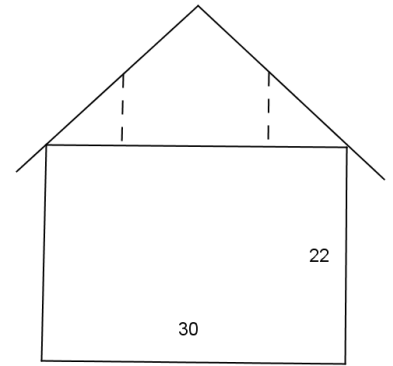


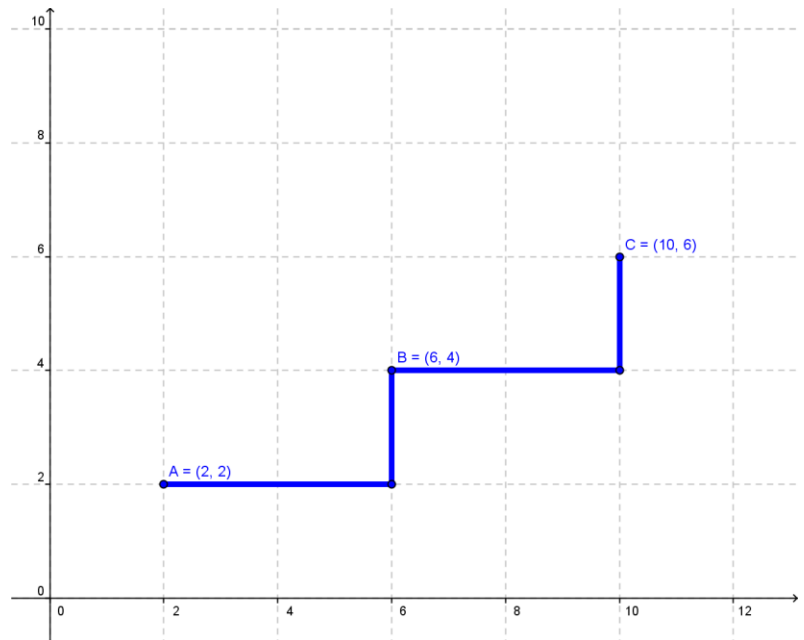
Motivational Problems on Slope, Distance and Lines
Schettino
Exeter Materials

1. A town's building code does not permit building a house that is more than 35 feet tall. An architect working on the design shown at right would like the roof to be sloped so that it rises 10 inches for each foot of horizontal length.
 - a. Given the other dimensions in the diagram, describe a way that the builder can build the house and stay within the town's building code?
 - b. Two vertical supports (shown dotted in the diagram) are to be placed 6 feet from the center of the building. How long should they be?



2. In 1986, renovations were made to the stairs in the Status of Liberty in an overall celebration of its bicentennial. Frank Schettino was the draftsman in charge of the calculations for the details of the structural steel for those stairs. He found that the stairs in the Lady's pedestal needed to have a vertical rise of 7.8 inches and a horizontal run of 9.75 inches. The more complicated spiral stairs the led up the body had a constant vertical rise of 9 inches and average horizontal run of 5.06 inches.
 - a. Using graph paper (or GeoGebra), create a graphical representation of these two sets of stairs.
 - b. Which flight of stairs do you think is steeper? Why?
 - c. Calculate the *rise / run* ratio for each flight and verify that the greater ratio belongs to the flight you thought was steeper.

3. (Continuation) A flight of stairs goes from point A(2,2) to B(6,4) to C(10,6) as shown in the diagram at the right. We define the *rise / run* ratio of the line AB made by the points on the stairs as the *slope* of the line. What's a way you would calculate that slope? What do you think the slope of the line AB is in this diagram?



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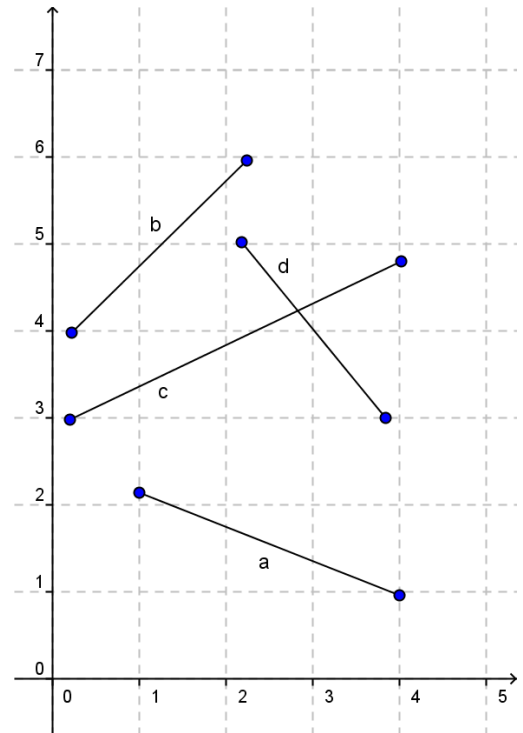
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4. Using GeoGebra, plot the points $P(3, 5)$, $Q(0, 0)$ and $R(-5, 3)$. Measure angle PQR , being careful to select the points in a clockwise manner. Create the segments PQ and QR . Use the Slope tool in the same toolbox as the Angle tool to find the slope of segment PQ . Do the same thing for segment QR . Make a conjecture about how these slopes are related. Verify by calculating the slopes by hand.

5. Find the slope of the line through
(a) $(3, 1)$ and $(3 + 4t, 1 + 3t)$; **(b)** $(m - 5, n)$ and $(5 + m, n^2)$.
6. Is it possible for a line $ax + by = c$ to lack a y -intercept? To lack an x -intercept? Explain.

7. Estimate the slopes of all the segments in the diagram at the right. Identify those whose slopes are negative. Find words to characterize lines that have negative slopes.



8. The sides of a triangle are formed by the graphs of $3x + 2y = 1$, $y = x - 2$ and $-4x + 9y = 22$. Use GeoGebra to discover if the triangle is isosceles. How do you know?

9. Consider the linear equation $y = 3.5(x - 1.3) + 2$. What is the slope of this line?

(b) What is the value of y when $x = 1.3$?

(c) This equation is written in *point-slope* form. Explain the terminology.

(d) Use your calculator or GeoGebra to graph this line.

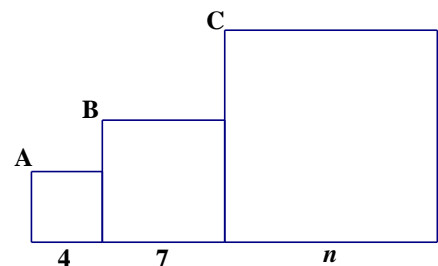
(e) Find an equation for the line through $(4.2, -2.5)$ that is parallel to this line. Leave your answer in point-slope form.

(f) Describe how you would graph by hand a line that has slope -2 and that goes through the point $(-7, 3)$.

10. A slope can be considered to be a *rate*. Explain this interpretation and give an example.

11. Explain the difference between a line that has *undefined slope* and a line whose slope is zero.

12. Three squares are placed next to each other as shown. The vertices A , B , and C are *collinear*. Find the dimension n .



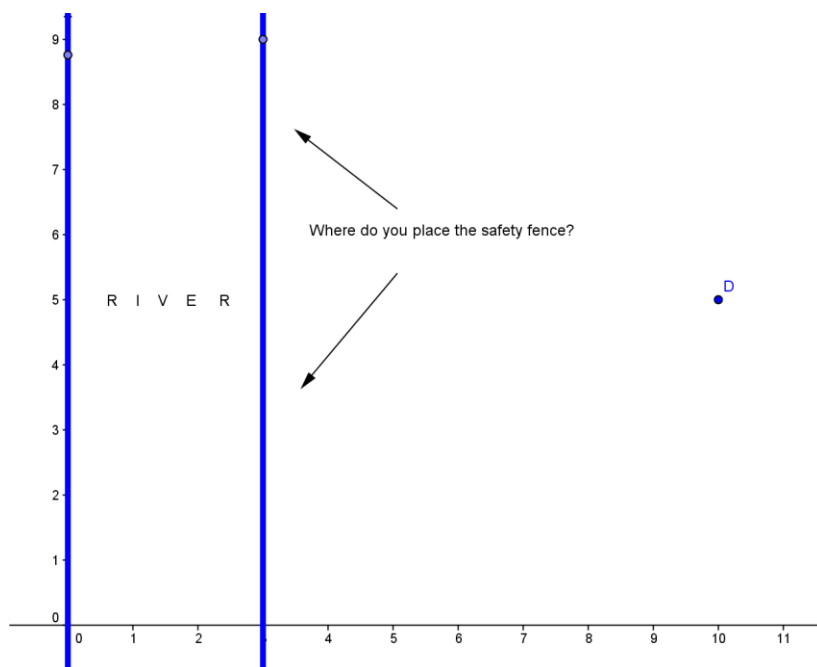
13. (Continuation) Replace the lengths 4 and 7 by m and k , respectively. Express k in terms of m and n .

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14. A five-foot tall Emma student casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?
15. (Continuation) How far from the streetlight should the student stand in order to cast a shadow that is exactly as long as the student is tall?
16. A river runs along the line $x = 3$ and a dog is tied to post at the point $D=(10,5)$. If the dog's leash is 25 units long (the same units as the coordinates), if a fence were going to be placed at the edge of the river along $x = 3$, name the two coordinates along the river where it would be safe for the fence to end so that the dog could not fall in the river.
- 17.
18. An airplane 27000 feet above the ground begins descending at the rate of 1500 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?
19. (Continuation) Graph the line $y = 27000 - 1500x$, using an appropriate window on your calculator. With the preceding problem in mind, explain the significance of the slope of this line and its two intercepts.
20. Find a way to show that points $A = (-4, -1)$, $B = (4, 3)$, and $C = (8, 5)$ are collinear.
21. An airplane is flying at 36000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.



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22. Golf balls cost \$0.90 each at Emma's Club, which has an annual \$25 membership fee. At Wendy & Marilyn's sporting goods store, the price is \$1.35 per ball for the same brand. Where you buy your golf balls depends on how many you wish to buy. Explain, and illustrate your reasoning by drawing a graph.
23. Given the line $y = \frac{3}{4}(x + 3) - 2$ and the point $(9, 2)$. Using point-slope form, write equations for the lines parallel and perpendicular to this line through the given point.
24. *Some terminology:* In a right triangle, the *legs* are the sides adjacent to the right angle. The *hypotenuse* is the side opposite to the right angle. Given the two points $A(3, 7)$ and $B(5, 2)$ find C so that triangle ABC is a right triangle with the right angle at C . How long are legs? How long is the hypotenuse?
25. Let $P = (a, b)$, $Q = (0, 0)$, and $R = (-b, a)$, where a and b are positive numbers. Prove that angle PQR is right, by introducing two congruent right triangles into your diagram. Verify that the slope of segment QP is the *negative reciprocal* of the slope of segment QR .
26. What is the slope of the line $ax + by = c$? Find an equation for the line through the origin that is perpendicular to the line $ax + by = c$.
27. Two different points on the line $y = 2$ are each exactly 13 units from the point $(7, 14)$. Draw a picture of this situation, and then find the coordinates of these points.
28. Given the points $A(1,3)$ and $C(5,7)$ find the coordinates of a point B that makes ABC a right triangle with B the right angle. What are the lengths of the sides of the triangle?
29. Can you describe a general method of finding the direct distance between two points in a coordinate plane? Why do you think this is referred to as the "Pythagorean distance"?
30. How would you proceed if you were asked to verify that $P = (1, -1)$ is the same distance from $A = (5, 1)$ as it is from $B = (-1, 3)$? It is customary to say that P is *equidistant* from A and B . Find three more points that are equidistant from A and B . By the way, to "find" a point means to find its *coordinates*. Can points equidistant from A and B be found in every *quadrant*?
31. Two groups of students from different classes are playing Capture the Flag on the playground at recess. One group's home base is at $A(7, 2)$ and the other group's home base is at $B(1, 6)$. Help them decide where to place the flag so that the game is fair and the both have to travel the same distance to get it.