1. Factor the following *perfect-square trinomials* :
(a) $x^2 - 12x + 36$ **(b)** $x^2 - 14x + 49$ **(c)** $x^2 + 20x + 100$
As suggested, these should all look like either $(x-r)^2$ or $(x+r)^2$. State the important connection between the *coefficients* of the given trinomials and the values you found for *r*.

2. (Continuation) In the following, choose *k* to create a perfect-square trinomial:
(a) $x^2 - 16x + k$ **(b)** $x^2 - 8x + k$ **(c)** $x^2 + 5x + k$ **(d)** $2x^2 + 16x + k$

3. As you may have learned in your Algebra I class, the *zero-product property* says that $a \bullet b = 0$ is true if $a = 0$ or $b = 0$ is true, and *only* if $a = 0$ or $b = 0$ is true. Explain this property in your own words (looking up the property if necessary). How is it necessary to solve quadratic equations?

4. The graph of a parabola crosses the x-axis at -3 and 0 and opens downwards. Sketch the graph of this parabola. It is said to have zeroes of -3 and 0. Write a possible equation for this parabola utilizing what you know about the zero product property. What is the possible parameter that could change the equation?

5. When an object falls, it gains speed. Thus the number of feet *d* the object has fallen is not linearly related to the number of seconds *t* spent falling. In fact, for objects falling near the surface of the Earth, with negligible resistance from the air, $d = 16t^2$. How many seconds would it take for a cannonball to reach the ground if it were dropped from the top of the Eiffel Tower, which is 984 feet tall? How many seconds would it take for the cannonball to reach the ground if it were dropped from a point that is halfway to the top?

6. If $\sqrt{2}$ can be expressed as a ratio $\frac{m}{n}$ of two whole numbers, then this fraction can be put in lowest terms. Assume that this has been done.
(a) Square both sides of the equation $\sqrt{2} = \frac{m}{n}$.
(b) Multiply both sides of the new equation by n^2. The resulting equation tells you that *m* must be an even number. Explain.
(c) Because *m* is even, its square is divisible by 4. Explain.
(d) It follows that n^2 is even, hence so is *n*. Explain.
(e) Thus both *m* and *n* are even. Explain why is this a contradictory situation.
A number expressible as a ratio of whole numbers is called *rational*. All other numbers, such as are $\sqrt{2}$ called *irrational*.

7. Sam was given a quadratic equation in factoring form as $(2x-3)(3x-4)=1$. How would she go about putting it in standard form as $ax^2+bx+c=0$ so that she could use the quadratic equation to solve it? Why can't she just use the zero product property to solve it if it is in a factored form as it is?

8. A worker accidentally drops a hammer from the scaffolding of a tall building. The worker is 300 feet above the ground. As you answer the following, recall that an object falls $16t^2$ feet in t seconds (assuming negligible air resistance).
(a) How far above the ground is the hammer after falling for one second? for two seconds? Write a formula that expresses the height h of the hammer after it has fallen for t seconds.
(b) How many seconds does it take the hammer to reach the ground? How many seconds does it take for the hammer to fall until it is 100 feet above the ground?
(c) By plotting some data points and connecting the dots, sketch a graph of h versus t. Notice that your graph is *not* a picture of the path followed by the falling hammer.

9. Using a driver on the 7*th* tee, Dale hits an excellent shot, right down the middle of the level fairway. The ball follows the parabolic path shown in the figure, described by the quadratic function $y = 0.5x - 0.002x^2$. This relates the height y of the ball above the ground to the ball's progress x down the fairway. Distances are measured in yards.
(a) Use the distributive property to write this equation in factored form. Notice that $y = 0$ when $x = 0$. What is the significance of this data?
(b) How far from the tee does the ball hit the ground?
(c) At what distance x does the ball reach the highest point of its arc? What is the maximal height attained by the ball?

10. Sketch the graphs of $y=x^2+5$, $y=x^2-4$, and $y=x^2-1$ on the same axes. What is the effect of the value of c in equations of the form $y=x^2+c$?

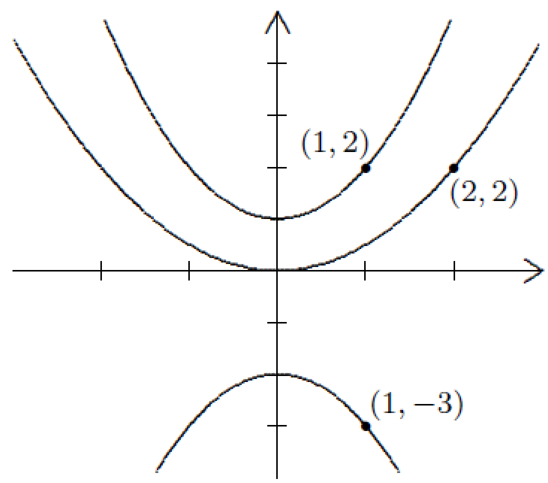
11. When asked to solve the equation $(x-3)^2=11$, Jess said, "That's easy — just take the square root of both sides." Perhaps Jess also remembered that 11 has two square roots, one positive and the other negative. What are the two values for x, in exact form? (In this situation, "exact" means no decimals.)

12. Alex was given the equation $x^2-8x+16=9$. Instead of subtracting 9 from both sides right away, Alex observed something special about the left hand side of the equation and realized there would be an easier way than factoring the equation. What could Alex do to solve this equation? Follow some steps that you think would leave to an exact answer.

13. (Continuation) When asked to solve the equation $x^2-6x=2$, Pat said, "Hmm. . .not so easy, but I think that adding something to both sides of the equation is the thing to do." This is indeed a good idea, but what number should Pat add to both sides? How is this equation related to the previous one? Continue with Pat's process that gives answers for x.

14. Jess, Pat and Alex realized they were onto something. "We hate factoring or using the quadratic formula," they agreed, " If we could just make a complete perfect square trinomial on the left side every time and make sure to add the right number to both sides, we'd be able to take the square root and get an answer." How would you advise these three budding mathematicians to secure the correct number to add to both sides in order to "complete the square"?

15. Solving a quadratic equation by rewriting the left side as a perfect-square trinomial is called solving by *completing the square*. Use this method to solve each of the following equations. Leave your answers in exact form.
(a) $x^2 - 8x = 3$ **(b)** $x^2 + 10x = 11$ **(c)** $x^2 - 5x - 2 = 0$ **(d)** $x^2 + 1.2x = .28$



16. Find a quadratic equation for each of the graphs pictured at the right. Each curve has a designated point on it, and the *y*-intercepts are all at integer values. Also notice that the *y*-axis is the axis of symmetry for all graphs.

17. With your graphing calculator, on the same set of axes, graph three quadratic equations, $y = x^2 - x$, $y = x^2 + 2x$ and $y = x^2 - 4x$. Make three observations about graphs of the form $y = x^2 + bx$ where b is a nonzero number.

18. A hose is used by the fire department shoots water out in a parabolic arc. Let x be the horizontal distance from the hose's nozzle and y be the corresponding height of the stream of water, both in feet. The quadratic function is $y = -.016x^2 + 0.5x + 4.5$

a. What is the significance of the 4.5 in the equation?
b. Use your graphing calculator to graph this parabola. What is the stream's greatest height?
c. What is the horizontal distance from the nozzle from where the stream hits the ground? Where is the maximum height of the stream in relation to the start and end points of the stream?
d. Will the stream travel over a six foot fence that is located 28 feet from the nozzle? Explain.

19. Tracey was given the graph of the parabola $y = x^2 + 10x + 23$ and was asked to think about how it was related to the parent graph of $y = x^2$. Tracey had just learned about graphical transformations in her Algebra class and wondered if the idea could be used to graph parabolas. Tracey said, "If I could just get it in that form of $a(x-d)^2 + b$ like when we learned the transformation, I could figure it out." What advice would you give Tracey?

20. Where is the vertex of a parabola in relation to the zeros? In relation to the axis of symmetry? In relation to the maximum and minimum?

21. How would you find the x-coordinate of the vertex of a parabola if you knew the two zeros of the graph? (Remember where it is and how you would find that point algebraically.)

22. (Continuation) Ayden was given the quadratic function $f(x) = 3x - 6x^2$. After factoring, Ayden said the zeros were at $x = 0$ and $x = \frac{1}{2}$. After doing the midpoint formula, however, Ayden thought the vertex should be at $x = -\frac{1}{2}$. What might have gone wrong?

23. Recall that when you use the quadratic formula for a quadratic equation you can get the two zeros given by the expressions $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$. Since the vertex is the midpoint of both of these zeros. Find the midpoint of x_1 and x_2 to come up with a general formula for the vertex of a parabola given in standard form $f(x) = ax^2 + bx + c$.

24. *The Quadratic Formula.* Now that you are pros at completing the square, we can even derive the quadratic formula with this method. We will begin with the standard form of the quadratic equation $y = ax^2 + bx + c$ and solve for x using completing the square.

a. Set this equation equal to zero as if you were solving for the zeros of the function.
b. Subtract *c* from both sides to isolate the constant *c*.
c. Divide everything by *a* so that you can complete the square.
d. Now complete the square. Be sure to add the square of half the coefficient of the *x* term to the $-\frac{c}{a}$ on the other side.
e. Factor the perfect square trinomial.
f. Combine the rational expressions on one side of the equation.
g. Take the square root of both sides of the equation.
h. Isolate the *x* because that's what you were solving for in the first place!

25. Rick is a student who dislikes factoring, using the quadratic formula, AND completing the square. "There must be a way to just use your graphing calculator to solve something like $2x^2 - 6x - 10 = 46$. That would be such a pain." Rick decided to graph the parabola $y = 2x^2 - 6x - 10$ and see where it intersected with the function that was always equal to 46. How did he do this?

26. Pat and Chris were given the equation $x^4 - 3x^2 + 2 = 0$ and Pat freaked out saying, "We've never done any equations with a power of four in it before, what are we supposed to do?" Chris said calmly, "Wait a minute; it's just a quadratic equation in disguise, silly." What did Chris mean and how did the pair solve this equation?

27. (Continuation) How could the equation $(2x-3)^2 - 5(2x-3) + 6 = 0$ be solved as a "quadratic equation in disguise?

28. (Continuation) What about $x^{\frac{1}{2}} - 3x^{\frac{1}{4}} + 2 = 0$?