Motivational Problems on Polynomial Functions

1. Consider the following functions – which of them are polynomial functions? Why or why not?

   a. $f(x) = x^6 - 3x^4 + 2x + 5$

   b. $g(x) = \sqrt{x+1} - 2x^2$

   c. $h(x) = x^{\frac{2}{3}} + 2x^{\frac{1}{3}} + 6$

   d. $k(x) = \left|2x^3\right| + 5$

   e. $j(x) = 5x^{-2} + 3x^{-3}$

2. In standard form, a polynomial functions' leading coefficient is the one with the largest exponent and is usually written first when the terms are written with exponents in descending order. Using your graphing calculator, fill in the following table with the behavior of the graphs of the following functions.

Function	Leading Term Exponent Even or Odd?	Leading Term Coefficient Positive or Negative?	Basic Shape of the graph? (note what happens at ends)
$f(x) = -x^4 - 2x^3 + 3x^2 + 4x - 4$			
$k(x) = 5x^4 - 20x^3 + 80x - 80$			
$g(x) = x^3 - 2x^2 - 9x + 18$			
$h(x) = -3x^5 + 15x^4 - 15x^3 + 3x^2 - 18x - 54$			

3. Graphically and numerically, explain the meaning of the expression: the zero of a polynomial.

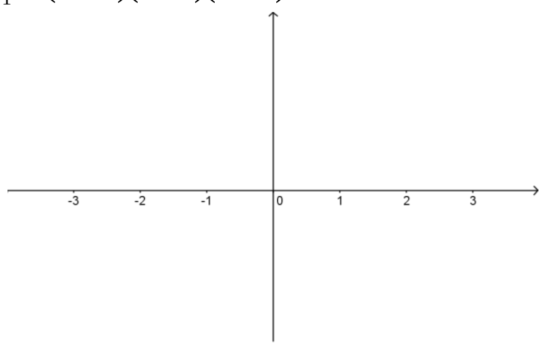
4. Using factoring by grouping, find the zeros of $k(x)$ and $g(x)$ from #2. How might you find the zeros of $f(x)$ and $h(x)$?

5. Generalize how you might determine the behavior or a polynomial when x becomes very large ($x \to \infty$) or when x gets very small ($x \to -\infty$). In other words, what has to be true about the standard form of a polynomial in order for you to determine its shape at the right hand side and the left hand side?

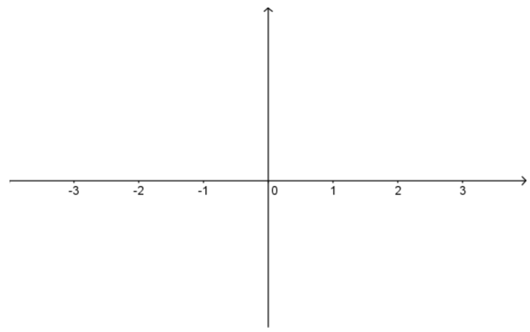
"Pass Through" and "Bounce" Points

Directions: Using your calculator, make a <u>quick sketch</u> of the graph of each of the following polynomial functions. Mark the *x*- and *y*-intercepts with their values. It is not important to have the heights drawn to scale. Make certain to draw smooth continuous curves. This is easiest with the Zoom 6 Window.

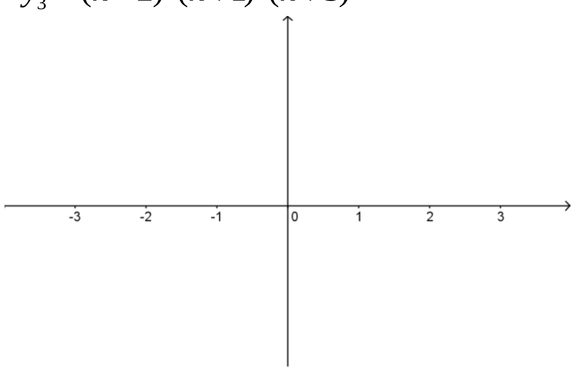
$y_1 = (x-2)(x+1)(x+3)$



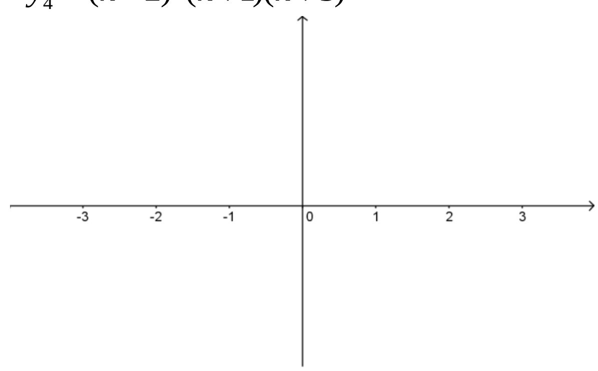
$y_2 = (x-2)^2(x+1)(x+3)$



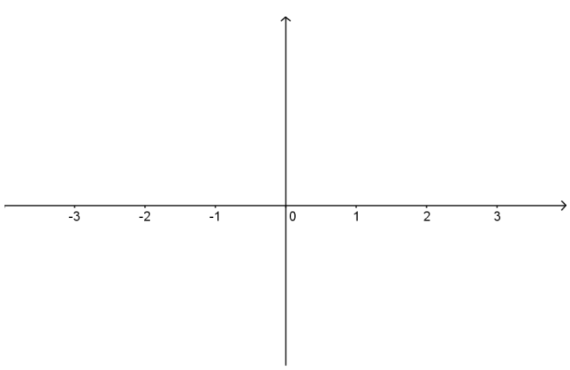
$y_3 = (x-2)^2(x+1)^2(x+3)$



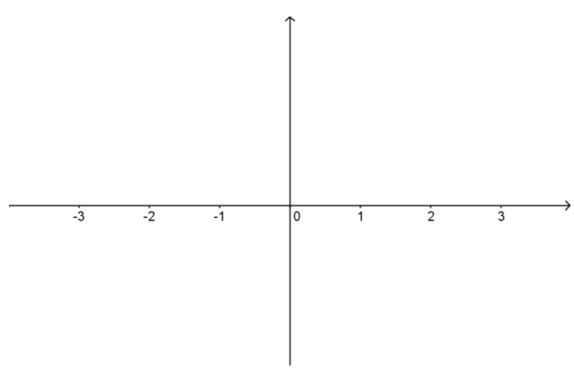
$y_4 = (x-2)^2(x+1)(x+3)^3$



$y_5 = (x-2)^2(x+1)(x+3)^2$



$y_6 = (x-2)(x+1)^4(x+3)$



Answer questions on next page.

I. When does the graph "pass-through" the x-intercept? Is there a visual difference between a first-degree pass-through versus a higher odd degree pass-through?

II. When does the graph "bounce" off the x-intercept?

III. Is the degree and leading coefficient of each polynomial consistent with its far-left and far-right behavior?

1. The ***multiplicity*** of a zero of a function is the exponent of its corresponding <u>linear</u> factor. For example, with $x^2 - 8x + 16 = (x-4)^2$, we see that the zero 4 has a multiplicity of 2 because its factor has an exponent of 2. Find the zeros of the polynomial functions below and state the multiplicity of each zero.

   (a) $B(x) = (x+3)(x-4)^2(x+1)^3$.

   (b) $M(x) = x^2(x-4)^3$

   (c) $T(x) = (2x-5)(x+3)^3$

   (d) $P(x) = (x^2-4)(x+3)^2$

2. (Continuation) Explain how the multiplicity of the factor determines if a graph will "pass-though" or "bounce" at the *x*-intercept. If you use your calculator, just focus on what happens close to the x-axis – it is not important to see the full function.

3. Give an example of a second degree polynomial equation that intersects the *x*-axis twice; one that intersects the *x*-axis exactly once; and one that does not intersect the *x*-axis. Include a sketch of each of your functions.

4. Give an example of a third degree polynomial equation that intersects the *x*-axis three times; one that intersects the *x*-axis twice; one that intersects the *x*-axis exactly once; and one that does not intersects the *x*-axis. Include a sketch of each of your functions.

5. If two polynomials has the same zeros, do the graphs of the polynomial functions look the same? Include a sketch supporting your conclusion.

6. Give an example of a fourth degree polynomial equation that intersects the *x*-axis four times; one that intersects the *x*-axis three times; one that intersects the *x*-axis twice; one that intersects the *x*-axis exactly once; and one that does not intersect the *x*-axis. Include a sketch of each of your functions.

7. Write a fourth degree polynomial with a zero of -1, a zero of 1 with multiplicity two, and a zero of 3 such that $P(0) = -6$.

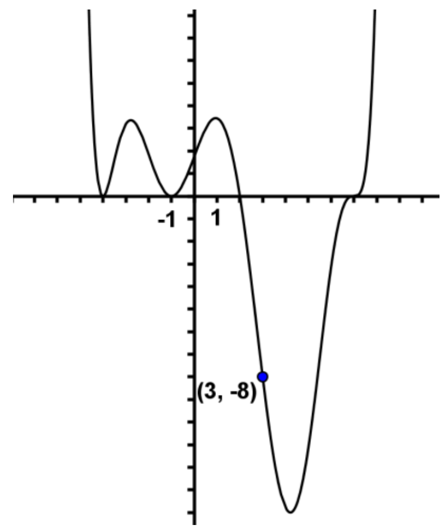
8. Write a fourth degree polynomial with a zero of 3 with multiplicity two, and a zero of -2 with multiplicity two such that $P(1) = 6$.

9. What information would be helpful before graphing a polynomial function?

10. Without your calculator, sketch a graph the following polynomial functions. It is not important to have the heights drawn to scale.

(a) $U(x) = -x^3 - 2x^2 + 3x$ **(b)** $S(x) = x^4 - 6x^3 + 8x^2$ **(c)** $P(x) = x^3 + 4x^2 - 4x - 16$

11. Given the polynomial at the right such that $P(3) = -8$ is on the graph, write an equation that represents this curve.



12. Write an equation of the polynomial with the lowest degree that represents the graph below.