Motivational Problems on Chapter 4 – Functions

1. In Algebra II, you defined domain and range. **Domain** was known to be the set of possible x values that could be used as input for an equation. The **Range** was defined as the set of possible y values that could be given as output of a function. In the following table of known functions state the domain and range as you understand them:

Function	Domain	Range
$y = x^2$		
$y = 5x + 2$		
$y = \sqrt{x+4}$		
$y = \sqrt{x} + 4$		
$y = \sin(x)$		
$y = \tan(x)$		
$y = \frac{2}{x-3}$		

2. *Function notation* – It is customary to refer to a function by a name, usually a letter like $f, g,$ or h. The name of the function is followed by the independent variable in parenthesis. For example, a function who's *independent variable* is x, and the name is f would be called $f(x)$ (said "f of x"). What would you call a function who's independent variable is t and the name is d? It is also customary to use $f(x)$ as another name for the *dependent variable* y, since that is what comes out of the function (that is what is defined by the function). So for the first function in the table above $y = x^2$, we could also say that $f(x) = x^2$, but this way we know that this one is called $f(x)$ and we won't confuse it with another function called $g(x) = 5x + 2$, since they all equal the y coordinates of the given function.

3. *The definition of a function* - an equation is said to be a function if for "every value in the domain there is only one value in the range". In other words, there can't be two points with the same x coordinate and different y coordinates. Draw an example of a graph of an equation that is not a function. (You don't necessarily have to have an equation for it.) Now give an example of something in real life that is a function and is not a function.

4. On the same system of coordinate axes, sketch both of the graphs $y = \cos(x)$ and $y = \sec(x)$. Explain why the secant graph has vertical asymptotes. What is the domain of secant? What is its range?

5. At a nearby video rental store, the annual dues are $15 and you pay $1.50 per video you rent. Write function notation for the total cost in terms of how many videos you buy.

6. Is the graph of $f(x) = x$ considered a function? Why or why not?

7. Explain how a rule called the "Vertical Line Test" is useful in determining whether a graph of an equation is for a function or not. (Relate the name of the test to the definition in #3).

8. Functions have operations like numbers have operations. Addition, Subtraction, Multiplication and Division are all operations that are defined (with certain restrictions) on functions. Let f(x)=cos(x) and g(x)=sin(x). Write functions for the Sum, Difference, product and quotient of these two functions. Make a sketch of the graph you see on your calculator. Do these graphs have anything in common? Any differences?

9. Given the conversion equation from Fahrenheit to Celsius $F = \frac{9}{5}C + 32$ find a formula that converts Celsius to Fahrenheit. Graph these two on your calculator and describe their graphical relationship.

10. Given $f(x) = \sqrt{2x}$ and $g(x) = x - 5$. What is the domain of $f(g(x))$? What is the domain of $g(f(x))$? Comment on any problems that arise.

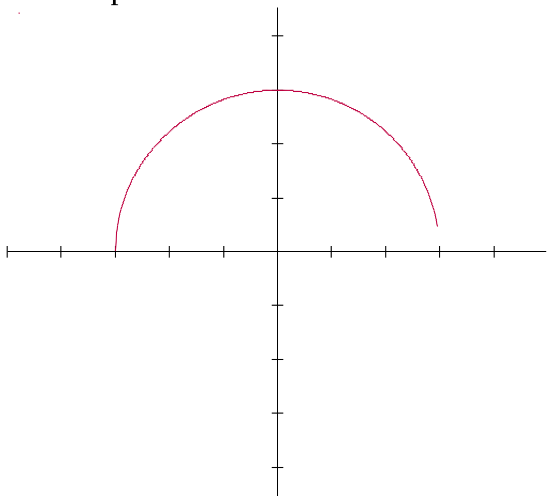
11. Sketch the graph of $f(x) = \sqrt{9 - x^2}$ for $-3 \le x \le 3$, then compare that graph with the graph of the following related functions. Be prepared to discuss the role of the constant "2" in each example. In particular, how does it affect the domain and range?

a. $y = 2\sqrt{9 - x^2}$ b. $y = \sqrt{(9 - (2x)^2)}$ c. $y = \sqrt{9 - x^2} + 2$ d. $f(x) = \sqrt{(9 - (x+2)^2)}$

12. What if $f(x) = \sin(x)$.
For each of the following expression write the expression with f(x) substituted in it. Graph each of the following with this definition of f(x) in mind.

a. $y = 2f(x)$ b. $y = f(2x)$ c. $y = f(x) + 2$ d. $y = f(x+2)$

13. The graph of $y = g(x)$ for $-3 \le x \le 3$ is shown below. Sketch the graph of each of the following related functions. Be prepared to discuss the role of the paramter 2 for each function. Write the domain and range of each expression.



a. $y = 2g(x)$ b. $y = g(2x)$ c. $y = g(x) + 2$ d. $y = g(x+2)$

14. Suppose that a function f has the property that $f(-x) = f(x)$ for all values of x. What does this tell us about the appearance of the graph of $y = f(x)$? Show that the function $f(x) = |x|$ has this property. Such a function is called an **even function**. Give 2 other examples.

15. Suppose that function f has the property that $f(x+6) = f(x)$ What geometric translation does the graph of f have? Give an example of such a function.

16. Suppose that a function f has the property that $f(-x) = -f(x)$ for all values of x. What does this tell us about the appearance of the graph of $y = f(x)$? Show that the function $f(x) = \sin x$ has this property. Such a function is called an **odd function**. Give 2 other examples.

17. You have met identities such as $f(-x) = f(x)$, $f(180 - x) = f(x)$, $f(-x) = -f(x)$, each of which describes a symmetry of the graph of y = f(x). Write an identity that says that the graph of y=f(x) a) has period 72; b)has reflective symmetry in the line x=72

18. Does the graph of $y = x^3 - x$ have half turn symmetry at the origin? Justify your answer.

11.The definition of an *Inverse Function* is as follows:
$f(x)$ and $g(x)$ are inverse functions if $g(f(x)) = f(g(x)) = x$ .Find the inverse function of $f(x) = \frac{x-4}{7}$. We denote the inverse function as $f^{-1}(x)$.

12. What is the difference between the functions $f(x) = (\sin x)^{-1}$ and $g(x) = \sin^{-1} x$? What are each of their domain and range?

13. In most mathematics books, the notation $\sin^2 A$ is often used in place of the clearer $(\sin A)^2$ or $\cos^3 B$ in place of $(\cos B)^3$. Why do you think that writers of mathematics fell into this strange habit? It is unfortunate that this notation is *inconsistent* with notation commonly used for inverse functions. Explain.

14. One famous pair of inverse functions are exponential functions and their related logarithmic function. The point (3,8) is on the graph of $y = 2^x$. What is the corresponding point on the graph of the inverse function $y = \log_2 x$? Give four other examples of this sort. What kind of symmetry does this imply between functions graphs and their inverses graphs?

15. We have agreed that in order for a relation to be a function it has to pass the Vertical Line Test. What kind of test could we perform to ensure that a function *has* an inverse? In other words, that its inverse would be a function?

16. A function that has an inverse function is called *one-to-one*. Give some examples of functions that are not one-to-one. There are many that we have studied.