NCTM Orlando 2017
Looking at PBL from a Thematic Perspective
Carmel Schettino, Ph.D.
Three Different Threads

Completing the Square Thread:

Other topics covered simultaneously while this thread is in development: Rate/Distance Problems, Absolute Value from a tabular perspective, basic properties of exponents, irrational/rational multiplication and simplification

Prior knowledge related to this thread: factoring quadratics, solving quadratics in the form of $x^2 = c$ by taking square roots, parabolas are the graphs of quadratics

1. When asked to solve the equation $(x-2)^2 = 11$, Jess said, “That’s easy — just take the square root of both sides.” Alex said to Jess, “Once you take the square root of both sides can you just add 2 to both side like a normal equation? Jess thought for a while and said, “I think so, so you just get x= $2+\sqrt{11}$.” Alex disagreed. Discuss Jess’ method and what might have gone wrong, if anything.

2. How many square roots does the number 28 have? What are they in “exact form”? What does that mean to you?

3. In each part, use the same number in each blank to make a true statement. Compare the number you put in the blanks with the original expression. What do you notice?
   a. $x^2 + 10x + 25 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$
   b. $x^2 + 12x + 36 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$
   c. $x^2 - 14x + 49 = (x - \_\_)(x - \_\_) = (x - \_\_)^2$

4. When asked to solve the equation $x^2 - 6x = 2$, excitedly Dylan said to Jess, “Let’s just take the square root of both sides again.” Jess hesitated with this suggestion. Try it – what might be problematic with Dylan’s suggestion? Why did it work in Jess’ problem?

5. When asked to solve the equation $x^2 - 6x = 2$, Tracy said, “Hmm . . . not so easy, but I think that adding something to both sides of the equation is the thing to do.” This is definitely a good idea, but what number should Tracy add to both sides? How is this equation related to the previous one? What are the similarities and differences?

6. *Definition of a function:* Equations such as $A = 40x - x^2$ and $h = 300 - 16t^2$ define quadratic functions. The word *function* means that assigning a value to one of the variables (x or t) determines a unique value for the other (*A* or *h*). It is customary to say that “*A* is a function of *x*.” In this example, however, it would be incorrect to say that “*x* is a function of *A*.” Explain.

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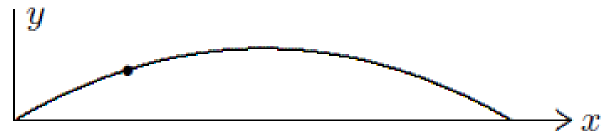
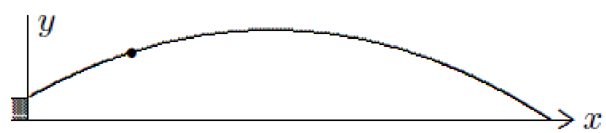
7. Playing golf, on the 7th tee, Taylor hits an excellent shot, right down the middle of the flat grass, called a fairway. The golf ball follows the parabolic path shown in the figure, described by the quadratic function, $y = 0.5x + 0.002x^2$. This relates the height, y, of the ball above the ground to the ball's distance, x, down the fairway. Distances are measured in yards.



   a. Use the distributive property to write this equation in factored form. Notice that y = 0 when x = 0. What is the significance of this data?
   b. How far from the golf tee (the starting point), does the ball hit the ground?
   c. At what distance x does the ball reach the highest point of its arc? What is the maximal height attained by the ball?

8. At the 8th tee, which is on a plateau 10 yards above the level fairway, Taylor hits another great shot. Explain why the quadratic function $y = 10 + 0.5x + 0.002x^2$ describes this parabolic trajectory, shown in the figure above.



   a. Why should you expect this tee shot to go more than 250 yards?
   b. Estimate the length of this shot, then using a graphing tool, find a more accurate value.
   c. How does this trajectory relate to the trajectory for the drive on the previous hole?

9. Find a number that you can add to the left side of this expression to make the quadratic expression a perfect square trinomial. Remember to keep the equation balanced and then attempt to solve with methods you know already.
   a. $x^2 + 10x + 24 = 0$
   b. $x^2 + 12x + 20 = 0$
   c. $x^2 - 14x + 24 = 0$

10. Previously, you calculated the length of Taylor's golf shot on the 8th tee using a graphing tool. To find the length of the shot by hand, you must set y equal to 0 and solve for x. Explain why, and show how to arrive at $x^2 - 250x = 5000$.
   a. The next step in the solution process is to add 125^2 to both sides of this equation. Why was this number chosen?
   b. Complete the solution by showing that the length of the shot is $125 + \sqrt{20625}$. How does this number, which is about 268.6, compare with your previous calculation?
   c. Comment on the presence of the number 125 in the answer. What is its significance?

11. The work at right shows the step-by-step process used by a student to solve $x^2+6x-5=0$ by the method of completing the square. Explain why the steps in this process are reversible. Apply this understanding to find specific values for a, b, and c to form a quadratic equation in the form $ax^2+bx+c=0$ whose solutions are $x=7+\sqrt{6}$ and $x=7-\sqrt{6}$.

$$x^2+6x-5=0$$
$$x^2+6x+9=5+9$$
$$(x+3)^2=14$$
$$x+3=\pm\sqrt{14}$$
$$x=-3\pm\sqrt{14}$$

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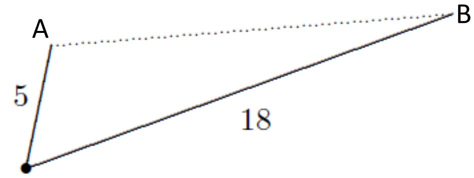
<u>Pythagorean Distance Development Thread:</u>

Other topics covered simultaneously while this thread is in development: new geometric definitions, equidistance, midpoint formula, Geogebra skills,

Prior knowledge related to this thread: Basic Pythagorean Theorem with right triangles, simplifying radicals, solving basic quadratics, absolute value

1. The diagram shows two steel rods hinged at one end. The other end is connected by a bungee cord (the dotted segment), whose unstretched length is 10 inches. The rods are 5 inches and 18 inches long. Use inequality symbols to describe all the possible lengths for the bungee cord, which stays being stretched.



2. At its longest stretch, how far apart are points A and B (the endpoints of the bungee chord)? At its shortest stretch, what is the distance between A and B? Justify your answer with a drawing or discussion.

3. On a number line, what is the distance between 6 and 6? between 18 and 5? between -5 and 18? between 18 and -5? Describe your method of answering this question of distance on a number line.

4. On a number line what is the distance from 28 to -4? From 4 to -4? From -3 to 7? From any number x to 4? Why is this last question harder to answer? Is the answer x - 4 or 4 - x ?

5. Finding the distance on a number is an absolute value calculation. Why do you think this is true? Use absolute value signs to express the distance between t and 4. What is the distance between the numbers a and b on the number line? What is the relationship between $|p - q|$ and $|q - p|$?

6. When asked to find the distance between two numbers on a number line six different times, Jamie responded with the following answers. What two numbers do you think Jamie was talking about?

(a) $|9 - 4|$ (b) $|9 + 4|$ (c) $|x - 7|$ (d) $|3 - x|$ (e) $|x + 5|$ (f) $|x|$

7. Rearrange the eight words “between”, “4”, “the”, “17”, “is”, “and”, “x”, and “distance” to form a sentence that is equivalent to the equation By working with a number line, find the values of x that fit the equation.

8. Draw a line through the origin with a slope of 0.4. Draw a line through the point (1, 2) with a slope of 0.4. How are these two lines related? What is the vertical distance between the two lines? Find an equation for each line.

9. For each of the following points, find the distance to the y-axis:

(a) (11, 7) (b) (−5, 9) (c) (4, y) (d) (x, −8)

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10. Plot a point near the upper right corner of a sheet of graph paper. Move your pencil 15 graph-paper units (squares) to the left and 20 units down, then plot another point. Use your ruler to measure the distance between the points. Because the squares on your graph paper are probably larger or smaller than the squares on your classmates' graph paper, it would not be meaningful to compare ruler measurements with anyone else in class. You should therefore finish by converting your measurement to graph-paper units.

11. (Continuation) Square your answer (in graph-paper units), and compare the result with the calculation 152 + 202.

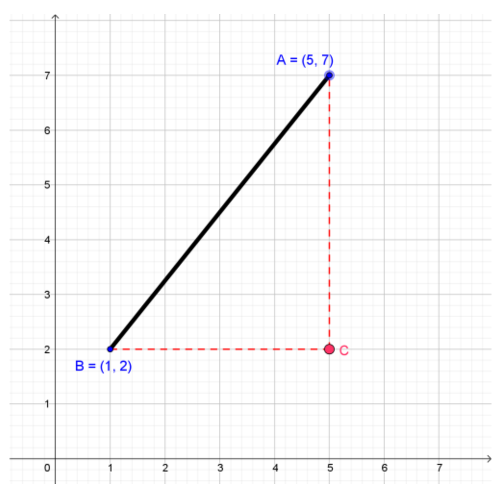
12. Repeat the entire process, starting with a point near the upper left corner, and use the instructions "20 squares to the right and 21 squares down." You should find that the numbers in this problem again fit the equation a2 +b2 = c2. These are instances of the *Pythagorean Theorem*, which is a statement about right-angled triangles. Write a clear statement of this useful result. You will need to refer to the longest side of a right triangle, which is called the *hypotenuse.*

13. If the hypotenuse of a right triangle is 12 and one of the legs is 4, what is the length of the other leg? What is the simplest form in which you can express your answer?

14. Two different points on the line y = 2 are both exactly 13 units from the same point (7, 14). Draw a picture of this situation, and then find the coordinates of these points.

15. Suppose two towns are located on a coordinate grid at the points A (5,6) and B (1,2). For construction purposes, the townspeople need to know how far away the towns are from each other. Find a way, with your graph paper, to calculate a measure of the distance from A to B. Try to be as precise as possible.

16. (Continuation) After considering the problem of the distance between town A and town B, Alex exclaims to the class, "we could just use the Pythagorean Theorem" and draws a picture like this:



What was Alex's method of finding the distance from A to B?

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17. The general notation in geometry is that points are labeled with capital letters and coordinates are defined with lowercase letters. Given the two points and (x_1, y_1) and (x_2, y_2) , what do the subscripts on *x* and *y* represent? If triangle ABC is right triangle with C being the right angle

a. Find possible coordinates for point C in terms of $x_1, x_2, y_1 \text{ and } y_2$.

b. How could you express the length of the side BC and AC in terms of $x_1, x_2, y_1 \text{ and } y_2$?

c. How could you represent the distance AB in terms of $x_1, x_2, y_1 \text{ and } y_2$?

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Right Triangle Trigonometry: Introducing the tangent function

Other topics covered simultaneously while this thread is in development: similarity, dilation, trapezoids, Three-Parallels Thm, Midpoint Connector Thm

Prior knowledge related to this thread: special right triangles, perpendicular bisectors, solving quadratics, Pythagorean Theorem

1. The sides of an equilateral triangle are 12 cm long. How long is an altitude of this triangle? The altitude divides the triangle into two right triangles. What are the measures of the angles in these right triangles? How does the short side of this right triangle compare with the other two sides? Please leave your lengths in simplest radical form.

2. A right triangle has a 24-cm hypotenuse which is twice as long as its shorter leg. In simplest radical form, find the lengths of all three sides of this triangle.

3. A line of positive slope is drawn so that it makes a 60-degree angle where it intersects the *x*-axis. What is the slope of this line?

4. Draw the lines $y = 0$, $y = \frac{1}{2}x$, y=x and $y = 3x$. Use GeoGebra (or a protractor if you have one) to measure the angle that the line y=$\frac{1}{2}x$ makes with the x-axis. Using your intuition, make a guess what the angle is that the line y=3x makes with the x-axis. Now measure it. Do this for other lines through the origin. Compare the angle made with the slope of the line. Describe any findings. Can you draw any conclusions about the relationship between the slope of a line and the angle it makes with the x-axis?

5. The altitudes of an equilateral triangle all have length 12 cm. How long are its sides?

6. Mark *A* = (0*,* 0) and *B* = (10*,* 0) on your graph paper or in GeoGebra and use your protractor to draw the line of positive slope through *A* that makes a 25-degree angle with *AB*. Calculate (approximately) the slope of this line by making suitable measurements.

7. (Continuation) Turn on your calculator, press the MODE button, and select the *Degree* option for angles. Return to the home screen, and press the TAN button to enter the expression TAN(25), then press ENTER. You should see that the display agrees with your answer to the preceding item.

8. How does the value of the tangent of an angle change as an angle increases from 0 to 90 degrees? Is there a direct relationship between the slope and the angle measure?

9. Standing 50 meters from the base of a fir tree, Rory measured an *angle of elevation* of 33° to the top of the tree with a protractor. The angle of elevation is the angle formed by the horizontal ground and an ant's line-of-sight ray to the top of the tree. How tall was the tree?

10. Find the tangent of a set of stairs in your house or your dorm. How could we use that tangent to tell whose stairs are the steepest in our class?

11. Alex and Skyler are in a park playing angry birds on their iPads one day and spy a bird in a tree that they know is 25 feet away from them. Alex decides to measure the angle of elevation to the top of the tree and sees that it is 60°. Skyler says, "I wonder how tall that tree is?" Alex replies, "Well it can't be more than 50 feet tall, that's for sure." How does Alex know this? (without using a calculator!)

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