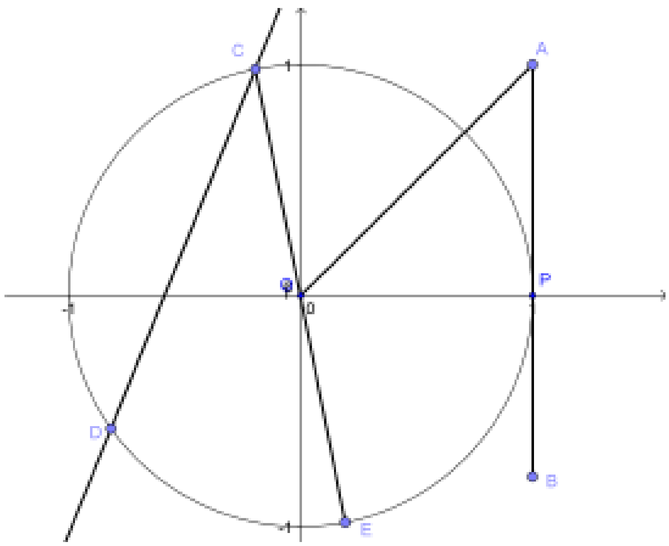
Motivational Problems on Circles

1. In the following diagram choose which you think fulfills the definitions of the terms that relate to a circle from your previous experience with circles, common sense or resources that you have - *radius, diameter, secant line, chord, tangent line, point of tangency, minor arc, major arc, central angle*



2. It is interesting that the line that intersects a circle at one point is named a tangent line and that is also the name of the function that mathematicians created to relate an angle to the slope created with the positive x-axis. Consider the diagram above and the triangle APO. Can you give an argument for why triangle APO must be a right triangle if AP is a tangent line? What is the relationship between $\angle AOP$ and the length of AP (to make the question easier, assume the radius of the circle is 1, as in the picture). Why do you think they named AP a tangent line?

3. If the central angle of one slice of 36 degrees, how many slices of pizza are there? If the pizza is round and the radius of the pie is 12 inches, how long would one crust be approximately? Explain your answer.

4. The vertices of a triangle are (-5,-12), (5,-12) and (5,12). Confirm that the *circumcenter* (the intersection of the perpendicular bisectors of the sides of the triangle) is the origin (0,0). Write the equation of the *circumscribed circle* (the circle that goes through all three vertices of the triangle.

5. A regular polygon is *inscribed* in a circle – all vertices of the polygon lie on the circle. (We can also say that the circle is *circumscribed* around the polygon). Go to the website http://www.mathopenref.com/polygoncircumcircle.html and increase the number of sides of the regular polygon. What happens to the polygon as the number of sides increases?

6. Draw a circle and several chords of that circle that are parallel. What do you think is true of the midpoints of all such chords?

7. An equilateral triangle ABC is inscribed in a circle centered at O. The portion of the circle that lies above chord AB is named arc AB ($\overset{\frown}{AB}$). Since ABC is equilateral, is would seem that $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{AC}$. If this is true, would do you conjecture is the measure of $\overset{\frown}{AB}$? Why? How are $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$ related to each other?

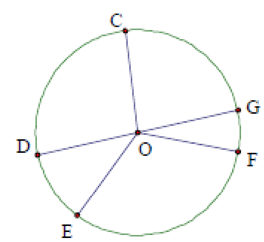
8. (continuation) Since a central angle is an angel whose vertex is at the center of a circle and whose sides are radii, what is the measure of $\angle AOB$? What do you conjecture is the relationship between a central angle and the angular size of the arc it intercepts?

9. What is the angular size of the arc that the diameter intercepts? What is the special name for this arc?

10. A regular hexagon has an inscribed circle of radius 4. Find the area of the hexagon.

11. Draw a circle and label a diameter AB. Choose any other point on the circle and call it C. What do you think you can say about the size of angle ACB? Verify your conjecture using GeoGebra. Construct a circle and then construct an angle whose endpoints are a diameter. Move the vertex around the circle and see if your guess was correct.

12. Circle *O* has diameter *DG* and central angles *COG* = 86, *DOE* = 25 and *FOG* = 15 . Find the angular size of minor arcs $\overset{\frown}{CG}, \overset{\frown}{CF}, \overset{\frown}{EF}$ and major arc $\overset{\frown}{DGF}$.



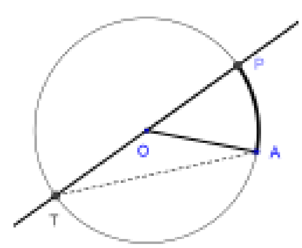
13. If two chords in the same circle have the same length, then their minor arcs will have the same length too. True or false? Explain. What about the converse of this statement?

14. A circle of radius 5 is circumscribed around a regular hexagon. Find the area of the hexagon.

15. The circle $x^2 + y^2 = 25$ goes through A=(5,0) and B=(3,4). The nearest tenth of a degree, find the angular size of the minor arc $\overset{\frown}{AB}$.

16. At what time during the day do the hands of an analog clock form a 120° angle? (there may be more than one answer)

17. On a circle whose center is O, using your protractor of GeoGebra, mark points P and A so that minor arc PA is a 46-degree arc. What does this tell you about angle POA? Extend PO to meet the circle again at T. What is the size of angle PTA? This angle is called an inscribed angle of the circle because its vertex is on the circle. Make a conjecture about arcs and the inscribed angles that *intercept* them.



18. If P and Q are points on a circle, then the center of the circle must lie on the

perpendicular bisector of the chord PQ. Explain. Which point on the chord is closest to the center and why?

19. In your own words, state a theorem about the relationship between a tangent line and a radius drawn to the point of tangency. Think about an *indirect proof* for this statement and what would you assume was not the case?

20. Draw a circle on GeoGebra with a two-inch radius (in other words make it rather large on the Graphics View). Construct four points (not evenly spaced) on the circle and name them consecutively G, E, O and M. Measure angles GEO and GMO. Do you notice anything about these two angles which we call *opposite angles*)? Could you have predicted your result based on what you know about circles and inscribed angles?\

21. A circular park 80 meters in diameter has a straight path cutting across it. It is 24 meters from the center of the park to the closest point on the path. How long is the path?

22. Triangle ABC is inscribed in a circle. Given that AB is a 40-degree arc and angle ABC is a 50 degree angle, find the sizes of the other arcs and angles in the figure.