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Problem-Based Learning in the High School Classroom

Motivational Problems on Matrices

1. A matrix can be used to display and process certain kinds of data. For example, last weekend, the local movie theater sold tickets to 186 adults on Friday, 109 adults on Saturday, 111 adults on Sunday, 103 children on Friday, 127 children on Saturday, 99 children on Sunday, 77 senior citizens on Friday, 67 senior citizens on Saturday, and 58 senior citizens on Sunday. This data is displayed in the 3 × 3 sales matrix **S** shown above. The descriptive labels given in the margin allow the reader to easily remember what all the numbers mean. Invent your own example of numerical data that can be displayed like this in a rectangular array.

$$\begin{array}{r c} & \begin{array}{ccc} child & adult & sr. \end{array} \\ \begin{array}{r} Fri \\ Sat \\ Sun \end{array} & \begin{bmatrix} 103 & 186 & 77 \\ 127 & 109 & 67 \\ 99 & 111 & 58 \end{bmatrix} \end{array}$$

2. This movie theater's ticket prices can be read from the 3 × 1matrix **P** shown below. Such a matrix is often called a *column vector*. The first row of matrix **S** is a 3-component *row vector*. What is the dot product of these two vectors? What does this "product" mean in the context of the problem? What about the dot products of **P** with the other rows of **S**?

$$P = \begin{array}{r} child \\ adult \\ sr. \end{array} \begin{bmatrix} 5.50 \\ 8.50 \\ 6.00 \end{bmatrix}$$

3. (Continuation) Matrix multiplication consists of calculating all possible dot products of row vectors from the first matrix and column vectors from the second matrix. How many dot products can be formed by multiplying matrix **S** times matrix **P**? How would you organize them into a new matrix, $S \bullet P$ called? What do the entries of $S \bullet P$ mean?

4. If $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $N = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 0 \end{bmatrix}$, find all possible products of two matrices. Some should work and some might not – try to articulate in a sentence why not.

5. Two matrices can be multiplied only if their sizes are compatible. Suppose that **U** is an *m* × *n* matrix, and that **V** is a *p* × *q* matrix. In order for **U.V** to make sense, what must be true of the dimensions of these matrices? Although matrix multiplication uses dot products, it is common to write **UV** without the dot, which will be done from now on.