1. Starting at the same spot on a circular track that is 80 meters in diameter, Hayley and Kendall run in opposite directions, at 300 meters per minute and 240 meters per minute, respectively. They run for 50 minutes. What distance separates Hayley and Kendall when they finish? There is more than one way to interpret the word *distance* in this question.

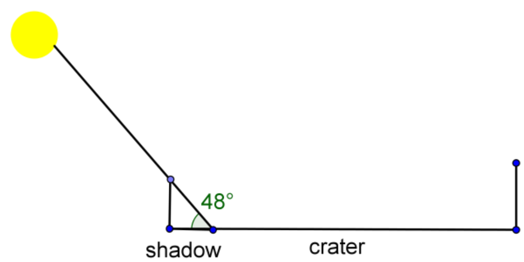
2. Find the side lengths of a square whose diagonal is length 1.

3. Find the diagonal of a square whose area is 10.

4. Commercial jets usually land using a 3 degree angle of descent. At what ground distance from the destination airport should the pilot begin the descent, if the cruising altitude of the jet is 37000 feet?

5. State the side ratios of the special right triangles, 30-60-90 and 45-45-90.

6. Scientists can estimate the depth of craters on the moon by studying the length of the shadow formed by the crater's wall. This shadow falls on the floor of the crater. Shadows' lengths can be estimated by measuring them on photographs. If the sun had an angle of elevation of 48 degrees and the shadow was estimated to be 400 meters, how deep was the crater?




7. State the exact value of the sine, cosine, and tangent of
   **(a)** 30 degrees **(b)** 60 degrees **(c)** 45 degrees

8. Write the components of a vector that is 8 units long and that makes a 30 degree angle with the x-axis.

9. The reciprocal of the sine is *cosecant* (abbreviated csc), useful for expressing answers to trigonometry problems without using a division sign. Use this function to express the length of the hypotenuse of a right triangle that has a 12.8 inch side opposite a 25 degree angle.

10. (Continuation) The *secant* is the reciprocal of the cosine: $\sec t = \dfrac{1}{\cos t}$. Use this function to express the length of the hypotenuse of the triangle from the previous problem.

11. Place the tail of the vector $\left[5, 5\sqrt{3}\right]$ at the origin. What is the angle formed with the x-axis? Hint: This is a special right triangle.

12. Without your calculator find:
    **(a)** $\sin 45^\circ \sin 30^\circ$ **(b)** $\cos 30^\circ + \cos 45^\circ$
    **(c)** $\tan 45^\circ \left(\cos 60^\circ + \tan 30^\circ\right)$

13. Its center at $O = (0,0)$, the *unit circle* $x^2 + y^2 = 1$ goes through $P = (1,0)$. The line $y$ = 0.6 intersects the circle at *A* and *B*, with *A* in the first quadrant. The angles *POA* and *POB* are said to be in *standard position*, because their *initial ray OP* points in the positive *x*-direction. (Their *terminal rays* are *OA* and *OB*.) Find the sizes of these angles. How are they related?

*14.* (Continuation) If we restrict ourselves to a single revolution, there are actually *two* angles in standard position that could be named *POB*. The one determined by minor arc *PB* is said to be *positive*, because it opens in the counterclockwise direction. Find its degree measure. The one determined by major arc *PB* is said to be *negative*, because it opens in the clockwise direction. Find its degree measure.

15. Place the tail of the vector $[-7,-7]$ at the origin.
    a. What is the acute angle formed with the x-axis?
    b. Going counter-clockwise, what is the angle formed with the *positive* x-axis?

16. Your clock is broken and the hour hand is stuck on the 3. If the minute hand is on the 10, what is angle formed by the hands?

17. (Continuation) Assuming now that your minute hand works, how many degrees does it rotate through when moving from the 3 to the 10? In what direction does it travel? How does your answer relate to the previous problem?

18. The point B is $(6,0)$ and A is $(0,0)$. Find a point P in the first and fourth quadrants that make angle PAB 60 degrees.

19. A *reference angle* is the positive, acute angle formed by the terminal side of the angle and the x-axis. Find the reference angles created by the following coordinates:
    **(a)** $(-3,2)$ **(b)** $(4,-3)$ **(c)** $(-2,-2)$

20. Barbara and Sue are contra dancing. They both start at $(5,0)$ and travel around in a circle, centered at the origin, going opposite directions and end at $(-4,-3)$. Barbara promenades clockwise and Sue promenades counterclockwise. Write an angle that describes each dancer's motion.

21. Gerry, Sandy, and Pat all start out at $(0,0)$ to explore a beach. Gerry goes 20 meters east and 10 meters north, Sandy goes 15 meters west and 45 meters south, and Pat goes 30

meters east and 23 meters south. Describe each of their positions relative to the positive x-axis using a positive angle, a negative angle, and a reference angle.

22. A merry-go-round in a playground has a radius of $\sqrt{13}$. You start just to the right of $\left(\sqrt{13},0\right)$, so that you are standing next to the merry-go-round, and your friend is at $(-3,2)$. Your friend wants to slap your hand each time as you go around, in a counterclockwise direction. How many degrees have you rotated at the moment to give your third high-five?

23. Explain the similarities and differences between positive and negative angles.

24. Take vector $\left[4,-4\sqrt{3}\right]$:
    a. This vector can be the radius of a circle centered at the origin. What is the radius length?
    b. Scale this vector so that the radius is 1. Recall that a vector of length 1 is called a *unit vector*. What do you think you can call this circle?
    c. Name three other vectors whose tails are at the origin and whose head is on the unit circle such that their reference angle is 60 degrees.

25. Name the four unit vectors whose reference angles are 45 degrees. What do you notice about their components?

26. Choose a positive number $\theta$ (*Greek* "theta") less than 90° and ask your calculator for sin $\theta$ and cos $\theta$. Square these numbers and add them. Could you have predicted the sum?

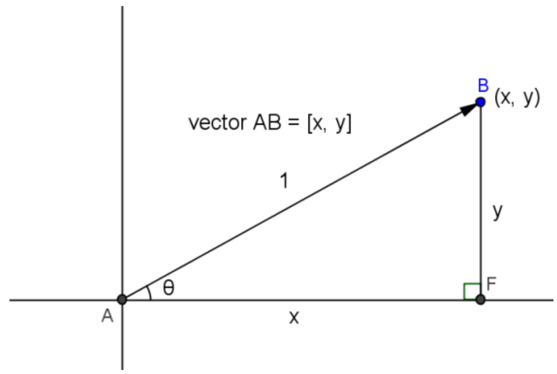
27. The point $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ lies on the unit circle.
    a. Name its angle and reference angle.
    b. Name another angle greater than 360 degrees that would also describe its position.
    c. Name the coordinates of the point on the unit circle in the second quadrant that have the same reference angle.

28. (Continuation) What is the cosine of 60 degrees? What is the sine of 60 degrees? How do the cosine and sine of 60 degrees relate to the coordinates of the point in the previous problem?

29. Determine whether the following points are on the circle $x^2 + y^2 = 1$

**(a)** $(0,1)$ **(b)** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ **(c)** $\left(\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right)$

30. A circular saw blade with a 5 cm radius is mounted so that one half of it shows above the table and is rotating at one degree per second. One tooth of the saw has been painted red. This tooth starts at $(5,0)$, what is the height of the saw tooth after 30 seconds? After 90 seconds? After 420 seconds?




31. Given the vector $AB = [x, y]$ with length 1 that makes angle $\theta$ with the x-axis, how do the coordinates of point B relate to the cosine and sine of angle $\theta$?

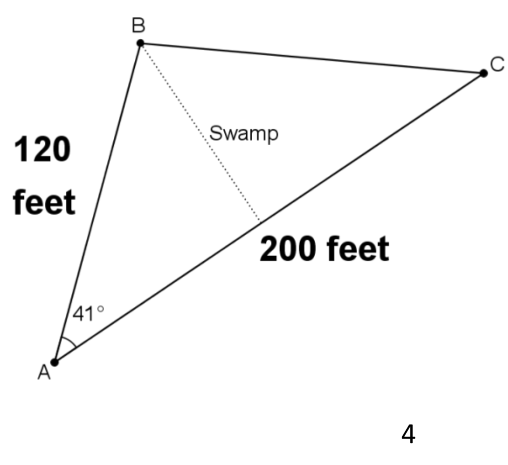
32. Although it may seem like a strange request, ask your calculator for sine and cosine values for a 120 degree angle. Try to make sense of the answers.

33. Two observers who are 5 km apart simultaneously sight a small airplane flying between them. One observer measures a 51 degree inclination angle, while the other measures a 40.5 degree inclination angle. At what altitude is the airplane flying? Include a diagram with your solution.

34. We have seen that when a vector of length 1 is in the first quadrant the cosine of $\theta$ is the x coordinate and the sine of $\theta$ is the y coordinate. What happens to the coordinates as the vector is placed in each of the quadrants?

35. Given that we have already learned that the tangent of $\theta$ is the slope of the line that forms the angle of θ with the x-axis, write an expression for tangent of $\theta$ in terms of $\cos\theta$ and $\sin\theta$ and in terms of x and y.

36. A triangular plot of land has the SAS description indicated in the figure shown at right. The SAS description means you are given two sides and the angle between them. Although a swamp in the middle of the plot makes it awkward to measure the altitude that is dotted in the diagram, it can at least be calculated. Show how. Then use your answer to find the area of triangle ABC, to the nearest square foot.



37. MacKenzie is on a Ferris wheel that is two decameters in radius. This Ferris wheel loads passengers at its lowest point, 1 m off the ground. What is MacKenzie's height off of the ground after the wheel has gone 120 degrees from where the ride began?
38. (Continuation) The Ferris wheel has rotated a total of five and a half times from the beginning of the ride and MacKenzie is now at the top of the wheel and wonders how many degrees total the wheel has rotated. Find the total number of degrees rotated.

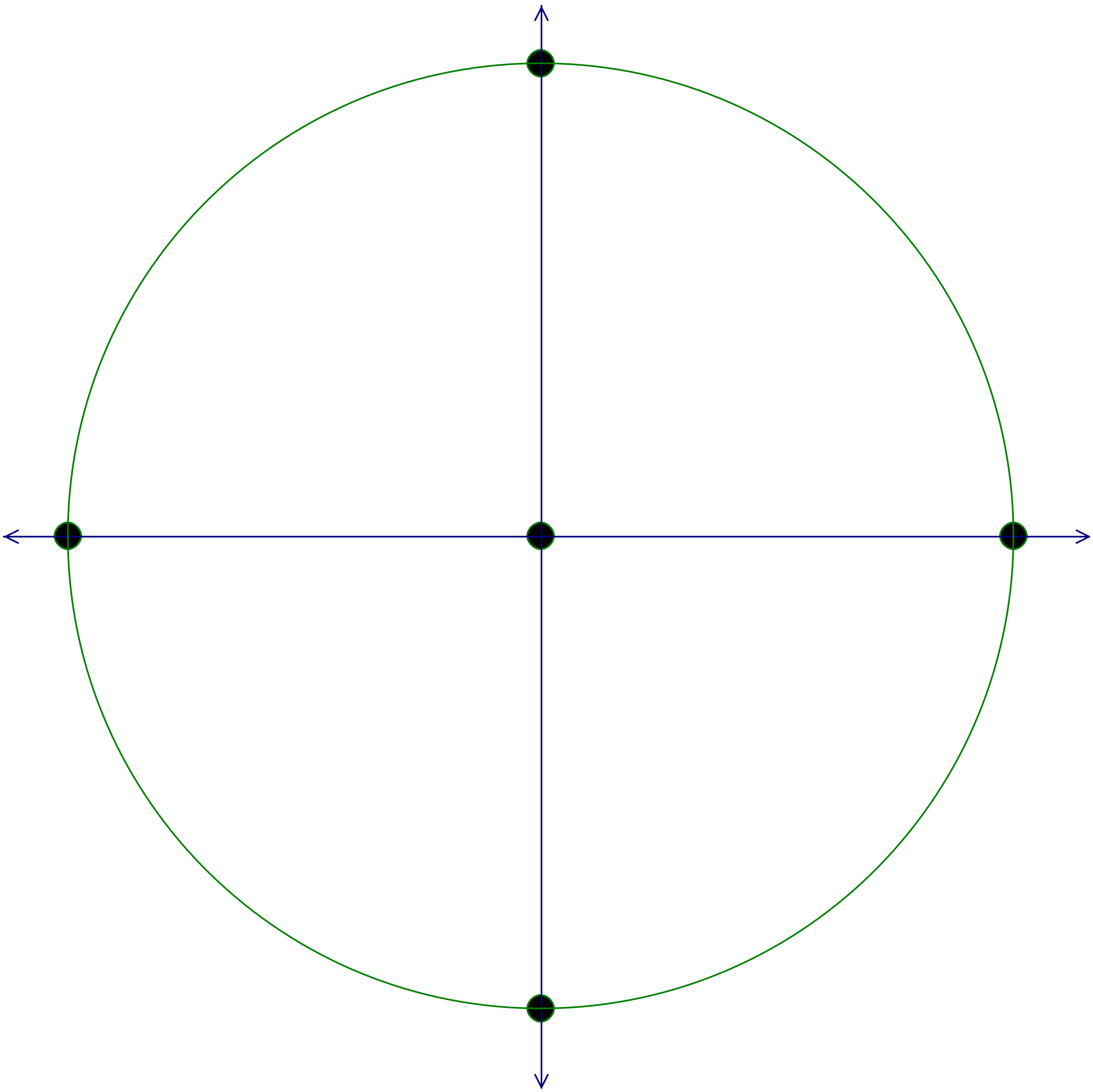
**Paper Folding Lab (Use Paper on next page)**

I. Fold your circle to create 45 degree angles – use the large dots to guide your folds so that you are folding a dot onto another dot. Trace your folds (radii) using a colored pen or pencil.

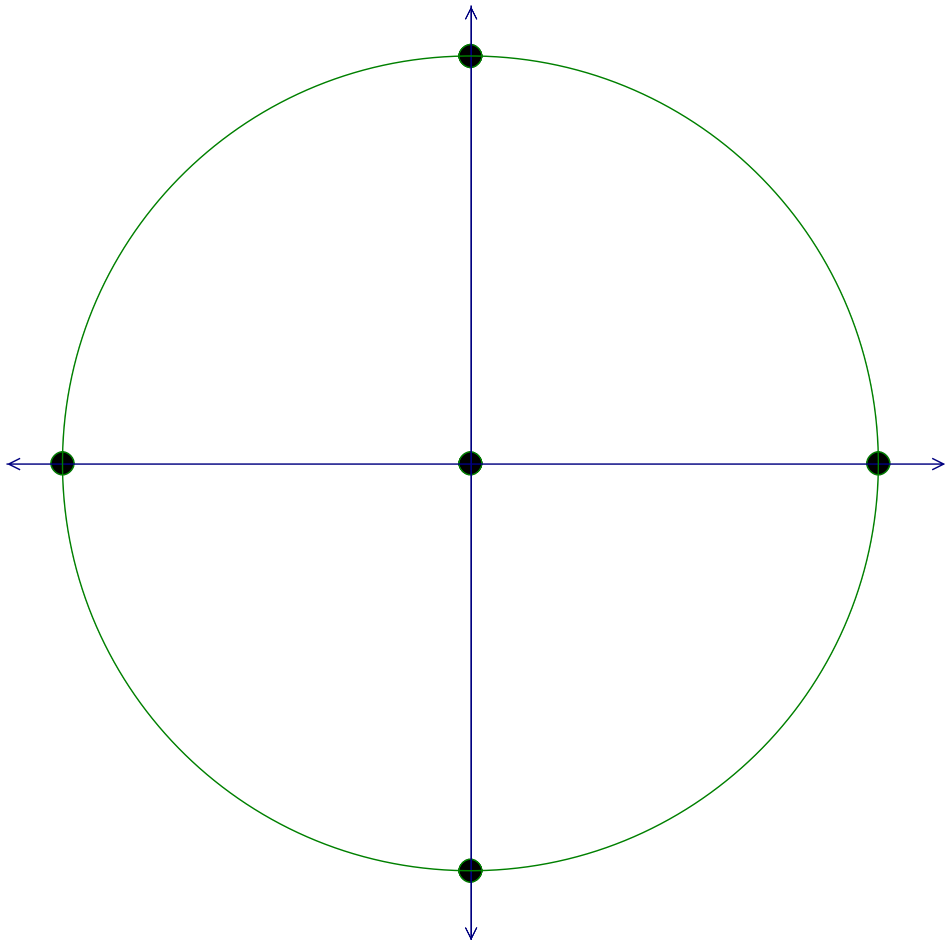
II. Fold your circle again, dot onto dot, this time to find the four 30 degree reference angles. You may want to consider the side ratios of a 30-60-90 triangle to help you make the folds. Again, the large dots will be useful, and you should use the dot at the origin somehow. Draw in the four radii using a different color.

III. Fold your circle a third time, dot onto dot – to find the 60 degree reference angles. Draw in the four radii using a third color.

IV. Label the angles and coordinates of each point on the unit circle using the same color pen as the radii, or use a pencil and circle with the corresponding color.



Motivational Problems on Trigonometric Ratios on Coordinate Axes



1. For the special right triangles on the unit circle:

   a. All short legs have length _______

   b. All 45-45-90 degree triangles have legs of length _______

   c. All long legs have length _______

   d. The hypotenuse always has length _______

2. The reference angle associated with — Degrees

   a. a long horizontal leg is _______

   b. a short vertical leg is _______

c. an isosceles right triangle is always _______

d. a short horizontal leg is _______

e. a long vertical leg is _______

3. List the $(x, y)$ and angle coordinate of the point on the Unit Circle that corresponds to:

| | $(x, y)$ | Positive Angle |
|---|---|---|
| a. a long horizontal leg and short vertical leg in the first quadrant | ________ | ________ |
| b. a long horizontal leg and short vertical leg in the second quadrant | _______ | _______ |
| c. a long horizontal leg and short vertical leg in the third quadrant | _______ | _______ |
| d. a long horizontal leg and short vertical leg in the fourth quadrant | _______ | _______ |
| e. equal length legs in the first quadrant | _______ | _______ |
| f. equal length legs in the second quadrant | _______ | _______ |
| g. equal length legs in the third quadrant | _______ | _______ |

h. equal length legs in the fourth quadrant _______ _______

i. a short horizontal leg and long vertical leg in the second quadrant _______ _______

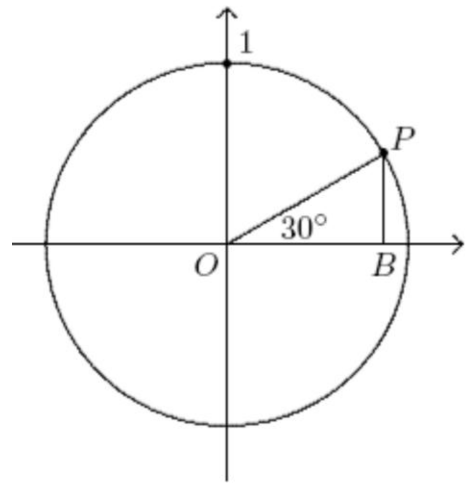
j. a short horizontal leg and long vertical leg in the third quadrant _______ _______

k. a short horizontal leg and long vertical leg in the fourth quadrant _______ _______

1. Evaluate:
   **(a)** $\sin 150^\circ$ **(b)** $\tan 300^\circ$
2. Find at least two values for $\theta$ that fit the equation $\sin\theta = \frac{\sqrt{3}}{2}$. How many such values are there?
3. The *Quadrantal Angles* on the unit circle are the angles that correspond to the points that are the intersection points of the unit circle and the coordinate axes. State and justify the sine, cosine and tangent of:
   **(a)** 0 degrees **(b)** 90 degrees, **(c)** 180 degrees, **(d)** 270 degrees, **(e)** 360 degrees
4. A wheel of radius one foot is placed so that its center is at the origin, and a paint spot on the rim is at $(1,0)$. The wheel is spun $\theta$ degrees in a counterclockwise direction. Now what are the coordinates of the paint spot, in terms of $\theta$?

5. Choose an angle $\theta$ and calculate $(\cos\theta)^2+(\sin\theta)^2$. Repeat with several other values of $\theta$. Explain the coincident results. It is customary to write $\cos^2\theta+\sin^2\theta$ instead of $(\cos\theta)^2+(\sin\theta)^2$.

6. Consider the unit circle, whose center is the origin *O* and whose radius is 1. Suppose that *P* is on the circle in the first quadrant, and that the angle formed by segment *OP* and the positive *x*-axis is 30 degrees. Mark *B* on the *x*-axis so that angle *OBP* is right. Find *PB* and *OB*, expressed in exact form. What are the *x*- and *y*-coordinates of *P*? Express the *x*- and *y*-coordinates of *P* using trigonometric functions of a 30 degree angle.




7. State the quadrant in which $\theta$ lies:
**(a)** $\sin\theta>0$ and $\cos\theta<0$ **(b)** $\tan\theta>0$ and $\cos\theta<0$

**(c)** $\sin\theta<0$ and $\cos\theta>0$

8. In most mathematics books, the notation $\sin^2 A$ is often used in place of the clearer $(\sin A)^2$, or $\cos^3 B$ in place of $(\cos B)^3$. Why do you think that writers of mathematics fell into this strange habit? It is unfortunate that this notation is *inconsistent* with notation commonly used for inverse functions. Explain.

9. Write without parentheses:
**(a)** $(xy)^2$ **(b)** $(x+y)^2$ **(c)** $(a\sin B)^2$ **(d)** $(a+\sin B)^2$

10. Use the unit circle to find $\sin 240^\circ$ and $\cos 240^\circ$, without using a calculator. Then use your calculator to check your answers. Notice that your calculator expects you to put parentheses around the $240^\circ$, which is because sin and cos are functions. Except in cases where the parentheses are required for clarity, they are often left out.

11. Given the following information,

**(a)** $\sin\theta=-\dfrac{1}{2}$ and $\cos\theta>0$. Find $\tan\theta$.

**(b)** $\tan\theta=1$ and $\sin\theta<0$. Find $\cos\theta$.

**(c)** $\cos\theta=-\dfrac{1}{2}$ and $\tan\theta<0$. Find $\sin\theta$.

**(d)** $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$. Find $\cos\theta$.

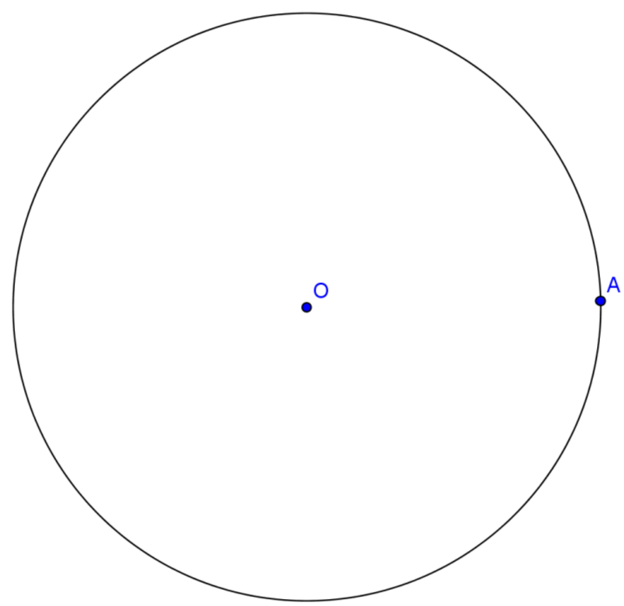
12. Quinn is running around the circular track $x^2 + y^2 = 10000$, whose radius is 100 meters. Quinn starts at the point $(100, 0)$ and runs in the counterclockwise direction for 1605 degrees. What are Quinn's coordinates?

13. If sin *A* is known to be 0.96, then what can be said about cos *A*? What if it is also known that *A* is an obtuse angle?

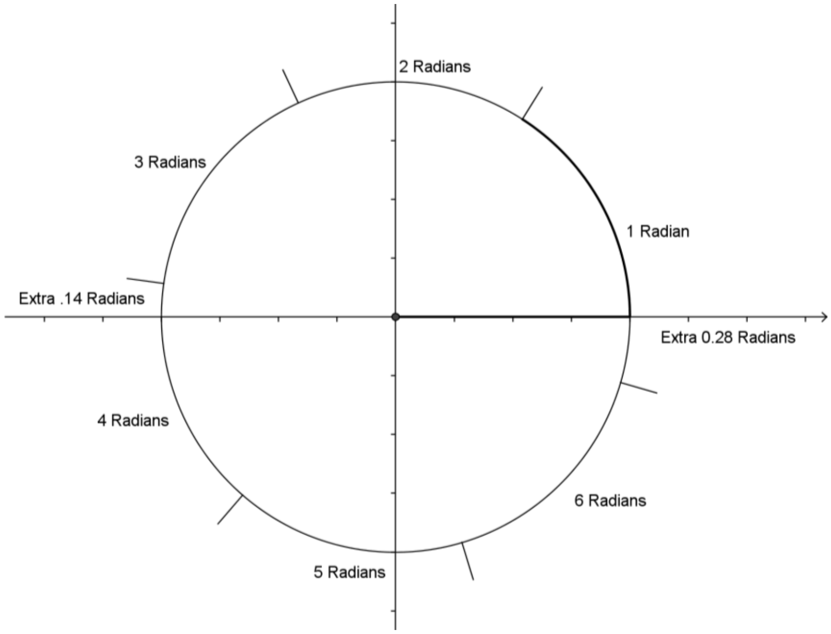
14. Find the exact value (no decimals) of each expression.
**(a)** $\sin 225^\circ$ **(b)** $\tan 330^\circ$ **(c)** $\cos 585^\circ$ **(d)** $\sin 510^\circ$ **(e)** $\cos 765^\circ$ **(f)** $\tan 1485^\circ$

15. Evaluate the following expressions. Please give an exact answer for each.
**(a)** $\sin(-180^\circ)$ **(b)** $\tan(-210^\circ)$ **(c)** $\cos(-570^\circ)$

16. The circle shown at right is centered at *O*. Use your headphone wire (or other appropriate measuring device) to find a point *B* on this circle for which minor arc *AB* has the *same length* as radius *OA*. Draw radius *OB* and use a protractor to measure the size of angle *AOB*. Your answer should be close to 60 degrees. By considering triangle *AOB* and the relation between the arc *AB* and its chord, explain why angle *AOB* must in fact be smaller than 60 degrees.
Angle *AOB* is an example of a *radian* – a central angle whose arc has the same length as the radius of the circle in which it is drawn.




17. How many 1-radian arcs does it take to fill a complete circle? First make an estimate using the headphone wire approach, then look for a theoretically exact answer. Do any of your answers depend on the size of the circle used?




18. Considering the formula for the circumference of a circle, how does this prove that there are $2\pi$ radians in the unit circle? Is this true for any circle?

19. (Continuation) How many radians are there in half a circle, i.e. 180 degrees = ________ radians. 90 degrees = __________ radians. Justify your answers.

20. (Continuation) 60 degrees is what fraction of half a circle? What is 60 degrees equivalent to in radians? 30 degrees? 45 degrees?

21. A 6-inch arc is drawn using a 4-inch radius. Describe the angular size of the arc **(a)** using radians; **(b)** using degrees

22. A 2.5-radian arc is drawn using a 6-inch radius. How long is the arc?

23. Find all solutions *t* between 360 and 720 degrees, inclusive:
**(a)** $\cos t = \sin t$ **(b)** $\tan t = -4.3315$ **(c)** $\sin t = -0.9397$

24. Given that $\tan\theta = 2.4$, with $180^\circ < \theta < 270^\circ$, find the values of $\sin\theta$ and $\cos\theta$. Are your answers *rational numbers*?

25. Find the following:

**(a)** $\sin\left(\frac{\pi}{3}\right)$ **(b)** $\tan\left(\frac{\pi}{4}\right)$ **(c)** $\cos\left(\frac{\pi}{6}\right)$

26. If $\csc A = \frac{5}{3}$, then what can be said about $\tan A$? What if I know that A is an obtuse angle?

27. Give an exact answer for each of the following:

**(a)** $\cos\frac{\pi}{2}$ **(b)** $\sin\frac{11\pi}{6}$ **(c)** $\sin\frac{4\pi}{3}$