Encouraging Reasoning and Sense-Making with Problem-Based Learning

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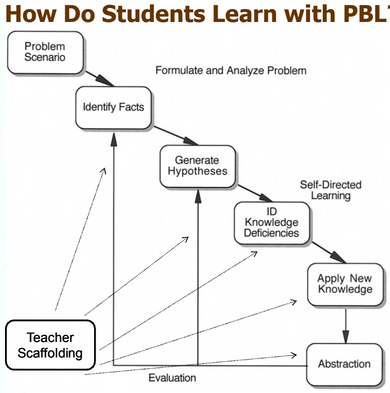
What is Problem-Based Learning?

- instructional approach of curriculum and pedagogy
- student learning and content material is constructed and co-constructed
- use, facilitation and experience of mostly contextual problems in a decompartmentalized, threaded topic format
- discussion-based, student-centered classroom
- student voice, experience, and prior knowledge are valued in construction of new knowledge

Schettino, (2010)

How Do Students Learn with PBL?



Hmelo-Silver, C.(2004). Problem-Based Learning: What and how do students learn? *Educational Psychology Review* ,16 (3), 235-266.

Problem-Based vs. Project-Based

- What's the difference?
- Direct Instruction vs. Student Constructed Learning
- Reasoning and Sense-Making of Curriculum and Content
- Buck Institute, New Tech Foundation, Illinois Math Science Academy PBL network, SIMMS, Phillips Exeter Academy

Where do you get problems?

- Problems need to
  - **motivate discussion**
  - **Create interest in the idea**
  - **Connect to prior knowledge**
  - **Inspire thinking**
  - **Allow for open communication & student presentation of ideas**
  - **Scaffold construction of new knowledge**
- Exeter Course Materials at http://www.exeter.edu/academics/72_6539.aspx
- AERA Special Interest Group for PBL

at http://tinyurl.com/aerasigpbl

- NCTM publications

Common Core Standards for Mathematical Practice & PBL

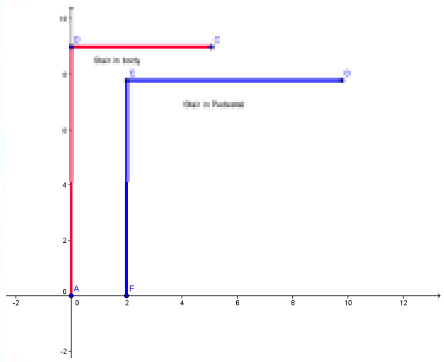
- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.
- Look for and make use of structure.

http://www.corestandards.org/the-standards/mathematics/introduction/standards-for-mathematical-practice/

Your packet and tasks

- Two strand problem packet
  - **Slope and Pythagorean Theorem**
- Work individually for about 15 minutes
- Present to each other for about 15 minutes
- Come together
- Work individually for about 15 minutes
- Present to each other for about 15 minutes
- Wrap Up

Sample Comparison for #3



Teaching for Reasoning and Sense-Making

- Provide tasks that require students to figure things out for themselves (NCTM, 2009)
- Resist the temptation to tell
- Stop teaching decontextualized content
- Stop giving students the final product of our thinking
- Problems first, teaching second
- Progressively withdraw from helping students
- Reevaluate evaluation

McCain, Ted (2005) Teaching for Tomorrow:Teaching Content and Problem-Solving Skills. Corwin Press, Thousand Oaks, CA (44).

Characteristics of a PBL Teacher

- Probing students for deep explanation
- Open-ended metacognitive questions
- Revoicing
- Summarizing
- Explicit Mapping between cause and effect
- Checking consensus on board
- Cleaning up board
- Encourage construction of visual representation

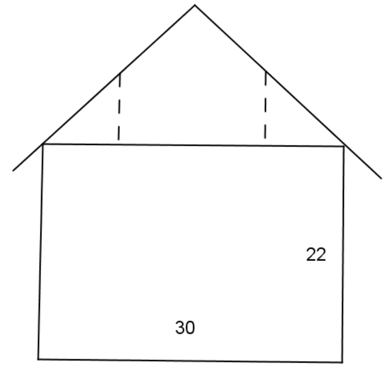
Hmelo-Silver & Barrows (2006). Goals and strategies of a PBL Facilitator. *Interdisciplinary Journal of Problem-Based Learning* , 1(1), 21-39

Last thoughts…

- Be open to the unknown
- Get comfortable with learning happening through discussion, not necessarily through planned instruction (that wasn't a guarantee anyway)
- Recycle through topics regularly - PBL
- Allow students to go with their ideas

Resources

- All of these hand outs are on my website at:
- www.carmelschettino.org
- carmel@carmelschettino.org
- Or cschettino@deerfield.edu
- Join a forum at my website

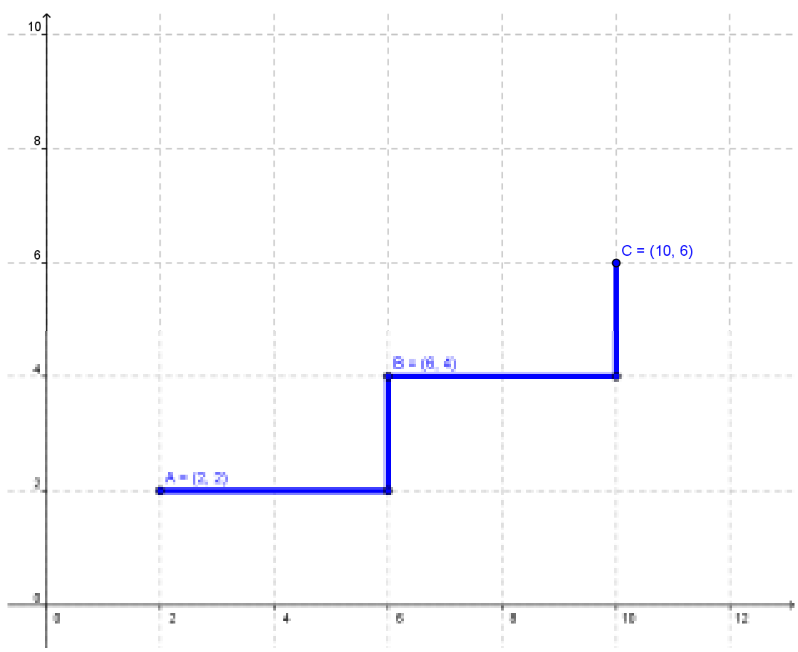


1. A town's building code does not permit building a house that is more than 35 feet tall. An architect working on the design shown at right would like the roof to be sloped so that it rises 10 inches for each foot of horizontal length.
   a. Given the other dimensions in the diagram, describe a way that the builder can carry out this plan?
   b. Two vertical supports (shown dotted in the diagram) are to be placed 6 feet from the center of the building. How long should they be?

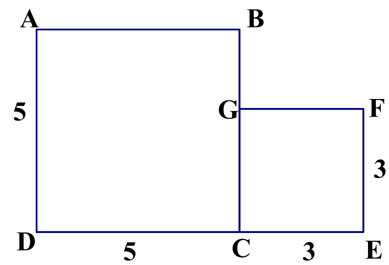
2. You are given the two squares and the sheets of paper (see handouts). Your task is to cut the two given squares into pieces and reform those pieces into a larger square. (Hint: Cut out the entire outline of the two squares, and cut along the dotted line shown, but do not cut the dashed line between the squares).
   a. Compare your "puzzle" to others' method. Is what you did to form the third square the same? Did you start with the same squares?
   b. Are you sure that the new shape that you formed is in fact a square? Can you justify that to your neighbor? What is enough evidence and reasoning? Write down your argument and then compare with another set of classmates.

3. In 1986, renovations were made to the stairs in the Status of Liberty in an overall celebration of its bicentennial. Frank Schettino was the draftsman in charge of the calculations for the details of the structural steel for those stairs. He found that the stairs in the Lady's pedestal needed to have a vertical rise of 7.8 inches and a horizontal run of 9.75 inches. The more complicated spiral stairs the led up the body had a constant vertical rise of 9 inches and average horizontal run of 5.06 inches.
   a. Using graph paper (or GeoGebra), create a graphical representation of these two sets of stairs.
   b. Which flight of stairs do you think is steeper? Why?
   c. Calculate the $rise/run$ ratio for each flight and verify that the greater ratio belongs to the flight you thought was steeper.

4. (Continuation) A flight of stairs goes from point A(2,2) to B(6,4) to C(10,6) as shown in the diagram at the right. We define the $rise/run$ ratio of the line AB made by the points on the stairs as the *slope* of the line. What's a way you would calculate that slope? What do you think the slope of the line AB is in this diagram?



5. A 5× 5 square and a 3 × 3 square can be cut into pieces that will fit together to form a third square.
   a. In the diagram at right, mark *P* on segment *DC* so that *PD* = 3, then draw segments *PA* and *PF*. Calculate the lengths of these segments.
   b. Segments *PA* and *PF* divide the squares into pieces. Arrange the pieces to form the third square.



6. Find the slope of the line through
   (a) (1, 0) and (2,-4) **(b)** (3, 1) and (3 + 4*t*, 1 + 3*t*); **(c)** (*m*− 5, *n*) and (5 + *m*, n^2).

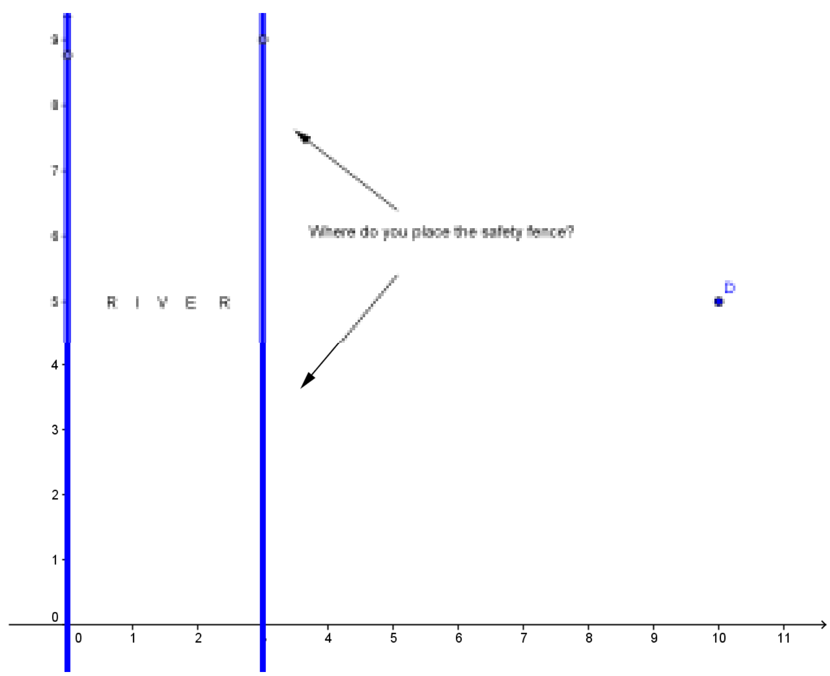
7. Consider the squares again - Change the sizes of the squares to *AD* = 8 and *EF* = 4, and redraw the diagram. Where should point *P* be marked this time? Form the third square again.

8. (Continuation) Will the preceding method *always* produce pieces that form a new square? If your answer is *yes*, prepare a written explanation. If your answer is *no*, provide a counterexample – two specific squares that can *not* be converted to a single square.

9. The main use of the Pythagorean Theorem is to find distances. Originally (6*th* century BC), however, it was regarded as a statement about *areas*. Explain this interpretation.

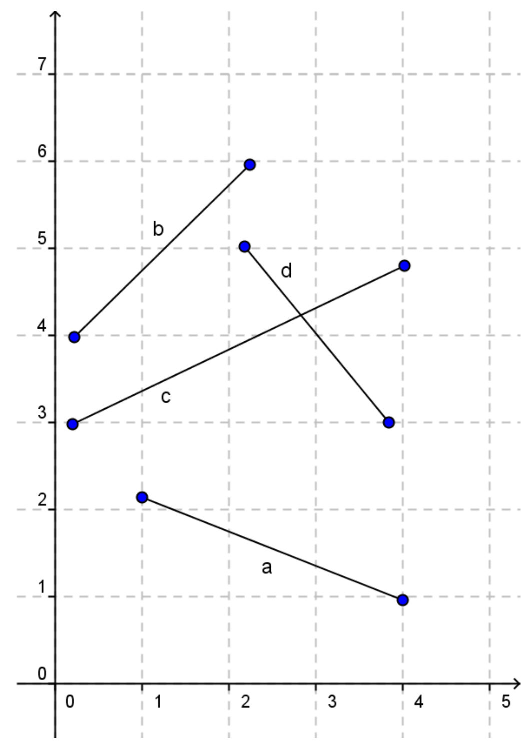
10. Draw the segment from (3, 1) to (5, 6), and the segment from (0, 5) to (2, 0). Calculate their slopes. You should notice that the segments are equally steep, and yet they differ in a significant way. Do your slope calculations reflect this difference?

11. A river runs along the line $x = 3$ and a dog is tied to post at the point D=(10,5). If the dog's leash is 25 units long (the same units as the coordinates), if a fence were going to be placed at the edge of the river along $x = 3$, name the two coordinates along the river where it would be safe for the fence to end so that the dog could not fall in the river.



12. The general notation in geometry is that points are labeled with capital letters and coordinates are defined with lowercase letters. Given the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ what do the subscripts on *x* and *y* represent? If triangle ABC is right triangle with C being the right angle, find expressions for all three sides using the distance formula.

13. A slope can be considered to be a *rate*. Explain this interpretation and give an example.



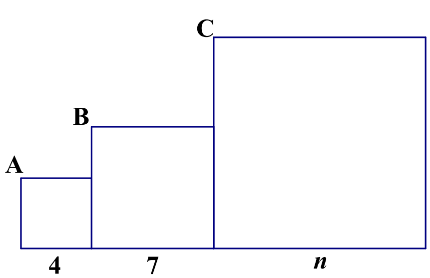
14. *Some terminology*: In a right triangle, the *legs* are the sides adjacent to the right angle. The *hypotenuse* is the side opposite to the right angle. Given the two points A(3, 7) and B(5, 2) find C so that triangle ABC is a right triangle with the right angle at C. How long are legs? How long is the hypotenuse?

15. Estimate the slopes of all the segments in the diagram at the right. Identify those whose slopes are negative. Find words to characterize lines that have negative slopes.

16. Explain the difference between a line that has *undefined slope* and a line whose slope is zero. What does it mean for the slope to be undefined?

17. Two different points on the line *y* = 2 are each exactly 13 units from the point (7, 14). Draw a picture of this situation, and then find the coordinates of these points.

18. Find the slope of the line containing the points (4, 7) and (6, 11). Find coordinates for another point that lies on the same line and be prepared to discuss the method you used to find them.



19. Three square buildings are built directly next to each other (as shown) in a city. The vertices *A*, *B*, and *C* are *collinear*. *Collinear* means that those three points are on the same line. Find the dimension *n*, the width of the third building.

20. (Continuation) Replace the lengths 4 and 7 by *m* and *k*, respectively. Express *k* in terms of *m* and *n*.

21. Given the two points A(-2, 1) and B(4, 7) describe two different methods to find the distance between A and B. Which method do you prefer?

22. A five-foot tall student casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

23. (Continuation) How far from the streetlight should the student stand in order to cast a shadow that is exactly as long as the student is tall?

24. How would you proceed if you were asked to verify that P = (1, -1) is the same distance from A = (5, 1) as it is from B = (-1, 3)? It is customary to say that P is *equidistant* from A and B. Find three more points that are equidistant from A and B. By the way, to "find" a point means to find its *coordinates*. Can points equidistant from A and B be found in every *quadrant*?

25. Two groups of students from different classes are playing Capture the Flag on the playground at recess. One group's home base is at A (7, 2) and the other group's home base is at B (1, 6). Help them decide where to place the flag so that the game is fair and the both have to travel the same distance to get it.

26. Using GeoGebra, plot the points P (3, 5), Q (0, 0) and R(-5, 3). Measure angle PQR, being careful to select the points in a clockwise manner. Create the segments PQ and QR. Use the Slope tool in the same toolbox as the Angle tool to find the slope of segment PQ. Do the same thing for segment QR. Make a conjecture about how these slopes are related. Verify by calculating the slopes by hand.

27. An airplane 27000 feet above the ground begins descending at the rate of 1500 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?

28. Let $P = (x, y)$ and Q = (1, 5). Write an equation that states that the slope of line PQ is 3. Show how this slope equation can be rewritten in the form $y-5 = 3(x-1)$. This linear equation is said to be in *point-slope form*. Explain the terminology. Find coordinates for three different points P that fit this equation.

29. Find a way to show that points A = (-4, -1), B = (4, 3), and C = (8, 5) are collinear.

30. Consider the linear equation $y = 3.5(x - 1.3) + 2$.
    a. What is the slope of this line?
    b. What is the value of y when $x = 1.3$?
    c. This equation is written in *point-slope* form. Explain the terminology.
    d. Use your calculator or GeoGebra to graph this line.
    e. Find an equation for the line through (4.2, -2.5) that is parallel to this line. Leave your answer in point-slope form.
    f. Describe how you would graph by hand a line that has slope -2 and that goes through the point (-7, 3).

31. An airplane is flying at 36000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.