Motivational Problems on Exponential and Logarithmic Functions
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Exeter/EWS Materials

1. Is the graph of $f(x) = b^x$ one-to-one? What are the restrictions on *b*? Explain.

2. (Continuation) Find the inverse of $g(x) = 3^x$ .Are you able to solve for *y* in terms of *x*?

3. (Continuation) Explain the phrase *y=the power of b that produces x*. How does this relate to your expression for the inverse function?

4. Find the exponent that gives you the answer when the base is raised to that power in each row of the table below:

Base (b)	Answer Produced (x)	Exponent(y)
3	27	
10	1000	
2	1/8	
25	5	
½	1/16	

5. John Napier (1550-1617) created the notation $\log_b x$ to replace the phrase *the power of b that produces x.* $\log_b x$ is read as "the logarithm (or log) base *b* of *x*." How would you write "4 is the power of 2 that produces 16?"

6. Rewrite the equation $\log_3 81 = 4$ as an exponential equation.

7. Write each equation in its exponential form.

(a) $\log_4 \frac{1}{16} = -2$ **(b)** $\log_2 16 = 4$ **(c)** $\log_2 8 = x$ **(d)** $\log_5 1 = x$

8. What is the inverse of $2^y = x$? Do you recognize this function? Can you graph it by reflecting it across the line y=x?

9. Express $3^2 = 9$ as a logarithm.

10. Change each exponential equation into a logarithmic form.

a. $5^{-2} = \frac{1}{25}$ b. $7^0 = 1$ c. $2^4 = 16$ d. $6^1 = 6$

11. If the logarithmic function $y = \log_b x$ is the inverse function of the exponential function $y = b^x$, what is the domain and range of $y = \log_b x$?

12. Explain my mantra "the answer is the exponent" when thinking about the equation $\log_b x$?

13. Given a positive number p, the solution to $10^x = p$ is called the *base-10 logarithm of* p, expressed as $x = \log_{10} p$, or simply $x = \log p$ (the 10 is implied when no base is written). For example, $10^4 = 10000$ means that 4 is the base-10 logarithm of 10000, or 4 = log 10000. The LOG function on your calculator provides immediate access to such numerical information. Using your calculator for confirmation, and remembering that *logarithms are exponents*, explain why it is predictable that
    (a) log 64 is three times log 4;
    (b) log 12 is the sum of log 3 and log 4;
    (c) log 0.02 and log 50 differ only in sign.

14. There is a LOG button on your calculator. Try LOG 3, LOG .001, LOG 100, LOG 10. Explain the meaning of this button. What assumption does your calculator make when evaluating the log?

15. Sketch the graph of $f(x) = \log_4 x$ by hand. Be sure to include at least 3 coordinate pairs. What is the domain and range?

16. Consider the graph of $f(x) = \log_4 x$ a "parent function" that you now know. You can graph transformation of this parent function just like others. Attempt to graph the following functions and then check with your calculator:

    a. $f(x) = \log_4(x+1)$
    b. $g(x) = \log_4 x - 5$
    c. $h(x) = \frac{1}{2}\log_4(x-2)$
    d. $k(x) = 6 - \log_4 x$

17. Without using your calculator, solve each of the following equations. Explain why they all have the same answer.

a. $8^x = 32$ b. $27^x = 243$ c. $1000^x = 100{,}000$

18. *What if the base of an exponential equation isn't 10* ? One way of solving an equation like $1.02^x = 3$ is to use your calculator's LOG function to rewrite the equation in the form $(10^{0.0086})^x = 10^{0.4771}$. First justify why these are the same equation, then solve $(10^{0.0086})^x = 10^{0.4771}$.

19. Rewriting 1.02 as $10^{0.0086}$ is the same as finding what logarithm? Rewriting 3 as $10^{0.4771}$ is the same as find what other logarithm?

20. How might you find $\log_5 8$ using only the common logarithm (or LOG function on your calculator which is base 10)?

21. Given that $10^{0.301} = 2$ and $10^{0.477} = 3$, solve without a calculator:

    a. $10^x = 6$ b. $10^x = 8$ c. $10^x = \frac{2}{3}$ d. $10^x = 1$

22. Give that $0.301 = \log 2$ and $0.477 = \log 3$, you should not need a calculator to evaluate

    a. $\log 6$ b. $\log 8$ c. $\log \frac{2}{3}$ d. $\log 1$

23. Given that $m = \log a$, $n = \log b$, and $k = \log(ab)$
    a. Express a, b, and ab as powers of 10
    b. Use your knowledge of exponents to discover a relationship among m, n, and k
    c. Conclude that $\log(ab) = \log a + \log b$
    d. Find a way to show that $\log(a^r) = r \log a$ where r is some exponent
    e. Similar to part c, Find a way to show that $\log\left(\frac{a}{b}\right) = \log a - \log b$

24. Another approach to solving an equation like $5^x = 20$ is to calculate base-10 logarithms of both sides of the equation. Justify the equation $x \log 5 = \log 20$, then obtain the answer in the form $x = \frac{\log 20}{\log 5}$. Evaluate the expression. Note that $\log_5 20 = \frac{\log 20}{\log 5}$.

25. Write an expression for $\log_b N$ that refers only to base-10 logarithms and explain.

26. Asked to simplify $\frac{\log 20}{\log 5}$, Brett replied "$\log 4$". What do you think of this answer?