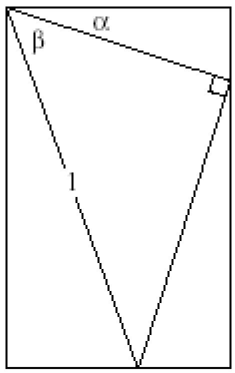
Motivational Problems on Trig Identities



1. The rectangle shown to the left has been formed by fitting together four right triangles. As marked, the sizes of two of the angles are α and β, and the length of one segment is 1. Find the two unmarked angles whose sizes are α and $\alpha+\beta$.

2. Once you know where $\alpha+\beta$ is in the picture, label the side of the rectangle that could be written as sin($\alpha+\beta$).

3. Consider the right triangle in the center with hypotenuse 1. How could you write expressions for the other two sides of that triangle?

4. (Continuation) Using those sides from #3, write expressions for the other parts of the rectangle.

5. Because rectangles have congruent opposite sides, come up with an equivalent expression for sin($\alpha+\beta$).

6. You have just found a trigonometric identity for the sine of a sum of two angles. When asked to find an expression that is equivalent to cos(a+b) , a student responded "cos a + cos b ". What do you think of this answer, and why?

7. Explain why the equation tanθ = 2 has solutions, but the equation sinθ = 2 does not.

8. Recall your work in problems 1-5. See if you can find an identity for the cos($\alpha+\beta$) using the same picture. Good Luck!

9. You need to find the sin (55) degrees. You just spilled soda on your calculator and the "4" and "5" buttons on your calculator are stuck. You can, however, find the sine and cosine of 10 degrees, and you know, from Algebra 2 Trig, the sin and cosine of 45. With just this information, how could you find the sin(55)?

10. If you are given that $\tan\theta = \frac{-3}{4}$, what are $\sin\theta$ and $\cos\theta$?

11. If α and β are angles in the first quadrant, draw a picture that represents the angle $\alpha-\beta$. If these angles represent angles for points on the unit circle, you now have a triangle with 2 sides that are one unit long and an angle in between them is $\alpha-\beta$. Using the law of Cosines, come up with an expression the distance AB (the third side of the triangle).

12. Let the point A =(cos α , sin α) and B=(cos β , sin β). Using the distance formula, find an expression for the distance AB. Notice that this distance should be the same as the third side of the triangle you came up with in #10.

13. You now have 2 expressions for the distance AB. Set them equal to each and solve for cos($\alpha-\beta$) to come up with an identity for cos($\alpha-\beta$).

14. You are given the expression $\sin\frac{5\pi}{12}\cos\frac{\pi}{12}+\cos\frac{5\pi}{12}\sin\frac{\pi}{12}$, and your calculator doesn't work. How could you still find the exact value for this expression?

15. Derive the identity for sin($\alpha-\beta$) yourself by substituting $-\beta$ in for β in the original identity for sin($\alpha+\beta$). You can do it!

16. You need to find the sin(-15) and don't have your calculator. How could you use one of the 4 trigonometric identities for sum or difference to find an exact value for sin(-15).

17. An isosceles triangle has two sides of length *w* that make a $2a$ -degree angle. Write down two different formulas for the *area* of this triangle, in terms of *w* and a . By equating the formulas, discover a relation involving *sin2a* , si*na* , and co*sa* .

18. Use the sum identity for cosine to derive the double angle identity for cosine. To do this, let $\cos(2\alpha)=\cos(\alpha+\alpha)$.

19. There are actually three useful forms of the double angle identity for cosine. Recall the Pythagorean identity $\sin^2\alpha+\cos^2\alpha=1$. How could you use these to substitute into the previous problem to obtain 2 more forms of the double angle identity for cosine.

20. Using the form of the double angle identity for cosine $\cos(2\alpha)=1-\sin^2\alpha$. Substitute $\alpha=\dfrac{x}{2}$ into this identity. With a little algebraic manipulation, you will get an expression for $\sin(\frac{x}{2})$.

21. Now use the other one, $\cos(2\alpha)=2\cos^2\alpha-1$ and again substitute $\alpha=\dfrac{x}{2}$. A little more algebra again, and you will get an expression for $\cos(\frac{x}{2})$.

22. Find an exact values for $\sin(\frac{\pi}{8})$ and $\cos(\frac{\pi}{8})$.