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Problem-Based Learning in High School

Motivational Problem on Apportionment

1. Students at Pascal High School have a 25- member student council, which represents the 2000 members of the student body. The class-by- class sizes are: 581 Seniors, 506 Juniors, 486 Sophomores, and 427 Freshmen. How do you think that the seats on the council should be distributed to the classes?

2. If you were President Washington, how would you distribute the 120 seats in the House of Representatives to the fifteen states listed below? The table shows the results of the 1790 census.

State	**Population**
Connecticut	236841
Delaware	55540
Georgia	70835
Kentucky	68705
Maryland	278514
Massachusetts	475327
New Hampshire	141822
New Jersey	179570
New York	331589
North Carolina	353523
Pennsylvania	432879
Rhode Island	68446
South Carolina	206236
Vermont	85533
Virginia	630560
Total	3615920

The Apportionment Problem

In every method of apportioning the House of Representatives, the *ideal quota* for a state is now calculated by the formula 435 · (state population/total population). Because this is not likely to be an integer, it is necessary to either round up to the *upper quota* or round down to the *lower quota* to obtain a meaningful result. A method of apportionment must specify exactly how this rounding is to be done.

Another quantity of significance in apportionment is the *ideal district size*, which is the total population divided by the total number of representatives. The 2000 census puts this figure at 646952 = 281424177/435. This is how many constituents each representative *should* have (and would have, if Congressional districts were allowed to cross state boundaries).

3. What do you get if you divide a state's population by the ideal district size?

The simplest method of apportionment was proposed in 1790 by Alexander Hamilton, and it is so intuitively appealing that you may have thought of it yourself already: Calculate each state's share of the total number of available seats, based on population proportions, and give each state as many seats as prescribed by the integer part of its ideal quota. The remaining fractional parts of the quotas add up to a whole number of uncommitted seats, which are awarded to those states that have the *largest fractional parts*.

Apply the Hamilton method to the following small, three-state examples. (The names of the states are simply A, B, and C.) You should notice some interesting anomalies.

4. Suppose that the populations are A = 453000, B = 442000, and C = 105000, and that there are 100 delegates to be assigned to these states on the basis of their populations.

5. Suppose that the populations are A = 453000, B = 442000, and C = 105000, and that there are 101 delegates to be assigned to these states on the basis of their populations.

6. Suppose that the populations are A = 647000, B = 247000, and C = 106000, and that there are 100 delegates to be assigned to these states on the basis of their populations.

7. Suppose that the populations are A = 650000, B = 255000, and C = 105000, and that there are 100 delegates to be assigned to these states on the basis of their populations.

Divisor Methods of Apportionment

According to the Hamilton method, quota-rounding decisions are made only after the *entire* list of quotas has been examined (and ranked in order of decreasing fractional parts).

There are several other methods for apportionment, each one characterized by a rounding rule that is meant to be applied to individual states, without specific reference to the quotas of other states. These methods are described next.

If an arbitrary (non-ideal) district size is used to divide the state populations, we obtain an *adjusted quota* for each state. What happens next depends solely on the rounding rule that is in effect.

The Jefferson method: All adjusted quotas are rounded down. Because all the fractional parts are being discarded, the divisor must be smaller than the ideal district size, if the target number of representatives (435) is to be hit exactly. This method, proposed by Thomas Jefferson, was approved by Washington and applied to the 1790 census, with a House size of 105 and 33000 as the divisor.

A significant amount of trial and error is necessary to carry out this divisor method (or any of the others). If the divisor is too small, the total number of assigned representatives will exceed 435; if the divisor is too large, the total will fall short of 435. For a project of this size (each trial divisor must be divided into *all fifty* state populations), it is desirable to use a computer to carry out the numerical work. (If you are interested, there is an open source program for PC called Windisc that can be downloaded at http://math.exeter.edu/rparris/windisc.html that has a demo that simplifies this example).

The Adams method: Adjusted quotas are rounded *up*. An acceptable divisor must be larger than the ideal district size. This method was proposed by John Quincy Adams. It has never been adopted.

The Webster method: Adjusted quotas are rounded *in the usual way* — to the nearest whole number. This method was proposed in 1831 by Daniel Webster, but not used until the 1840 census.

8. Pascal High School has a 25-member student council to represent its 2000 students. There are 581 Seniors, 506 Juniors, 486 Sophomores, and 427 Freshmen. Apply the Jefferson, Adams, and Webster methods to apportion the seats on the council.

For more Motivational Problems on Discrete Math see Exeter's Discrete Math PBL Curriculum at

http://www.exeter.edu/documents/discrete.pdf