of your equation fits the given description. When you check answers with your neighbor, is it expected that you will both have found the same equation?

6. Working in degree mode, find both values of n between 0 and 360° for which $\sin 7672^\circ = \sin n$. A calculator is not needed.

7. The figure at right shows the graph of a periodic function $y = f(x)$. The graph, whose period is 8, is built from segments and semicircular arcs. Notice the values $f(3) = -1$ and $f(5) = -1$.

   (a) Calculate $f(11)$, $f(13)$, $f(16)$ and $f(19)$.

   (b) What does the graph of $y = f(x-3) - 1$ look like?



1. Working in degree mode, graph $y = \sin(x+90)$. On the basis of your graph, suggest a simpler expression that is equivalent to $\sin(x+90)$.

2. (Continuation) Write the radian-mode version of the question, then answer it.

3. Rewrite The Pythagorean Identity, $\cos^2 x + \sin^2 = 1$, in terms of $\sec x$ and $\tan x$.

4. (Continuation) Repeat, in terms of $\csc x$ and $\cot x$.

5. Show that $y = 3\cos 2x$ can be rewritten in the equivalent form $y = a\sin b(x+c)$, thereby confirming that a cosine curve is sinusoidal, i.e. can be written as a sine curve.

6. Consider the function $f(x) = |\sin x|$. Draw its graph, and find the period. Explain why this graph is not sinusoidal.

7. Consider the function $f(x) = \sin^{-1}(\sin x)$. Draw its graph, and find the period.

8. Working in degree mode, find the periods for the three functions $f(x) = \sin(60x)$, $g(x) = \sin(90x)$, $h(x) = \sin(30\pi x)$.

9. Another common name for the inverse sine function is *arcsin*, which is an abbreviation of *find the arc whose sine is*. Explain this terminology.

10. Work in radian mode for this one. The line $y = 0.5x$ intersects the sine curve in three places. What are the coordinates of these intersection points?

11. Recall that the conjugate of a binomial expression changes the sign of the second term. They are the factorization of the difference of two squares. For instance, $(\sin x + \cos x)(\sin x - \cos x) = \sin^2 x - \cos^2 x$. Use this to verify that $\dfrac{\sin x}{1-\cos x} = \csc x + \cot x$ by:
    (a) Manipulating the left side only to get it to match the right side
    (b) Manipulating the right side only to get it to match the left side.

12. Working in radian mode, graph both $y = \sin^{-1} x$ and $y = \dfrac{\pi}{2} - \sin^{-1} x$ on the same coordinate- axis system. Choose a viewing window that allows you to see both graphs in their entirety. What is the customary name for the second function?

13. Write a sine function whose period is 10 and whose values oscillate between -15 and 3.

1. By means of a formula, invent an example of a periodic function *f*, whose period is 50, and whose values *f(x)* oscillate between the extremes -3 and 7.

2. The graph of $f(x) = \sin^2 x - 1$ intersects the x-axis an infinite number of times. Find 3 of its x-intercepts.

3. Verify that $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$. Work with one side only.

4. Write an equation for the sinusoidal curve that has a crest at the point $(50, 7)$ and an adjacent valley at $(150, -7)$.

5. Write equations for at least two sinusoidal curves that have a crest at the point $(150, 7)$ and an adjacent valley at $(200, -7)$.

6. Recall that $\tan x = \dfrac{\sin x}{\cos x}$. Use this to verify that $\dfrac{\sin x + \tan x}{1+\cos x} = \tan x$, making sure to manipulate only one side of the equation.

7. Find periods for $f(x) = \sin(7.2x)$, $g(x) = \sin(4.5x)$, and $h(x) = \sin(1.7x)$.

8. Verify each identify. Remember, you can only work with one side of the equation.
   (a) $\dfrac{1}{\sin x} - \dfrac{1}{\cos x} = \dfrac{\cos x - \sin x}{\sin x \cos x}$ **(b)** $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$ **(c)** $\csc x = \dfrac{\cot x + \tan x}{\sec x}$