Motivational Problems on Graphs of Trigonometric Functions

1. Devon's bike has wheels that are 27 inches in diameter. After the front wheel picks up a tack, Devon rolls another 100 feet and stops. How far above the ground is the tack?

2. (Continuation) How many degrees does the wheel turn for each foot that it rolls?

3. Avery is riding a Ferris wheel that turns once every 24 seconds, and whose radius is 8 meters. The function $h(t) = 9 - 8\cos(15t)$ describes Avery's distance from the ground (in meters) after t seconds of riding. For example, $h(8) = 13$ means that Avery is 13 meters above the ground after 8 seconds of riding. By the way, "*h* of 8" or "*h* at 8" are two common ways to read $h(8)$.

(a) Evaluate $h(0)$, and explain its significance.
(b) Explain why $h(16) = h(8)$.
(c) Find a value for t that fits the equation $h(t) = 10$. Interpret this t-value in the story.
(d) Explain why $h(t+24) = h(t)$ is true, no matter what value t has.
(e) What is the complete range of values that $h(t)$ can have?

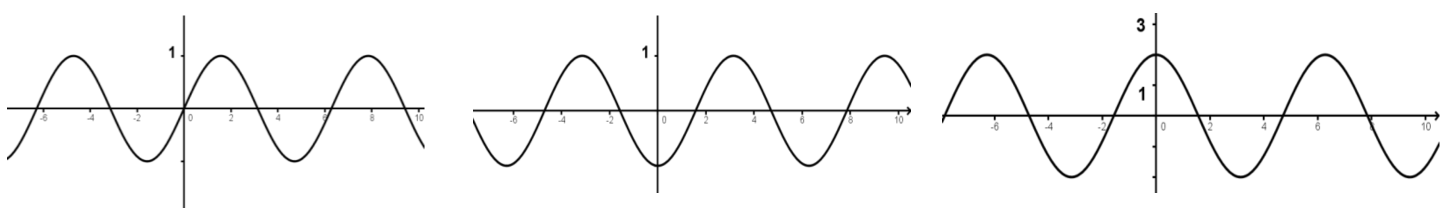
4. The function $f(x) = \tan bx$ has period of 4. Write an equation of the function.

5. What is the amplitude, period, domain, and range of $f(x) = 2 - 7\sin 45x$?

6. Without using a calculator, simplify the following (in degrees):

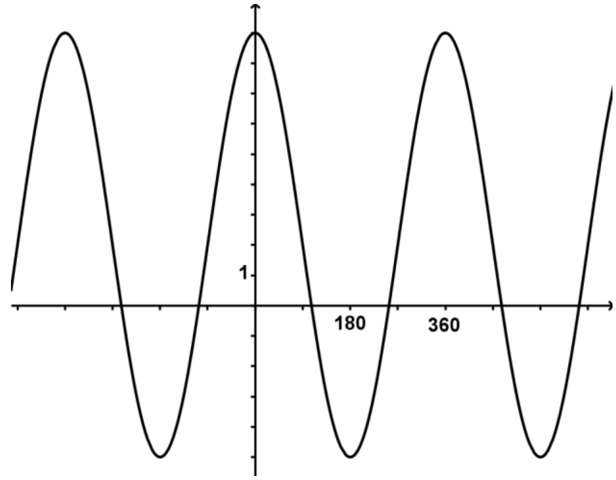
(a) $\sin\left(\sin^{-1} 0.32\right)$ (b) $\tan\left(\tan^{-1} 1.61\right)$ (c) $\cos^{-1}\left(\cos 225^\circ\right)$ (d) $\sin^{-1}\left(\sin 120^\circ\right)$

7. The sinusoidal (shaped like a sine or cosine curve) graphs shown below appeared on a calculator that was operating in radian mode. Find equations that might have produced the graphs. The period of each curve is 2π.



8. Draw the angle described by $\tan^{-1}\left(\dfrac{12}{5}\right)$.

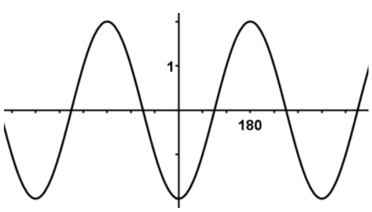
9. If the cosine of an acute angle is some number k, then what is the *sine* of the same angle? In terms of k, what is the cosecant of this angle? Simplify the expression $\tan\left(\cos^{-1} k\right)$.

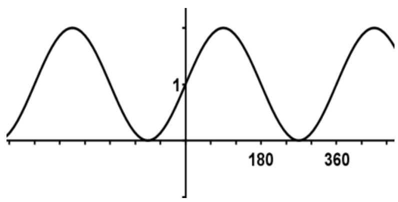


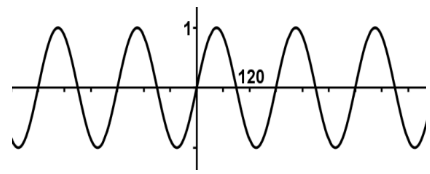
10. The equation whose graph is shown at right has the form $y = k + a\cos x$. Working in degree mode, find believable values for the coefficients *a* and *k*, and explain how these numbers affect the appearance of the graph.

11. An object moves around $x^2 + y^2 = 25$, which represents a circle whose radius is 5 meters, at a constant speed. At time *t* = 0 seconds, the object is at $(5,0)$. When *t* = 1, it is at $(4,3)$. Where is the object when *t* = 2? When *t* = 3? When *t* = *n?* What is the object's speed? At what time does the object return to $(5,0)$?

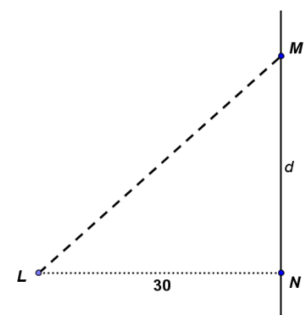
12. Working in degree mode, find plausible equation for each of the sinusoidal graphs below:







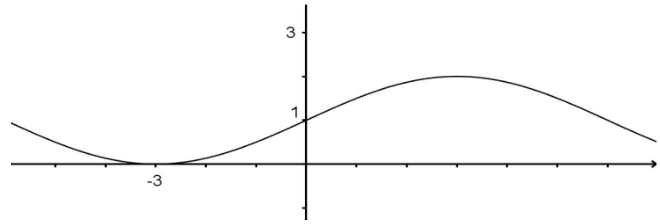
13. A prison guard tower is 30 feet from the nearest wall of the prison. The diagram shows this arrangement from above, as if the viewer were in a helicopter. The spotlight *L* on top of the tower rotates counterclockwise, once every six seconds, casting a moving beam of light onto the wall. Let *N* be the point on the wall that is nearest the spotlight. Let *M* be the moving spot. Let *d* be the distance from *N* to *M*, and let *t* be the time, in seconds, since *M* last passed *N*. Find *d* when *t* = 0.00, *t* = 0.30, *t* = 0.75, and *t* = 1.49. Are *d* and *t* related linearly? What does the graph of this relationship look like?



14. Tarzan is swinging back and forth on his grapevine. As he swing, he goes back and forth across the riverbank, going alternately over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the nmber of meters Tarzan is from the riverbank. Assume that y varies sinuisoidally with t, and that y is positive when Tarzan is over water and negative when he is over land. Jane finds that whne t=2, Tarzan is at one end of his swing, where y=-23. She find when t=5, he reaches the other end of his swing and y=17.
    a. Sketch a graph
    b. Write an equation expressing Tarzan's distance from the riverbank in terms of t
    c. Predict y when t=2.8 and t=15
    d. Where was Tarzan when Jane started the stopwatch?

15. What is the relationship between the graph of $y=\sin x$ and the graph of $y=\sin\left(x+90^\circ\right)$? Work in degree mode.

16. The graph to the right touches the *x*-axis at -3. Write an equation that could have produced it. Does it affect your answer whether you choose to work in degree mode or radian mode?



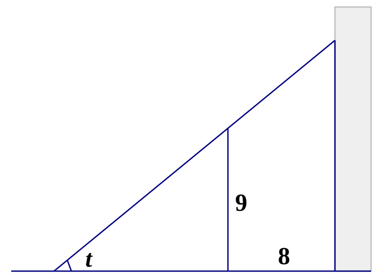
17. Verify that $1-2\sin^2 x = 2\cos^2 x - 1$ by manipulating one side of the equation to look like the other side.

18. A fence that is 9 feet tall is situated 8 feet from the side of a tall building. As the figure at right shows, a ladder is leaning against the building, with its base outside the fence. It so happens that the ladder is touching the top of the fence. Find the length of the ladder, given that

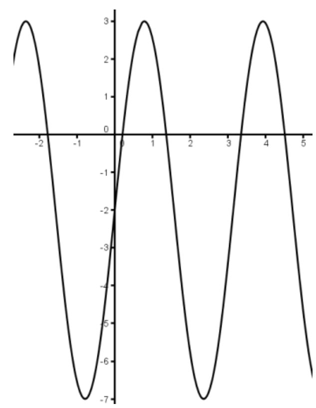
(a) it makes a 60 degree angle with the ground

(b) it makes a *t* degree angle with the ground.

(c) Apply your calculator to the answer for part (b) to find the length of the shortest ladder that reaches the building from outside the fence.

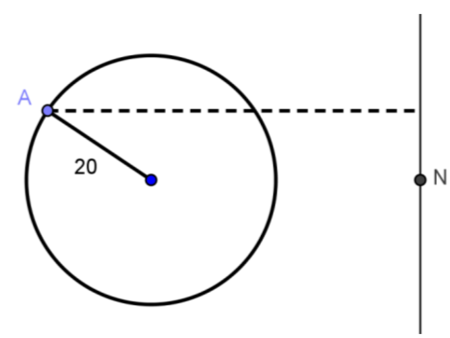


19. The equation whose graph is shown at right has the form $y=k+a\sin 2x$. To three decimal places, one of the *x*-intercepts is 1.365. Working in radian mode, find values for the other *x*-intercepts shown in the figure.

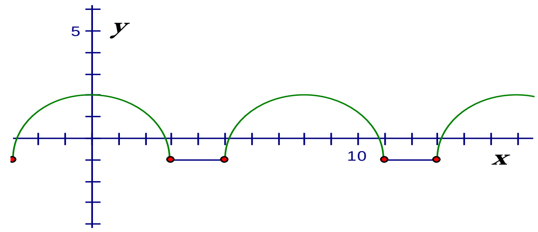


20. Write an equation of the form $y=k+a\sin bx$ for a curve that has a maximum point at $(30,6)$ and that has 2 as its *y*-intercept. Use a calculator to check that the graph of your equation fits the given description. Because you have the freedom to work in either degree mode or radian mode, and because there are many curves that fit the given description, there are many correct answers to this question. Can you find another one?

21. Illuminated by the rays of the setting Sun, Andy rides alone on a merry-go-round, casting a moving shadow on a wall. The merry-go-round is turning 40 degrees per second. As the top view shows, Andy is 20 feet from its center, and the Sun's rays are perpendicular to the wall. Let *N* be the point on the wall that is closest to the merry-go-round. Write an equation that represents the shadow's distance from *N* as a function of time.



22. The figure at right shows the graph of a periodic function $y = f(x)$. The graph, whose period is 8, is built from segments and semicircular arcs. Notice the values $f(3) = -1$ and $f(5) = -1$.



(a) Calculate $f(11)$, $f(13)$, $f(16)$ and $f(19)$.

(b) What does the graph of $y = f(x-3)-1$ look like?

23. Write a sine function whose period is 10 and whose values oscillate between -15 and 3.

24. Researchers find a creature from an alien planet. Its body temperature varies sinusoidally with time. 35 minutes after they start timing, it reaches a high of 120 degrees F. 20 minutes after that it reaches its next low, 104 degrees F.

    a. Sketch a graph.
    b. Write an equation express body temperature in terms of minutes since the beginning of the timing.
    c. What was its temperature when they first started timing?
    d. Find the first three times after they started timing at which the temperature was 114 degrees F.