Motivational Problems on Counting

1. Suppose there are 23 people in attendance at a meeting. There are three door prizes to give away. The 23 people will put their names in a hat (one slip of paper per person). The names will be drawn out of the hat to award the prizes.

   a. This is a three step process. Each prize drawing will count as one step. Use the Multiplication Principle of Counting to determine the number of ways the door prizes can be awarded if it is possible for the same person to be awarded more than one prize? In this setting, once a name is pulled from the hat, it is replaced before the next drawing.

      # of ways to award Prize 1 = _____

      # of ways to award Prize 2 = _____

      # of ways to award Prize 3 = _____

      # of ways to award the three prizes = _____.

   b. The situation above is not typical. The typical situation is one in which a person's name is removed from the hat and <u>not</u> replaced once he/she has won a prize. How many ways are there to award the three door prizes in this setting?

      # of ways to award Prize 1 = _____

      # of ways to award Prize 2 = _____

      # of ways to award Prize 3 = _____

      # of ways to award the three prizes = _____.

   c. The situations in the parts a and b above are often categorized as selection "with replacement" and selection "without replacement". Describe in your own words what the main difference is in a counting problem where the selection is done without replacement.

2. Suppose you run a business, and you need to issue passwords to your customers, so they can access an online system. Your passwords will have the form of two upper case letters (chosen from the 26 letter alphabet) followed by three digits (chosen from values 0 through 9). Here's one example of a possible password: BZ022. Letter and digit choices can be repeated.

   a. How many steps are involved in the task of selecting a password?

b. List the steps and the number of choices for each.
c. Use the Multiplication Principle of Counting to determine how many passwords can be made using this format.
d. If your company has more than 1 million customers, will you have enough passwords using the format above? If not, suggest a change in the format of the passwords that would generate enough possibilities to cover 1 million or more customers? How many passwords can be generated using your new suggested format?

3. Six friends (Andrew, Barb, Cami, David, Erin, and Fran) are about to occupy six seats in a movie theater row. The task of seating the group is a six-step task.

    a. If one wishes to count the number of ways the friends can occupy the seats. Would this be a problem counted "with replacement" or "without replacement"? Explain.
    b. Use the Multiplication Principle of Counting to find out how many ways the six friends can occupy the six seats? Show your thought process.
    c. If Barb absolutely insists on occupying seat #1, how many ways can the seating be accomplished? Show your thought process.
    d. Barb is still determined to sit in seat #1. Tension develops and David refuses to sit next to Barb. How many ways can the friends be seated given these requirements? Show your thought process.

4. If you have a group of n objects, what is the total number of ways you can arrange the whole group? For example, if I have 5 books on my shelf, how many different ways can I arrange those books on the shelf? What if I have n books?

5. You get the chance to create your own pizza. You must first choose the size, (small, medium or large). Then you must choose the crust (thin or thick). Finally, you must choose only one topping. The topping choices are (pepperoni, sausage, anchovies, mushrooms, peppers, and black olives).

    a. How many different pizzas can you order?
    b. Great news!! You get two toppings. If you choose two <u>different</u> toppings then how many different pizzas can you order?
    c. Suppose the second topping could be the same as your first (for example you could order double anchovies). How many different pizzas are there now?

6. There are 15 people in a class and the teacher needs to choose three to be on a special project together. How many different groups can the teacher make?
    a. Use the Multiplication Principle of Counting to do this problem in one way. What do you have to take into consideration in order to make sure the count is accurate?

b. Another way to look at this problem is to think about the original groups as *permutations* of the class. The group would be a permutation (mixture or grouping) of 15 students taken 3 at a time. (Usually written ${}_{15}P_3$). In this case, if I had three students, A, B and C the group ABC would be the same group as ACB but they would be different permutations of each other.

c. However, if we really wanted the number of <u>different</u> groups of three students we would need to eliminate all of those repeated permutations of the same kids. How many repeated permutations are there of three students?

d. We say that the number ${}_{15}C_3$ represents the number of Combinations of 3 students chosen from 15 when the order of the group is NOT accounted for. So in other words, we have eliminated the repeated groups that are the same. How do you think you would calculate this? Try to come up with a calculation.

7. Read the following two problems. Do not try to solve them. Which problem would be appropriate for the Multiplication Principle of Counting? Which problem would be over counted by the Multiplication Principle of Counting? Explain your choice.

Problem 1: A combination lock has whole numbers 0 through 39. A combination to unlock this lock consists of three numbers dialed in a specific order. Numbers may be reused. How many combinations are there?

Problem 2: A lottery drawing is performed by selecting one ball from each of three containers. There are ten balls in each container labeled with digits 0 through 9. The order in which the balls are does not matter. (This means 9, 1, 3 is the same as 3, 1, 9 or 1, 3, 9, etc). How many <u>different</u> three-ball drawings are possible?

8. A 5 member team consists of Bill, Ted, Juan, Shaq, and Kobe. It is necessary to choose 2 different members to go to a special camp. You need to find out how many possible pairs there are to send to camp.

a. You decide to approach this problem as a two step process. First, you will choose one of the guys, and then you will choose a different guy. Apply the Multiplication Principle of Counting in this manner to see how many ways there are to choose one person followed by a different person to go to the camp.

b. List out all the possibilities counted in part a.

c. Looking at your list, you can see that the number of pairs to send to camp has been over counted. How many different pairs of players are there to send to camp?

d. What situation was present in this problem that caused the Multiplication Principle of Counting to over count the possible outcomes?